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ABSTRACT

This guide for grade 12 mathematics has been developed as part of an overall revision of the mathematics program for Manitoba, Canada schools and is designed to be used in coordination with existing guides for grades K-12. It is the framework upon which grade 12 teachers may build their mathematics programs. This guide consists of four main sections including a general overview of the program, the Mathematics 304 course outline, the Mathematics 301 course outline, and the Mathematics 300 course outline. The outlines are based on the aims of mathematics which are to foster maximum and continuous growth in three behavioral areas: (1) the cognitive or factual; (2) the affective which includes feelings and perceptions; and (3) the psychomotor or movements. The mathematical content of this program has been chosen to promote the above goals. (KR)

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This guide consists of four main sections including a general overview of the program, the Mathematics 304 course outline, the Mathematics 301 course outline and the Mathematics 300 course outline.

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overview

INTRODUCTION

The Mathematics Working Party (K-12) MWP(K-12) was established in conjunction with the Articulation Council and the Elementary, Middle Years and Senior High Program Review Committees in March 1977 by the Department of Education. Its terms of reference were defined this way: "to review the goals, content, methodology, and resources of all currently approved and developing provincial programs in mathematics and to establish the necessary committees/curriculum structures to complete the revision of the K-12 programs in mathematics within an articulated K-12 framework."[1]

Meetings of senior high mathematics teachers were held in Winnipeg, Thompson, The Pas, Dauphin, Brandon, Morris, and Teulon in the spring of 1978 to obtain a sample of opinion on the following topics: the structure of high school mathematics programs, of geometry, consumer mathematics, business-vocational mathematics, and the mathematical needs of non-college-bound students. Responses to questionnaires distributed to teachers, superintendents, and school trustees indicated agreement of the following points:

- Some form of streaming is necessary in high school mathematics to provide for the varying levels of ability, needs and interest in students. Response to this issue from the Manitoba Association of School Superintendents

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[1] "'Terms of Reference,' K-12 Working Parties in Mathematics and Science" Discussion Paper prepared for the December 11, 1976 meeting of the Senior High Program Review Committee.

(MASS) states that "subject matter appropriate for different students will vary. It is not just the speed at which material can be assimilated by different pupils, but the actual subject matter itself which is required for differing needs."

- The 01 program requires immediate modification. Instead of paralleling the existing 00 programs, it should meet the needs of students not intending to pursue post-secondary study in mathematics by closer application to practical problems that arise in everyday life.

- The 00 program should remain substantially as it is. The amount and rigour of formal geometry should be reduced, although geometry is still to be retained as an important part of a revised course.

- Consumer mathematics should be studied by all students.

STRUCTURE FOR SENIOR HIGH SCHOOL MATHEMATICS

The Proposal for High School Mathematics Revision, prepared by the Mathematics Working Party (K-12) and presented to the Curriculum Policy Review Council in January 1979, identified three levels of senior high mathematics (04, 01, and 00), each containing consumer mathematics. This proposal, which reflected data obtained from the previously mentioned survey, from interface meetings with post-secondary education representatives, and from the MWP (K-12) membership, was approved and forms the basis of the current 10-12 Mathematics revision. The following chart outlines the course offerings for senior high school mathematics.

	GRADE 10	GRADE 11	GRADE 12
COURSE "04"	Mathematics 104	Mathematics 204	Mathematics 304
COURSE "01"	Mathematics 101	Mathematics 201	Mathematics 301
COURSE "00"	Mathematics 100	Mathematics 200	Mathematics 300
			Ad. Math. 305
			Statistics and Prob. 305
			Intro. to Calculus 305

DESCRIPTION OF THE SENIOR HIGH COURSES

04 Mathematics - full credit courses (60% core, 40% options in each of grades 10, 11 and 12) designed to concentrate on the application of mathematical skills and concepts to everyday situations. Skills covered include information processing, estimation and the use of calculators, computation, and problem solving in areas encountered by the student at home, at work, and at leisure. These courses are intended as an option for any student with or without work education. However, students intending to pursue post-secondary education should be advised against taking them.

01 Mathematics - full credit courses (50% core, 50% options in each of grades 10 and 11, and 75% core, 25% options in grade 12)designed to include practical applications to current problems from the business, consumer, trade, and vocational areas. The current 01 program amalgamates previous 01

(general), 02 (business), and 03 (vocational) mathematics courses. It is truly a general mathematics program, incorporating the best from several branches of mathematics, and leading to a range of post-secondary education programs, vocational training and employment, as well as to more general life skills. Since it is a general program, not algebra-based, competency in algebra is not essential for success. 01 Mathematics is designed to expand future student options by introducing a variety of mathematics topics, to benefit smaller schools by making business and vocational skills part of the 01 Mathematics package, and to allow larger schools the option of identifying separate classes of 01 Mathematics with options dedicated to business, vocational or general mathematics.

00 Mathematics - full credit courses (85% core, 15% options in each of grades 10 and 11, and 80% core, 20% options in grade 12) designed for students wishing to pursue further studies in either mathematics or science-related programs at post-secondary institutions. Both Mathematics 100 and 200 contain 50% algebra, 40% geometry, and 10% consumer math. Mathematics 300 contains 80% core and 20% option topics selected from algebra, analytic geometry and trigonometry. To make these courses more meaningful and relevant, a greater emphasis has been placed on problem solving and on practical applications of content under consideration.

USE OF MATHEMATICS

It is important to study mathematics because of its many applications to everyday life and to specialized scientific work. A knowledge of mathematics is necessary if students are to have an awareness and an understanding of their environment and to enjoy a fuller life.

With this in mind, the Mathematics Working Party (K-12) suggests that mathematics be considered:

1. A mode of communication, with a language all its own. For students to communicate successfully in mathematics, they must learn the basic concepts without having to meet demands for logical rigour. The language acquired should be used to describe relationships observed in the real world. At a later stage, symbols should be introduced to record these relationships.

2. A tool. After a mathematical system has been introduced as a language, it should be used as a tool to answer practical questions, thus demonstrating to students that to function in society they need basic skills in arithmetic, algebra, geometry and statistics. Students should be capable of identifying a problem and employing an organized process leading to its solution. This involves applying fundamental operations and information processing skills--such as collecting, organizing and interpreting data--to familiar and unfamiliar contexts. Realistic problems relating to the students' environment should be sought to foster consumer awareness, and vocational and business applications should be stressed at every opportunity.

3. A source of esthetic enjoyment. Mathematics can be considered an esthetic discipline like music. Many students can develop an appreciation of mathematics through the discovery of patterns in numbers and in created or naturally occurring designs. Students should be encouraged to reproduce these as well as to create

their own. Methodologies and techniques conducive to developing positive inquiring attitudes toward mathematics should be employed.

Students who are most successful in developing skills and understanding concepts in mathematics have learned to recognize and appreciate the three ways in which the subject can be used. Classroom emphasis on these uses cannot always be equal, however, but must vary depending on the nature of the topic and the interests and abilities of the students.

GENERAL GOAL OF MATHEMATICS

The aim of mathematics is to foster maximum and continuous growth in the following behavioral areas: the cognitive (factual), the affective (feelings, perceptions), and the psychomotor (movements). This growth will be accomplished when mathematical content is used to develop processes such as the following:

1. The process of communicating: using mathematical language and symbolism
2. The process of recalling mathematical skills
3. The process of comprehending mathematical principles
4. The process of seeing relationships in mathematics
5. The process of generalizing: finding common relationships among mathematical concepts
6. The process of making applications: applying generalizations to new situations
7. The process of learning self-direction: searching for mathematical ideas
8. The process of discovering: recognizing mathematical ideas

9. The process of creating: generating mathematical ideas
10. The process of appreciating the place of mathematics: understanding its widespread applications to our daily activities
11. The process of problem solving: identifying, hypothesizing, interpreting, relating, evaluating, concluding

OBJECTIVES FOR GRADE 11 MATHEMATICS

It is important that grade 12 mathematics teachers become familiar with the mathematics programs in grades 10 and 11, since the programs outlined in this guide are intended to build on the skills possessed by students graduating from grade 11. There are three full credit course levels in grade 10 and grade 11 mathematics: 104, 101 and 100; and 204, 201, and 200. Following are the objectives specific to the grade 11 courses.

MATHEMATICS 204 60% Core (C) - 40% Option (Op.)

C.I. Accommodation: Finding a Rental Unit (½ module)

The student should be able:

1. To identify potential monthly living expenses.
2. To demonstrate an understanding of standard terms and abbreviations used in classified ads for rental accommodation.
3. To read and to interpret a local or city street map in order to locate available housing.

C.II. Accommodation: Rental Costs (1 module)

The student should be able:

1. To select an accommodation to suit a given person's lifestyle.
2. To determine the maximum amount of one's salary that should be spent on rent each month.
3. To read and to complete a Tenancy Agreement.
4. To read electrical meters and to compute the number of kilowatt-hours given readings for consecutive months.
5. To calculate the number of watt-hours of energy that are used by different appliances given the rating of each appliance and its length of operation.
6. To convert the electrical units of watts to kilowatts.
7. To determine the cost of telephone service.
8. To discuss the insurance coverage for personal property.
9. To determine the cost of basic furnishings for an apartment.

C.III. Food (½ module)

The student should be able:

1. To understand metric units used in purchasing foods.
2. To find unit prices when buying:
    a) by single items,
    b) by weight,
    c) by volume.
3. To determine the cost of multiple items.
4. To determine the "better buy."
5. To calculate the cost of a grocery list.
6. To calculate the saving when given the regular price and the sale price.

7. To find the cost of items when coupons are used.
8. To identify items in a given grocery list which are taxable.

C.IV. Graphing (½ module)

The student should be able:

1. To read and to interpret information from tables as well as bar graphs and line graphs.
2. To read and to interpret information from pictographs.
3. To read and to interpret information from circle graphs.
4. To gather data and to arrange it for graphing.
5. To construct a bar graph.
6. To construct a line graph.

C.V. Measurement and Geometry (1½ modules)

A. The Circle

The student should be able:

1. To demonstrate an understanding of the terminology associated with a circle.
2. To demonstrate an understanding of the relationship between radius and diameter.
3. To demonstrate an understanding of the relationship between circumference and diameter.
4. To solve problems involving circumference given the formula.
5. To demonstrate an understanding of the concept of area by counting the number of unit squares.
6. To calculate the area of a circle using the formula $A = \pi r^2$.

7. To solve problems involving the area of circles.

B. Angles

The student should be able:

1. To identify the arms and the vertex of an angle.
2. To measure angles from 0° to 360°.
3. To estimate the degrees in a given angle and to check the measure with a protractor.
4. To sketch an angle of a given number of degrees without a protractor and then to check it with a protractor.
5. To construct angles less than or equal to 360° using a protractor.

C.VI. Mental Arithmetic (½ module)

The student should be able:

1. To perform mental calculations using whole numbers in addition, subtraction, multiplication and division.
2. To multiply and to divide by powers of ten (limit to 1, 10, 100, 1000).
3. To perform mental calculations using decimal fractions in addition, subtraction, multiplication and division.
4. To round off and to estimate cost calculations.
5. To complete a given sequence.
6. To respond to oral arithmetic questions.
7. To perform calculations involving order of operations.
8. To calculate the amount of change for a sales transaction.

C.VII. Money Management: Banking (1 module)

A. Reconciling a Chequing Account

The student should be able:

1. To identify the ending balance on a bank statement.
2. To identify and to list any "outstanding cheques."
3. To identify and to list any "outstanding deposits."
4. To identify and to record the "service charge."
5. To reconcile a specific chequing account given the procedure.

B. Finding Simple Interest

The student should be able:

1. To define the following terms: interest, principal, rate and time.
2. To calculate interest given the simple interest formula.

C. Finding Compound Interest

The student should be able:

1. To define "compound interest" and "compound amount."
2. To calculate the compound interest and the compound amount.

C.VIII. Retail Sales (½ module)

The student should be able:

1. To complete sample sales slips using a catalogue or price list.
2. To determine sales tax.
3. To calculate the total cost of one item.
4. To calculate the total cost of a series of items.
5. To make change given the cost of the item and the amount offered in payment.

Op.I. Accommodation: Buying a Home (1 module)

1. To calculate the maximum to be spent on a home.
2. To calculate the maximum P.I.T. for a given salary.
3. To calculate the down payment given the purchase price of a home.
4. To calculate the total mortgage required after the minimum down payment.
5. To determine the monthly payments on a mortgage loan.
6. To calculate the total cost of a mortgage over a given period of time given the monthly payment.

Op.II. Food (Nutrition) (½ module)

The student should be able:

1. To identify the recommended daily caloric needs for males and females at different ages.
2. To identify the number of calories used by the body during certain activities.

3. To identify the quantity of vitamins, calories and proteins in different kinds of foods.
4. To establish a menu given caloric restrictions.

Op.III. Measurement/Geometry: Orienteering (1 module)

The student should be able:

1. To demonstrate a knowledge of the parts of a compass.
2. To demonstrate an understanding of the magnetic compass.
3. To find the bearing of a visible object.
4. To follow a bearing.
5. To measure distances using the number of paces (double steps) required for 100 metres.
6. To find a station, given a starting point, a bearing and a distance.
7. To follow an orienteering trail from one station to the next given the bearing and a distance between each station.

Op.IV. Money Management: Consumer Credit and Savings (1 module)

A. Credit Cards

The student should be able:

1. To complete a credit card application form.
2. To read and to interpret a statement identifying the previous balance, the payments, the interest, the new charges, the new balance, and the minimum payment.

B. Personal Loans

The student should be able:

1. To complete a sample loan application.
2. To determine assets, liabilities, and net worth for a given problem.
3. To determine the collateral or security offered for a given loan.
4. To calculate the cost of borrowing a specified amount given the monthly payment and the number of payments.

C. Savings Plans

The student should be able to identify some common savings plans.

Op.V. Money Management: Stock Market (½ module)

The student should be able:

1. To identify various investment possibilities.
2. To discuss advantages and disadvantages of each investment possibility.
3. To read and interpret stock market quotations.
4. To calculate the cost of stock purchases.
5. To calculate expected dividends on preferred stock.
6. To calculate dividends on common stock.

Op.VI. Personal Income Tax (1 module)

The student should be able:

1. To describe the method by which income tax is paid by an employee.

2. To complete a sample TD1 form.
3. To complete the identification section of the T1 General tax return.
4. To complete the Calculation of Total Income section of the T1 General form.
5. To complete the Calculation of Taxable Income section of the T1 General form.
6. To complete the Summary of Tax and Credits section and to determine amount of refund or balance due.

Op.VII. Recreational Mathematics (½ module)

The student should be involved in:

1. Solving mathematical problems.
2. Playing mathematical games.
3. Participating in geometric activities.

Op.VIII. Statistics: Averages (½ module)

The student should be able:

1. To identify three types of "averages."
2. To calculate the mode(s) for a set of data.
3. To calculate the median for a set of data.
4. To calculate the arithmetic mean for a set of data.
5. To identify the most suitable average in a given situation.

MATHEMATICS 201 50% Core (C) - 50% Option (Op.)

C.I. Consumer Mathematics (1½ modules)

A. Income

The student should be able:

1. To calculate gross wages.
2. To calculate commissions earned:
   a) given the amount of sales and the consumption rate,
   b) given the categories to which various rates apply and the commission rate.
3. To calculate the gross salary earned given the regular salary, amount of sales and commission rate on sales.
4. To calculate the claim code and to fill in a TD1 form.
5. To use CPP, UIC and income tax tables in the calculation of net pay:
   a) given salary,
   b) given rate of pay, hours worked and overtime rates,
   c) given a or b and other deductions; i.e., union dues, dental plan, bonds, charitable donations, etc.

B. Income Tax

The student should be able:

1. To identify the four main sections of an income tax return given a blank copy of a return.
2. To fill out the appropriate information in all four sections of a T1 General form given a blank form, a completed T4 slip and a completed T5 slip.
3. To calculate the refund or the balance due for an individual given a blank T1 General Income Tax Form and completed T4 and T5 information slips.

C. Savings

The student should be able:

1. To calculate the amount of simple interest given the formula I = PRT.
2. To calculate principal, rate or time, given the simple interest and two other variables from the formula I = PRT.
3. To calculate the compound amount when interest is compounded:
   a) annually,
   b) semi-annually.
4. To use compound interest tables to calculate the amount when interest is compounded:
   a) annually,
   b) semi-annually.

D. Consumer Credit

1. Installment Buying

   The student should be able:

   a) To determine the cost of credit (carrying/finance charges) for installment purchases, given the cash price, down payment, number of payments and the amount of each payment.
   b) To calculate the rate by which the installment price exceeds the cash price.

2. Credit Cards

   a) To calculate the annual interest rate on credit card purchases given the monthly rate of interest.
   b) To find the interest charge given the previous balance and the schedule of interest rates on the unpaid balance.
   c) To compute the minimum monthly payment given the previous balance, the interest rate and the schedule of minimum monthly payments.
   d) To find the minimum monthly payment given the previous balance, the interest rate, the purchases made during the month and the schedule of minimum monthly payments.
   e) To calculate the minimum monthly payment given the balance on the previous statement, the payment made, the interest rate, the purchases made during the month and the schedule of minimum monthly payments.

C.II. Geometry (1½ modules)

A. Circles

The student should be able:

1. To demonstrate an understanding of the definitions and terminology associated with circles.

2. To understand and to use the relationship between the circumference and the diameter of a circle.
3. To understand and to use the formula for the area of a circle.
4. To understand and to use the formula for the surface area of a cylinder.
5. To understand and to use the formula for the volume of a cylinder.

B. Pythagorean Theorem

The student should be able:

1. To state the Pythagorean Theorem and to recognize situations in which it can be used.
2. To use the Pythagorean Theorem to find one side of a triangle given the other two sides.

C. Scale Drawings

1. Interpretation

The student should be able to interpret scale drawings.

2. Analysis

The student should be able to analyze a floor plan.

3. Construction

The student should be able to construct a drawing to scale given actual dimensions.

4. Enlarging Patterns

The student should be able to enlarge or reduce a pattern using a grid system.

C.III. Algebra (2 modules)

A. Algebraic Operations

The student should be able:

1. To multiply a monomial by a monomial with integral coefficients, common bases and natural number exponents.
2. To multiply a polynomial of two or more terms by a monomial.
3. To multiply a binomial by a binomial.
4. To divide a monomial by a monomial with integral coefficients, common bases and natural number exponents, where the exponent in the numerator is greater than or equal to the exponent in the demonimator.
5. To divide a polynomial of two or more terms by a monomial, where the exponent in the numerator is greater than or equal to the exponent in the denominator.

B. Evaluation of Formulae

The student should be able:

1. To evaluate formulae:
   a) by substituting numerical values for letters in a given formula,
   b) by substituting numbers for letters where letters are identified by words,
   c) by translating from a word statement to a formula.

C. Coordinate Geometry

The student should be able:

1. To demonstrate an understanding of the terminology associated with coordinate graphing.
2. To plot a point given its coordinates and to write the coordinates for a point given its location on a graph.
3. To determine, using experimental data, graphs of direct relationships involving a straight line passing through the origin.
4. To define slope.
5. To determine, from experimental data, the graph of direct relationships involving a straight line not passing through the origin.
6. To define vertical or y-intercept.
7. To sketch the graph of a straight line given its equation in the form $y = mx + b$ where $m > 0$.

D. Equations Involving Proportions

The student should be able:

1. To recognize and to write ratios.
2. To reduce ratios to equivalent forms.
3. To solve a given proportional equation.
4. To set up proportional equations from word problems of either a direct or inverse nature and, given sufficient information, to solve for an unknown quantity.

OP.I. Operation and Application of the Ten-Key Printing Electronic Calculator (1 module)

The student should be able:

1. To organize and to maintain an efficient work area.
2. To use correct posture at the calculator.
3. To care for and to maintain the calculator.
4. To identify and to explain the purpose of the operating parts of the calculator.
5. To identify the number keyboard.
6. To use correct key-stroking techniques.
7. To perform the four basic mathematical operations.
8. To apply the skills of adding, subtracting, multiplying and dividing to business problems.

Op.II. Financial Operations (1 module)

A. Promissory Notes and Bank Discount

The student should be able:

1. To calculate the value of an interest-bearing note at maturity.
2. To calculate the proceeds of an interest-bearing note discounted at the bank.
3. To calculate the proceeds of a non-interest-bearing note discounted at the bank.

B. Bad Debts

The student should be able:

1. To calculate a bad debt loss based on net credit sales.
2. To calculate a bad debt loss based on aging of Accounts Receivable.

C. Bankruptcy

The student should be able:

1. To calculate the amount of money available to creditors.
2. To calculate the percent rate of creditors' claims that a trustee can pay.
3. To calculate how many cents on the dollar creditors receive on their claims.
4. To calculate the amount that a creditor would receive from a bankrupt concern.
5. To calculate the original amount of an individual's claim given the percent rate paid to creditors and the amount paid to the individual creditor.

Op.III. Income Statements (1 module)

A. Completing an Income Statement

The student should be able:

1. To explain the following terminology:
    a) sales returns and allowance,
    b) net sales,
    c) cost of goods,
    d) cost of goods sold,
    e) gross profit on sales,
    f) operating expenses,
    g) net income.

2. To calculate net sales given gross sales and sales returns and allowances.
3. To calculate the cost of goods sold given the beginning inventory, the purchases during the period and the ending inventory.
4. To calculate gross profit/loss given net sales and cost of goods sold.
5. To calculate net income/net loss.

B. Analysing an Income Statement

The student should be able:

1. To calculate the percent rate of the accounts on an income statement based on net sales.
2. To prepare a comparative income statement for two accounting periods using percent rate of net sales.
3. To prepare a comparative income statement showing:
   a) increases or decreases between periods,
   b) the percent rate of increase or decrease between items on the income statement.

OP.IV. Division of Business Profits/Losses (1 module)

A. Division of Partnership Profits/Losses

The student should be able:

1. To calculate the share of net income or loss when the net income or loss is divided equally.
2. To calculate the share of net income or loss when the net income or loss is shared in a fixed ratio.

3. To calculate the share of net income or loss when the net income or loss is shared by a a fixed percent rate.
4. To calculate the share of net income or loss when the net income or loss is shared in proportion to the partner's investment.
5. To calculate the share of net income or loss when the partners receive interest on their investments and then divide the remaining net income or loss equally.
6. To calculate the share of net income or loss when the partners receive a salary and then divide the remaining net income or loss equally.
7. To calculate the share of net income or loss when the partners receive a salary and interest on their beginning capital and then divide the remaining net income or loss equally.

B. Division of Corporation Profits/ Losses

The student should be able:

1. To calculate the percent rate of dividend if the entire net income is distributed to shareholders.
2. To calculate the dividend in dollars and cents per share if the entire net income is distributed to shareholders.
3. To calculate the total dividend payable to shareholders if the directors retain a portion of the net income.
4. To calculate the percent rate of dividend payable to shareholders if the directors retain a portion of net income.

5. To calculate the dividend in dollars and cents per share if the directors retain a portion of net income.

C. Division of Cooperative Profits/Losses

The student should be able:

1. To calculate the patronage dividend.
2. To calculate the dividend on capital stock.
3. To calculate the total dividend.

OP.V. Calculations in the Vocational Areas (1 module)

A. Numerical Calculations

1. The student should be able to solve problems pertaining to trade or vocational areas involving:
   a) whole numbers and decimal fractions,
   b) percent,
   c) capacity and mass,
   d) direct proportion,
   e) combinations of whole numbers, decimals, percents, capacity, mass and direct proportion problems.

B. Geometric Calculations

1. The student should be able to solve problems involving:
   a) perimeter/circumference,
   b) area/surface area, volume of rectangular, triangular and cylindrical solids,
   d) Pythagorean theorem,
   e) combinations of perimeter/circumference, area, volume and Pythagorean theorem.

OP.VI. Mathematics of Specific Vocational Trades (1 module)

A. Carpentry

The student should be able:

1. To determine the number of sheets of playwood to cover a given surface area.
2. To calculate the amount of lumber to be purchased when a waste factor is included.
3. To calculate the amount of concrete to pour the foundation of a building given appropriate specifications.
4. To calculate the number of bundles of asphalt shingles for a roof given the dimensions.
5. To calculate the length of a common rafter given the pitch of the roof and the overhang allowance.
6. To determine the stringer length for a stairway with a given rise and run.
7. To determine the cost of lumber given the dimensions and price per m^3.

B. Masonry

The student should be able:

1. To state the number of standard bricks required to cover 1 m^2 of surface with 100 mm veneer.
2. To calculate the number of standard bricks required to build a veneer on four rectangular walls.
3. To calculate the waste factor allowed as a percent rate when ordering extra bricks.

4. To determine the width of the rough opening to be allowed for a door if the width is to be equivalent to a given number of modular nominal bricks.
5. To determine the width of a masonry (veneer) panel between two windows when the width is equivalent to a given number of bricks.
6. To calculate the number of standard concrete blocks required to build a garden wall of concrete blocks.
7. To calculate the number of standard bricks, piano blocks and standard concrete blocks required to build a garden wall of concrete blocks with an American bond veneer.
8. To determine the number of standard bricks required to build a chimney, given the height of the chimney and the size of the flue lining.
9. To determine the ratio of face brick to backup brick in a brick wall given the bond pattern.
10. To determine the amount of water to be added to a given muriatic acid solution to reduce the concentration to the desired level.

C. Mechanical Trades

The student should be able:

1. To differentiate between linear, surface and volume measurements.
2. To calculate the volume or capacity of a cylinder (pipe, hose) given π and the SI dimensions.
3. To explain the graduation on a micrometer.

4. To calculate the mechanical advantage for each of the three classes of levers.
5. To calculate the mechanical advantage of a simple machine.
6. To calculate the rate of rotation of any gear in a train.
7. To compute allowance of materials.
8. To compute length and width of stretchouts.

B. Plumbers and Pipefitters

The student should be able:

1. To calculate pipe lengths given certain information.
2. To express the gradient of a pipe as a percent rate given the rise and the run.
3. To determine how many pipes of smaller radius can be fed by a larger supply pipe given their respective radii.
4. To determine the change in the rate of flow in a pipe when the pipe diameter changes.

Op.VII. Electrical Trades (1 module)

The student should be able:

1. To explain the meaning of each of the terms: "voltage," "current" and "resistance," and to write the letter representing each.
2. To write the word and symbol of the unit by which each of voltage, current and resistance is measured.
3. To draw the circuit schematic representation of each.
4. To use Ohm's Law to compute any one of I, E or

R when two of the three quantities are known, given a voltage source connected to a single resistive device.

5. To explain the meaning of the term "series circuit."
6. When given a voltage source connected to two or more resistive devices in series:
    a) to compute the total circuit resistance,
    b) to compute the current I,
    c) to compute the voltage drop across each resistive device.
7. To determine the voltage drop across any element of a series circuit when the voltage drops of all others are known.
8. To explain the meaning of the term "parallel circuit."
9. To determine the following, given a parallel circuit of two or more branches with the source voltage and resistances known:
    a) the branch currents,
    b) the total line current.
10. To determine the following, given a series-parallel circuit and sufficient information:
    a) the total resistance,
    b) the total current,
    c) the current through any resistive device,
    d) the voltage drop across any resistive device,
    e) the source voltage.
11. To determine the electrical power expended in a series, parallel or series-parallel circuit, when:
    a) voltage and current(s) are known or can be found,
    b) current and total resistance are known or can be found.

Op.VIII. Problem-Solving Strategies (1 module)

The student should be able:

1. To identify various strategies for solving problems.
2. To solve problems using strategies and to identify the strategy used.

Op.IX. Probability (½ module)

A. Introduction

The student should be able:

1. To discuss the importance of probability, both historically and in today's society.
2. To define probability.
3. To calculate the probability of a given event.
4. To distinguish among certain, uncertain and impossible events.
5. To identify the probability of a certain event, an uncertain event, and an impossible event.
6. To translate a statement of probability to a statement of odds.

B. Experimental Probability

The student should be able:

1. To calculate relative frequency using experimental data.
2. To observe that relative frequencies vary with repetition of the experiment.
3. To estimate probability using simple experiments.

C. Tree Diagrams

The student should be able:

1. To construct one- and two-stage tree diagrams.
2. To calculate probability using a tree diagram.

D. Simulation

The student should be able to construct a Monte Carlo simulation of some experiment or process.

Op.X. Owning and Operating an Automobile (½ module)

The student should be able:

1. To calculate the cost of buying a car.
2. To calculate the costs of operating a car.

Op.XI. Costs of Living on Your Own (½ module)

The student should be able:

1. To calculate an income based upon minimum wage.
2. To identify rents that are manageable within one's income.
3. To estimate the cost of furnishing a one-bedroom apartment.
4. To estimate the cost of stocking a kitchen.
5. To prepare a budget for costs of establishing and living in an apartment.
6. To identify/discuss alternatives in the event costs exceed income.

Op.XII. Leisure Mathematics (½ module)

A. Leisure Activities

The student should be able:

1. To interpret and to complete mathematical exercises associated with a number of leisure activities.

B. Recreational Mathematics

The student should be able:

1. To participate in solving mathematical problems.
2. To participate in playing mathematical games.
3. To participate in activities involving geometry.

MATHEMATICS 200 85% Core (C) - 15% Option (Op.)

C.I. Coordinate Geometry and Systems of Equations (1 module)

A. Coordinate Graphing in The Plane

The student should be able:

1. To demonstrate an understanding of coordinate graphing by:
   a) drawing a straight line given its equation in the form $y = mx + b$, $y = k$ or $x = k$,
   b) writing the equation of a straight line in the form $y = mx + b$, $y = k$ or $x = k$ given the appropriate information in words or on a graph.

2. To determine the equation of a straight line given two points on the line.
3. To determine the coordinates of the vertices of a rectangle with sides parallel to the axes, given the coordinates of the end-points of one diagonal.
4. To find the distance between two points on the same vertical or horizontal line.
5. To determine the distance between any two points on the coordinate plane using the Pythagorean Relation.
6. To derive the formula for the distance between two points.
7. To determine the mid-point of the line segment joining two given points.
8. To discover the relationship between the slopes of:
   a) parallel lines,
   b) perpendicular lines.
9. To find the equation of a line passing through a specified point and:
   a) parallel to a given line, or
   b) perpendicular to a given line.

B. Systems of Equations

The student should be able:

1. To solve a system of two first-degree equations in two unknowns using:
   a) graphing,
   b) addition-subtraction,
   c) substitution methods.
2. To solve a system of three first-degree equations in three unknowns using the addition-subtraction method.
3. To solve word problems requiring the use of:
   a) a system of two first-degree equations,
   b) a system of three linear equations.

C.II. Linear Inequalities (½ module)

The student should be able:

1. To recognize and to use the appropriate notation for inequalities.
2. To solve and to graph linear inequalities.
3. To graph inequalities involving compound conditions and to express their solutions algebraically.
4. To graph linear inequalities on a plane.
5. To graph systems of two or more linear inequalities on a plane.

C.III. Graphs of Equations and Functions (1 module)

A. Graphs of Equations

The student should be able:

1. To graph an equation of the form $y = mx + b$.
2. To graph an equation of the form $y = ax^2$.
3. To graph an equation of the for $y = \frac{k}{x}$.
4. To graph the equation $y = [x]$.
5. To graph the equation $y = k\sqrt{x}$ $(k > 0)$.
6. To graph the equation $y = |x|$.
7. To graph the equation $x^2 + y^2 = 1$.

B. Functions and Functional Notation

The student should be able:

1. To identify the domain and range for each type of graph in section A.
2. To determine which of the seven types of equations graphed in section A represent functions.

3. To recognize equivalent ways of describing functions.
4. To use functional notation.

C. Zeroes

The student should be able to determine the zeroes of each of the functions dealt with in section A of this unit.

C.IV. Quadratic Functions (1 module)

The student should be able:

1. To determine from the graph of a quadratic function:
   a) the coordinates of the vertex,
   b) the equation of the axis of symmetry,
   c) the maximum or minimum value.
2. To graph quadratic functions of the type $y = ax^2 + k$.
3. To graph quadratic functions of the type $y = a(x-h)^2$.
4. To graph quadratic functions of the type $y = a(x-h)^2 + k$.
5. To complete the square of a general quadratic function.
6. To graph quadratic functions of the type $y = ax^2 + bx + c$.
7. To solve maximum and minimum problems.

C.V. Quadratics (1½ modules)

The student should be able:

1. To find roots of a quadratic equation (zeroes of a quadratic function) using the techniques

of common factoring, difference of squares and trinomial factoring.

2. To use the quadratic formula,

$$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$, to determine the roots of a quadratic equation.
3. To determine the relationship between the roots of $f(x) = 0$ and the factors of $f(x)$ where $f(x)$ is a quadratic function.
4. To determine a quadratic equation which has two given real numbers or roots.
5. To use the discriminant to determine the number of roots of a quadratic equation.
6. To determine the relationship between the x-intercepts (zeroes) of a quadratic function and the roots of a quadratic equation.
7. To sketch quadratic functions by finding the vertex and x-intercepts.
8. To solve a system of one first-degree equation and one second-degree equation using graphical methods.
9. To solve a system of one first-degree equation and one second-degree equation using substitution.
10. To solve equations of the following types:
    a) equations involving radicals which can be solved by quadratic methods,
    b) fractional equations expressible as quadratic equations.
11. To solve problems which involve quadratic equations.

C.VI. Parallels (½ module)

The student should be able:

1. To demonstrate an understanding of the term

"transversal."
2. To identify corresponding angles in a diagram with two lines and a transversal.
3. To define parallel lines.
4. To use standard notation for parallelism.
5. To identify:
   a) alternate interior angles,
   b) consecutive interior angles.
6. To determine the validity of the following statements:
   a) for any transversals, if alternate interior angles are congruent, then the lines are parallel,
   b) for any transversal, if consecutive interior angles are supplementary, then the lines are parallel.
7. To determine the validity of the following statements:
   a) if lines are parallel, corresponding angles are congruent,
   b) if lines are parallel, alternate interior angles are congruent,
   c) if lines are parallel, consecutive interior angles are supplementary.
8. To construct a line passing through a given point and parallel to a given line.
9. To recognize and to use the fact that the sum of the three angles of a triangle is 180°.

C.VII. Quadrilaterals (1½ modules)

The student should be able:

1. To recognize those plane figures which are quadrilaterals.

2. To identify the parts of a quadrilateral.
3. To define parallelograms.
4. To prove the following properties of a parallelogram and to use them in proofs.
   a) opposite sides congruent,
   b) opposite angles congruent,
   c) diagonals bisect each other.
5. To prove that a quadrilateral is a parallelogram if:
   a) both pairs of opposite sides are congruent,
   b) two sides are parallel and congruent,
   c) the diagonals bisect each other.
6. To define "rectangle," "rhombus," and "square."
7. To prove certain properties of special parallelograms and to use these in proofs.
8. To define "trapezoid."

C.VIII. Circles (½ module)

A. Terminology

The student should be able:

1. To identify center, radius, diameter, chord, secant and tangent lines of a circle.
2. To identify congruent and concentric circles.

B. Perpendicular Bisector of Chords

1. To recognize that the perpendicular bisector of a chord passes through the center of the circle.
2. To recognize that the bisector of a chord which passes through the center is perpendicular to the chord.
3. To recognize that congruent chords are equidistant from the center.

4. To use the above properties in solving problems.

C. Drawing Tangents

The student should be able to draw a tangent to a circle, at a given point on the circle.

D. Arcs

The student should be able:

1. To recognize a central angle and its intercepted arc.
2. To recognize inscribed angles.
3. To recognize the relationship between an inscribed angle and its central angle, and to solve related problems.

C.IX. Similarity (½ module)

The student should be able:

1. To express a ratio in simplest form.
2. To solve a proportion.
3. To define similar polygons.
4. To identify similar polygons and to solve for parts of those polygons.
5. To recognize similar triangles using the following criteria:
    a) AA similarity,
    b) SSS similarity,
    c) SAS similarity.
6. To prove that a line parallel to one side of a triangle and intersecting the other two sides cuts off proportional segments.
7. To solve problems using similar triangles.

C.X. Trigonometry (½ module)

The student should be able:

1. To identify hypotenuse and the sides opposite and adjacent to any of the acute angles in a right-angled triangle.
2. To write sin θ, cos θ and tan θ as ratios of the sides of a right triangle.
3. To determine the value of sin θ, cos θ and tan θ if given a right-angled triangle whose sides are known.
4. To determine the value of sin θ, cos θ, tan θ if given a right-angled triangle with known angles.
5. To use trigonometric ratios to find unknown sides in right triangles.
6. To find the angle measure if given the trigonometric ratio.
7. To find unknown angles in a right-angled triangle.

GEOMETRY OPTION

Op.I. Indirect Proof (½ module)

The student should be able to formulate simple indirect proofs in geometry.

Op.II. Mathematical Reasoning (½ module)

The student should be able:

1. To give an explanation of the terms, "intuitive," "inductive" and "deductive" thinking.
2. To solve problems involving intuition, induction and/or deduction.

Op.III. Vectors (½ module)

A. Vectors

The student should be able:

1. To add two vectors.
2. To find the opposite of a vector.
3. To find the difference of two vectors.

B. Vector Problems

The student should be able to solve problems where the sum vector is given and one of the constituent vectors is unknown.

CONSUMER MATHEMATICS

Op.IV. Consumer Geometry (1 module)

The student should be able:

1. To calculate the perimeter of a polygon given the dimensions of the sides.
2. To calculate the circumference of a circle using $C = 2\pi r$ or $C = \pi d$.
3. To calculate the area of:
   a) a rectangle using $A = bh$,
   b) a triangle using $A = \frac{1}{2}bh$,
   c) a parallelogram using $A = bh$,
   d) a trapezoid using $A = \frac{1}{2}h\ (b_1 + b_2)$.
4. To calculate the area of a circle using $A = \pi r^2$.
5. To calculate the volume of a rectangular solid using $V = lwh$.
6. To calculate the volume of a sphere using

$$V = \frac{4}{3}\pi r^3.$$

7. To calculate the volume of a cylinder using $V = \pi r^2 h$.
8. To solve a series of word problems involving perimeter, area and volume.

Op.V. Investments and Annuities (1 module)

A. Stocks and Banking Investments

The student should be able:

1. To list ways money can be invested through the banking system.
2. To list advantages and disadvantages of saving money at a trust company rather than at a bank.
3. To outline the procedure for the purchasing of gold and silver through the major dealers of Manitoba.
4. To develop a working knowledge of the stock market.
5. To develop a working knowledge of the mathematics involved in acquiring stocks.
6. To interpret financial records and graphs.

B. Annuities

The student should be able:

1. To show that, if the rate of interest per period is i and the original principal is P, the accumulated amount at the end of n periods is given by $S = P(1 + i)^n$.
2. To solve problems using the formula $S = P(1 + i)^n$.

3. To understand terms involved in dealing with annuities.
4. To write the sum which gives the accumulated amount of an annuity, making repeated use of the the formula $S = P(1 + i)^n$.
5. To solve annuity problems using the formula

$$S = R\left[\frac{(1 + i)^n - 1}{1}\right].$$

OBJECTIVES FOR GRADE 12 MATHEMATICS

There are three full credit courses in grade 12 mathematics: 304, 301 and 300. Following are the objectives specific to each level.

MATHEMATICS 304 60% Core (C) - 40% Option (Op.)

C.I. Accommodation (1 module)

A. Utility Costs

The student should be able:

1. To interpret telephone bills.
2. To interpret hydro bills.
3. To interpret water bills.
4. To interpret heating bills.

B. Decorating and Redecorating

The student should be able to:

1. To calculate the amount of paint needed and the total cost of painting a room, an apartment or a home.

2. To calculate the amount of wallpaper needed to cover the walls of a certain room.
3. To calculate the amount and the total cost of carpet needed to cover the floor of a certain room.
4. To calculate the number of tiles needed to cover a floor or ceiling.
5. To construct a scale drawing of a room and of the furniture to be bought.

C.II. Algebraic Applications - Formulae (½ module)

The student should be able:

1. To evaluate formulae:
    a) by substituting numerical values for letters in a formula,
    b) by substituting numerical values for letters where letters are identified by words.
2. To translate from a word statement to an algebraic formula.

C.III. Mental Arithmetic (½ module)

The student should be able:

1. To perform mental calculations using whole numbers in addition, subtraction, multiplication and division.
2. To multiply and divide by powers of ten (limit to 1, 10, 100, 1000).
3. To perform mental calculations using decimal fractions in addition, subtraction, multiplication and division.

4. To round off and estimate cost calculations.
5. To complete a given sequence.
6. To respond to oral arithmetic questions.
7. To perform calculations involving order of operations.
8. To calculate the amount of change for a sales transaction.

C.IV. Insurance (1 module)

A. Automobile Insurance

The student should be able:

1. To identify the main parts of the basic Autopac plan.
2. To identify the main types of insurance use.
3. To calculate, given the appropriate rates or tables, the cost of insuring a given vehicle in a given situation.

B. Homeowner's or Rental Insurance

The student should be able:

1. To distinguish between physical damage to property (Section 1) and legal liability of the insured (Section II).
2. To define "replacement cost coverage."
3. To identify "personal articles floater."
4. To calculate a total annual premium given particular rates.

C. Unemployment Insurance

The student should be able:

1. To define "unemployment insurance."
2. To calculate unemployment insurance deductions given a weekly, semi-monthly or monthly salary.

D. Life Insurance

The student should be able:

1. To identify the main types of life insurance.
2. To list some advantages/disadvantages of life insurance policies.
3. To calculate annual premiums.

E. Long-Term Salary Continuance

The student should be able:

1. To define "long-term salary continuance."
2. To calculate the weekly or monthly deduction, given the annual premium.

F. Supplementary Health Benefit Plan

The student should be able:

1. To identify some of the features of a supplementary health benefit plan.
2. To calculate the annual premium for a single person and a family given the monthly rates.
3. To identify and to calculate the premium for additional travel health insurance.

G. Other Types of Insurance

The student should be able to identify some other types of insurance.

C.V. Money Management: Consumer Credit (1 module)

The student should be able:

1. To identify the role of debit cards.
2. To identify basic consumer credit agreements:
   a) promissory note,
   b) chattel mortgage,
   c) conditional agreement (time),
   d) overdraft privileges.
3. To identify types of personal loans available from banks:
   a) installment loans,
   b) demand loans,
   c) single payment loans,
   d) student loans,
   e) home improvement loans.
4. To calculate the true cost of borrowing money.
5. To identify some positive and detrimental factors involved in establishing a credit standing.
6. To identify methods used to establish a good credit standing.
7. To identify the responsibilities of the borrowers in a credit transaction.
8. To have an understanding of the "Manitoba Personal Investigation Act."
9. To identify some of the Manitoba laws regarding credit cards.
10. To calculate the cost of buying on credit.

C.VI. Ratio, Rate and Percent (2 modules)

A. Ratios

The student should be able:

1. To recognize and to express ratios.
2. To interpret three-term ratios.
3. To reduce ratios.
4. To determine equivalent ratios.
5. To use equivalent ratios to solve problems involving:
   a) scale drawings,
   b) map scales.

B. Rates

The student should be able:

1. To recognize and to express rates.
2. To calculate equivalent rates.
3. To use equivalent rates to solve problems relating to:
   a) comparison shopping,
   b) fuel economy.

C. Percent

The student should be able:

1. To determine the percent of a diagram which is/is not shaded.
2. To convert a percent to a decimal fraction and vice-versa.

3. To solve percent problems involving finding the percent of a number.

Op.I. Improving Your Residence (½ module)

The student should be able to complete at least two of the proposed projects.

Op.II. Algebraic Applications - Coordinate Graphing (½ module)

The student should be able:

1. To demonstrate an understanding of terminology involved in a coordinate system.
2. To describe the location of a given point.
3. To plot points given the ordered pair.
4. To construct, using data, graphs of direct relationships involving a straight line passing through the origin.
5. To define slope.

Op.III. Business Management (½ module)

The student should be able:

1. To identify methods of marketing goods.
2. To explain the method by which a business buys its products.
3. To interpret payment date on an invoice.
4. To calculate discount and discount series on an invoice.
5. To define F.O.B. terms.
6. To calculate total cost given an invoice form.

Op.IV. Computer Awareness (1 module)

A. History

The student should be able:

1. To be aware of the historical development of the electronic computer.
2. To identify four generations of digital computers.

B. Parts of a Computer

The student should be able:

1. To describe the parts of the computer and how they work.

C. Vocabulary

The student should be able:

1. To understand common computer terms.

D. Advantages, Limitations, Role and Impact; Career Opportunities

The student should be able:

1. To understand the main advantages of the computer.
2. To understand its limitations.
3. To recognize specific uses of computers in some fields.
4. To have some understanding of the impact the computer has on the individual and society.

5. To learn about computer-career opportunities.

E. The Computer

The student should be able:

1. To have "hands on" experience with the microcomputer.
2. To use the microcomputer in recreational and educational activities.
3. To create simple programs in either Logo or BASIC.

Op.V. Graphing (1 module)

The student should be able:

1. To read and interpret multiple bar graphs.
2. To read and interpret component bar graphs.
3. To read and interpret multiple line graphs.
4. To read and interpret component line graphs.
5. To interpret and to construct a circle graph.
6. To recognize distortion in graphs.

Op.VI. Leisure/Sports Activities (½ module)

A. Direct Individual/Team Participation

The student should be able:

1. To develop diagrams or tables comparing various leisure/sports activities.
2. To calculate the cost of participating in various leisure/sports activities.

B. Hobbies

The student should be able to identify costs in hobby involvement and to perform calculations for a given hobby.

C. Spectator Activities

The student should be able to compile and to interpret statistical tables.

Op.VII. Money Management - Foreign Exchange (½ module)

The student should be able:

1. To recognize situations when foreign exchange transactions are necessary.
   To identify the unit of currency of several countries.
3. To list the current value of foreign currencies in terms of Canadian dollars.
4. To complete calculations comparing Canadian dollars to currencies of other countries.

Op.VIII. Probability (½ module)

The student should be able:

1. To calculate the probability of an event as the ratio of desired outcomes to possible or total outcomes.
2. To determine relative frequency using data obtained experimentally.
3. To determine probabilities in games of chance.

Op.IX. Taxation (1 module)

A. Property Tax

1. To identify methods of collecting property tax.
2. To identify services covered by property tax.
3. To calculate homeowner's property tax.

B. Manitoba Tax Credit Plan

The student should be able:

1. To read and interpret Manitoba Tax credit plans.
2. To calculate cost of living credit.
3. To calculate Manitoba Property Tax credit both as renter and as homeowner.

C. Other Types of Taxation

The student should be able to identify other forms of taxation.

MATHEMATICS 301 75% Core (C)- 25% Option (Op.)

C.I. Consumer Mathematics (2½ modules)

The student should be able:

1. To define and to explain the purpose of budgeting.
2. To calculate the amounts a family would spend on an annual basis in various income categories, given the annual take-home pay and a recommended breakdown of the income dollar.

3. To calculate the amounts a family would spend on a monthly basis in the various income categories, given the annual income and the breakdown of the average income dollar.
4. To explain the various sources of income.
5. To define and to explain fixed expenses.
6. To define and to explain variable expenses.
7. To define and to explain a reserve fund.
8. To identify the steps required to organize a budget.
9. To prepare, on a budget form, a monthly budget for a family.

B. Taxation

The student should be able:

1. To identify the four basic sections on an income tax return, given a blank copy of the return.
2. To fill out the appropriate information in all four sections of a T1 General Form.
3. To calculate the refund or the balance due for a family.

Real Property Tax

The student should be able:

1. To calculate the rate of property tax in cents on a dollar, percent rate and mills, given the total budgetary requirement and the total assessment for a municipality.
2. To calculate property tax on residential property, given the assessed value and the rate of taxation.

Hidden Taxes

The student should be able:

1. To calculate federal sales tax.
2. To calculate customs duties (tariffs) given a table of customs tariffs.
3. To calculate excise taxes given a table of excise taxes.

C. Insurance

Life Insurance

The student should be able:

1. To explain and use the insurance vocabulary.
2. To demonstrate an understanding of the four types of life insurance policies.
3. To define and to understand the nature of each type of life insurance policy, including:
   a) advantages,
   b) benefits received,
   c) characteristics.
4. To calculate premium payments on an annual basis.
5. To calculate semi-annual, quarterly and monthly payments using a Premium Rate Table and a Periodic Premium Table.

Property Insurance

6. To explain and use the vocabulary concerning property insurance.

7. To calculate premiums of a Fire Insurance Policy using appropriate tables.
8. To calculate premiums of a Homeowner's Policy using appropriate tables.
9. To calculate premiums using Tenants' and Condominium Owners' Content Package Rates.
10. To calculate the loss paid using the co-insurance clauses in insurance policies.

Automobile Insurance

11. To explain the various factors affecting the cost of automobile insurance.
12. To calculate premiums using V.I.C. Tables and Premium Tables.
13. To compare rates paid for different territories and/or other variables such as use codes, third party liability, underinsured motorist coverage and deductibles.

D. Mortgages

The student should be able:

1. To explain the meaning of terms associated with mortgages.
2. To demonstrate an understanding of financing the purchase of a home.
    a) define the related terms,
    b) understand sources of mortgage loans:
        i) conventional first mortgage loans (75% of appraised value),
        ii) National Housing Act (NHA) Loans (95% of property value),
    c) explain methods of financing:
        i) cash down payment and mortgage balance,
        ii) cash down payment and seller carries mortgage,

        - iii) cash down payment and agreement of sale,
        - iv) carry existing mortgage and second mortgage on balance.
3. To calculate payments of mortgage loans using:
    - a) the decreasing payments plan,
    - b) equal payments plan:
        - i) calculations without the use of tables,
        - ii) calculations using tables.
4. To calculate and prepare a Statement of Adjustments.

C.II. Algebra (3 modules)

A. Algebraic Operations (4 classes)

The student should be able:

1. To perform operations with bases having:
    - a) positive exponents,
    - b) negative exponents.
2. To perform operations of addition, subtraction, multiplication and division with polynomials.
3. To write large numbers in scientific notation.
4. To write numbers between zero and one in scientific notation.
5. To convert numbers from scientific notation to expanded notation.

B. Equation Solving (16 classes)

The student should be able:

1. To solve linear algebraic equations in one variable (3 classes).

2. To solve word problems by setting up a linear equation in one variable (2 classes).
3. a) To determine a solution for a linear equation of the form $ax + by = c$,
   b) To determine whether an ordered pair is a solution for a system of linear equation (1 class),
   c) To determine the solution of a system of two linear equations by graphing (3 classes),
   d) To solve algebraically a system of two linear equations (4 classes).
4. to solve word problems by setting up a system of 2 linear equations (4 classes).

C. Graphing (12 periods)

Direct Linear Relations (4 periods)

The student should be able:

1. To obtain data experimentally for a direct linear relation.
2. To graph the data obtained in the previous objective.
3. To identify reasons for deviations from the mathematical model.
4. To interpolate and to extrapolate beyond the data obtained in Objective 1.
5. To express an equation for the data obtained in Objective 1.
6. To graph linear relations in the form $y = mx$ where "m" is slope.

Partial Linear Relations (6 periods)

The student should be able:

1. To obtain data experimentally for a partial linear relation.

2. To graph the data obtained in Objective 1.
3. To identify reasons for deviations from the mathematical model.
4. To interpolate and to extrapolate beyond the data in Objective 1.
5. To express an equation for the data in Objective 1.
6. To graph linear relations in the form $y = mx + b$ where "m" is slope and "b" is y- intercept.
7. To graph a linear relation in the form $Ax + By + C = 0$ after converting to the form $y = mx + b$.
8. To write the equation for a linear relation in the form $y = mx + b$ where "m" is the slope and "b" is the y- intercept.

Non-Linear Relations (2 periods)

The student should be able to sketch simple non-linear relations given appropriate data.

D. Variation

Direct Variation

The student should be able:

1. To recognize direct variations expressed as:
   a) word statements,
   b) charts,
   c) graphs,
   d) equations.
2. To determine the constant of variation.
3. To solve problems involving direct variation.
4. To change one form of direct variation to another.

Partial Variation

The student should be able:

1. To recognize partial variations expressed as:
   a) word statements,
   b) charts,
   c) graphs,
   d) equations.
2. To determine the constant of variation.
3. To solve problems involving partial variation.

Inverse Variation

The student should be able:

1. To recognize inverse variations expressed as:
   a) word statements,
   b) charts,
   c) graphs,
   d) equations.
2. To determine the constant of variation.
3. To solve problems involving inverse variation.

Types of Variation

The student should be able:

1. To identify the type of variation (direct, partial or inverse) from word problems and solve them.

2. To identify the type of variation from graphs.

C.III. Trigonometry (1 module)

The student should be able:

1. To recognize similar triangles.
2. To identify corresponding parts of similar triangles.
3. To recognize that the ratio of two sides of a triangle is equal to the ratio of corresponding sides of a triangle similar to the first.
4. To define the trigonometric ratios, sin θ, cos θ, and tan θ, as ratios of the sides of a right triangle.
5. To determine the numerical value of sin θ, cos θ, and tan θ, given θ, $0° < \theta < 90°$.
6. To find all parts of a right triangle given one angle other than the right angle and one side.
7. To determine the angles, $0° < \theta < 90°$, given the numerical value of sin θ, cos θ or tan θ, or the lengths of the sides of a right triangle.
8. To find all parts of a right triangle, given any two sides.
9. To determine the numerical value of sin θ, cos θ and tan θ, given $90° < \theta < 180°$.
10. To apply the Law of Sines.
11. To apply the Law of Cosines.
12. To solve problems which require the use of either the Law of Sines or the Law of Cosines or right triangle relationships.

C.IV. Statistics (1 module)

A. Areas of Application - Uses of Statistics

The student should be able:

1. To identify areas in which statistics are used.

B. Data Gathering

The student should be able:

1. To distinguish between a census and a sample.
2. To list various methods of obtaining data.
3. To recommend methods of obtaining a representative sample.

C. Ungrouped or Raw Data

The student should be able:

1. To calculate from ungrouped data measures of central tendency:
   a) mode,
   b) mean,
   c) median.
2. To select the measure of central tendency most representative of the group of data.
3. To calculate measures of dispersion of a set of ungrouped data:
   a) range,
   b) mean deviation.

D. Grouped Data

The student should be able:

1. To organize ungrouped data in a frequency distribution table.
2. To construct a histogram to illustrate the information displayed in a frequency distribution table.
3. To construct a frequency polygon to illustrate the information in a frequency distribution table.
4. To calculate from grouped data measures of central tendency including:
   a) mode,
   b) mean,
   c) median.
5. To calculate the following measures of dispersion (or variability):
   a) range,
   b) mean deviation.

E. Percentiles

The student should be able:

1. To calculate the percentiles for a given set.

Op.I. Annuities (½ module)

The student should be able:

1. To calculate the compound amount of a given principal, invested for a specified time, given the annual interest rate and the interest conversion period.

2. To calculate the present value (initial principal required) to be invested in order to develop a projected amount, given the term of investment, the annual interest rate and the interest conversion period.
3. To list three types of annuities as they are classified by term.
4. To list three types of annuities as they are classified by date of payments.
5. To explain the difference between a simple and a complex (general) annuity.
6. To compute the amount (Sn) of an ordinary simple annuity.
7. To compute the present value of an ordinary simple annuity, given the formula.
8. To compute the amount of an ordinary complex annuity, given the formula.
9. To compute the present value of an ordinary complex annuity, given the formula.

Op.II. Depreciation and Distribution of Overhead Costs (1 module)

A. Depreciation

The student should be able:

1. To define the terms:
    a) original cost,
    b) disposal, scrap or salvage value,
    c) estimated useful life.
2. To calculate the total amount of estimated depreciation.
3. To calculate annual or monthly depreciation expense, using the straight-line method.
4. To define the term "book value".
5. To calculate the book value.
6. To calculate the percent rate of depreciation.

7. To calculate the annual depreciation expense, using the declining-balance method.
8. To calculate the annual depreciation expense, using the sum-of-the-years digits method.
9. To compare the annual depreciation charge, using the above three different methods.

B. Distributing Overhead Costs

The student should be able:

1. To allocate overhead costs according to floor space occupied.
2. To allocate overhead costs according to the number of employees.
3. To allocate overhead expenses based on sales volume.

Op.III. Digital Logic (1 module)

The student should be able:

1. To draw the symbols of AND, OR, NAND and NOR gates.
2. To indicate all possible arrangements of 1 and 0 at the input of 2-, 3- and 4-input gates.
3. To explain the function of AND, OR, NAND and NOR gates.
4. To state the controlling input of AND, OR, NAND and NOR gates.
5. To prepare a truth table for a 2-input gate.
6. To indicate the symbol and the function of an inverter.
7. To recognize the decimal numbers 0 to 15 as coded in the binary system.
8. To prepare a truth table for a 4-input gate.

9. To prepare a truth table showing the inputs and outputs at all stages in a combinational circuit.
10. To explain the function of an inverter.
11. To prepare a truth table to show the output of a logic circuit which includes gates with inverted inputs.
12. To prepare a truth table to test and compare the final output of two different logic circuits.
13. To differentiate between combinational and sequential logic circuits.
14. To recognize the schematic symbol of a latch and explain its main function.
15. To sketch and explain the function of the internal gate combination of a simple latch.
16. To indicate the output of a simple latch, on a wave diagram, given the wave diagrams of the inputs.
17. To prepare a schematic of a four-bit binary counter.

Op.IV. Investments (Stocks, Bonds, Real Estate) (1 module)

A. Stocks

The student should be able:

1. To demonstrate an understanding of stock quotations as listed in a newspaper.
2. To calculate the cost of buying stock, using stock quotations.
3. To calculate the commission that must be paid in stock transactions.
4. To calculate the total cost of purchasing stock.
5. To calculate the net proceeds from selling stock.

6. To calculate the capital gain or loss on the sale of shares.
7. To calculate the yearly dividend from stocks if the dividend was paid quarterly, semi-annually or annually.
8. To calculate the annual rate of return on stock.

B. Bonds

The student should be able:

1. To calculate the market price of a bond.
2. To calculate cost of a bond including commission.
3. To calculate bond income.
4. To calculate the rate of return on bonds.

C. Real Estate

The student should be able:

1. To calculate the annual net income from an investment in real estate.
2. To calculate the rate of return on real estate.

Op.V. Interpretation of Graphs (½ module)

The student should be able to discuss and to analyze a given graph.

Op.VI. Quadratic Equations (½ module)

The student should be able:

1. To recognize quadratic equations.
2. To solve a quadratic equation of the type $ax^2 + c = 0$.

3. To express quadratic equations in standard form: $ax^2+bx+c=0$.
4. To identify the values of a, b, c, in quadratic equations.
5. To solve quadratic equations of the type $ax^2 + bx + c = 0$, using the quadratic formula: $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$.

6. To solve problems involving quadratic equations.

Op.VII. Permutations and Combinations (½ module)

A. Permutations

The student should be able:

1. To define a permutation.
2. To construct a tree diagram to illustrate possible permutations.
3. To define the Fundamental Principle.
4. To apply the Fundamental Principle.
5. To simplify factorial notation.
6. To solve permutations using the formula: ${}_nP_r = \frac{n!}{(n-r)!}$
7. To solve permutations of "n" objects, some of which are the same.

B. Combinations

The student should be able:

1. To define the term "combination."

2. To solve combinations using the formula:

$$C(n,r) \text{ or } {}_nC_r = \frac{n!}{(n-r)!r!}$$

Op.VIII. Statistics (Standard Deviation and the Normal Curve) (½ module)

The student should be able:

1. To calculate for a set of ungrouped data:
    a) the variance,
    b) the standard deviation.
2. To calculate for a set of grouped data:
    a) the variance,
    b) the standard deviation.
3. To examine some characteristics of the curve of normal distribution.

MATHEMATICS 300 80% Core (C) - 20% Option (Op.)

C.I. Polynomial and Rational Functions and Their Graphs (2 modules)

The student should be able:

1. To solve problems using function terminology learned in Math 200.
2. To define the expression "graph of a relation."
3. To define the term "inverse" for a relation.
4. To find the inerse of a given relation.
5. To graph a relation and its inverse using one set of axes.

6. To describe how various transformations affect the graph of a function.
7. To identify linear and quadratic functions and to sketch their graphs.
8. To determine algebraically the range of a quadratic function.
9. To find a polynomial function f which has a given set of numbers as its zeroes.
10. To find f(a) by substitution.
11. To sketch the graph of a polynomial function given in factored form.
12. To review operations on polynomials and rational expressions.
13. To factor sums and differences of cubes.
14. To understand the statement of the Factor Theorem.
15. To use the Factor Theorem to factor certain polynomials.
16. To sketch the graphs of simple polynomial functions which are readily factorable.
17. To sketch the graphs of functions which are quotients of polynomials.
18. To determine algebraically the equations of asymptotes for functions involving quotients of polynomials.

C.II. Circular Functions and Trigonometry (3 modules)

The student should be able:

1. To write the equation and sketch the graph of the unit circle.

2. To calculate the circumference of the unit circle.
3. To change an arc length into the equivalent number of revolutions for unit circles.
4. To sketch an arc on the unit circle where length is not necessarily less than one revolution and to show the terminal point T.
5. To find the coordinates of $P(\theta)$ for $\theta = 0$, $\pm \frac{\pi}{2}$, $\pm \pi$, $\frac{\pm 3\pi}{2}$, $\pm 2\pi$, and for larger multiples of $\frac{\pi}{2}$.
6. To define:
   a) the function cosine - $\cos: \theta \to \cos\theta$ where $\cos \theta$ is the x-coordinate of the point $P(\theta)$,
   b) the function sine - $\sin: \theta \to \sin\theta$ where $\sin \theta$ is the y-coordinate of the $P(\theta)$.
7. To find $\sin \theta$, $\cos \theta$ when $\theta = 0, \pm \frac{\pi}{2}, \pm \pi; \pm \frac{3\pi}{2}, \pm 2\pi$.
8. To show that $\cos^2\theta + \sin^2\theta = 1$.
9. To find $\sin \theta$ or $\cos \theta$, given the value of one of them.
10. To define:
    a) $\tan \theta = \frac{\sin \theta}{\cos \theta}$,
    b) $\csc \theta = \frac{1}{\sin \theta}$,

c) $\sec \Theta = \dfrac{1}{\cos \Theta}$,

d) $\cot \Theta = \dfrac{1}{\tan \Theta}$

e) $\cot \theta = \dfrac{\cos \theta}{\sin \theta}$

11. To find $\tan \Theta$, $\csc \Theta$, $\sec \Theta$, $\cot \Theta$ when $\Theta = 0 \pm \frac{\pi}{2}, \pm \pi, \pm \frac{3\pi}{2}, \pm 2\pi$.

12. To identify the quadrants in which $\sin \Theta$, $\cos \Theta$ and $\tan \Theta$ are positive or negative.
13. To find the values of the other five circular functions if given one value.
14. To determine the exact values of $\sin \Theta$, $\cos \Theta$ and $\tan \Theta$ when $\Theta = \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}$.

15. To determine the exact values of $\sin \Theta$, $\cos \Theta$ and $\tan \Theta$, where Θ = multiples of $\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}$ and $-2\pi < \Theta < 2\pi$.

16. To draw the graphs of $\sin \theta$, $\cos \Theta$, $\tan \Theta$ vs Θ in the interval $-2\pi < \Theta < 4\pi$.
17. To draw the graphs in the interval $-2\pi < \Theta < 4\pi$ of functions similar to those shown in the accompanying examples.
18. To determine radian measure and to obtain the approximate values for the six circular functions in the interval $0 < \Theta < 2\pi$.
19. To define inverse sin w, inverse cos w and inverse tan w.
20. To find all values of Θ in the interval $0 < \Theta < 2\pi$ when given $\sin \Theta = w$, $\cos \Theta = w$ or $\tan \Theta = w$

where w is a real number.

21. To derive the three Pythagorean Relations:
$\sin^2\theta + \cos^2\theta = 1$
$1 + \tan^2\theta = \sec^2\theta$
$1 + \cot^2\theta = \csc^2\theta$.
22. To use the reciprocal properties, the quotient properties and the Pythagorean Relations to prove circular functions identities.
23. To solve "linear" circular function equations.
24. To solve "quadratic" equations involving circular functions.
25. To define degree measure.
26. To solve oblique triangles.
27. To solve problems involving solution of triangles.

C.III. Analytic Geometry (1 module)

The student should be able:

1. To define the angle of inclination of a line and to determine the relationship between the angle of inclination and the slope of a line.
2. To determine the angle of inclination of a line given the equation of the line.
3. To solve problems involving midpoint, distance and slope.
4. To apply the formula for the distance from a point to a line.
5. To determine the equation of a set of points, given the conditions which govern their locus.
6. To sketch the graph of the equation of a given locus.
7. To recognize the conic section, given its equation in standard form.
8. To sketch the conic section given, its equation in standard form.

9. To determine the radius and centre of a circle, given its equation.

C.IV. Exponents and Logarithms (1 module)

A. Exponents/Exponential Functions

The student should be able:

1. To understand and to use the following terminology:
   a) base,
   b) exponent,
   c) power,
   d) coefficient,
   e) term/monomial,
   f) law of exponents.
2. To express a term containing negative exponents in its equivalent positive form and vice versa.
3. To express powers containing fractional exponents as radicals and vice-versa.
4. To evaluate and to simplify expressions involving zero exponents, negative exponents and fractional exponents.
5. To locate points on the graph of a specific exponential function such as $y = 2^x$ where x is real.
6. To graph given exponential functions and to determine domain, range, zeroes and when the function increases/decreases.
7. To solve simple exponential equations without using calculators.

B. Logarithms/Logarithmic Functions

The student should be able:

1. To graph the inverse of a specific exponen-

tial function such as $y = 2^x$.
2. To define a logarithm as an exponent.
3. To graph given logarithmic functions and to determine domain, range, zeroes and when the function increases/decreases.
4. To convert equations from logarithmic to exponential form and vice-versa.
5. To identify special types of logarithms.
6. To use the logarithmic theorems to express products, quotients or powers in the form $\log x = m \log a \pm n \log b \pm p \log c$ where $a, b, c > 0$.
7. To write an expression of the form $\log p = n \log a \pm m \log b$ as $p = a^n b^{\pm m}$ and solve for "p".
8. To solve simple logarithmic equations without using calculators.
9. To evaluate logarithms and inverse logarithms using calculators.
10. To solve exponential equations using logarithms.
11. To solve applications involving exponential or logarithmic equations.

C.V. Sequences and Series (1 module)

The student should be able:

1. To write a given sequence of numbers as a function rule.
2. To represent a finite sequence in three alternate forms: t_k or $f(k)$ or $f:k \rightarrow$.
3. To represent a finite sequence via a recursion formula.
4. To use a recursion formula and to generate the first "m" terms.
5. To identify arithmetic and geometric sequences (A.P.'s and G.P.'s), and understand terminology associated with them.

6. To find the fourth unknown given the other three in an A.P. (i.e., t_1, t_n, n,d).
7. To find the fourth unknown given the other other three in a G.P. (i.e., t_1, t_n, n,r).
8. To insert "k" arithmetic means in any A.P.
9. To insert "m" geometric means in any G.P.
10. To use 'sigma notation' in representing a sum.
11. To verify and to apply the formula used for finite arithmetic series:

    $S_n = \frac{n}{2}(t_1 + t_n)$ or

    $S_n = \frac{n}{2}\left[2t_1 + (n-1)d\right]$

12. To derive and to use the formula for finite geometric series ($r \neq 1$):

    $$S_n = \frac{t_1(1-r^n)}{1-r}$$

    or multiplying top, bottom by -1

    $$S_n = \frac{t(r^n-1)}{r-1}$$

13. To verify intuitively and to use the formula for the sum of an infinite geometric series.

    $S = \frac{t_1}{1-r}$ if $|r| < 1$.

14. To solve problems involving A.P.'s, G.P.'s and infinite G.P.'s.

Op.I. The Binomial Theorem (½ module)

The student should be able:

1. To simplify factorial expressions.
2. To define $C(n,r) = \frac{n!}{r!(n-r)!}$
3. To expand $(a + b)^n$ where n is a whole number, using Pascal's triangle to determine the coefficients.
4. To determine the relationship between the C(n,r) values and the numbers in Pascal's triangle.
5. To find a given term in any $(a + b)^n$ expansion.
6. To find the term containing a^k or b^k for any $(a + b)^n$ expansion.

Op.II. Compound Interest, Annuities, Amoritization and Mortgages (1 module)

The student should be able:

1. To use and to apply the compound interest formula.
2. To use and to apply the annuity formulae.
3. To apply the concept of "present value" to problem situations.
4. To use the concept of "amortization" in problems such as loans and mortgages.

Op.III. Graphs of Sinusoidal Functions (½ module)

The student should be able:

1. To sketch the graph of a sinusoidal function of the form $y = a \sin \Theta$.
2. To sketch the graph of a sinusoidal function of the form $y = \sin b\,\Theta$.
3. To sketch the graph of a sinusoidal function of the form $y = \sin(\Theta + c)$.
4. To sketch the graph of a sinusoidal function of the form $y = \sin \Theta + d$.
5. To sketch the graph of a sinusoidal function of the form $y = a \sin b(\Theta + c) + d$.
6. To determine the equation in the form $y = a \sin b(\Theta + c) + d$ for the function represented by a sinusoidal curve.
7. To write a function for a problem involving sinusoidal variation.
8. To sketch the graph of the function above in #7.

Op.IV. Circular Functions of Two Angles (½ module)

The student should be able:

1. To prove the formula $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$.
2. To solve problems by means of the formula in Objective #1.
3. To derive the following formulae from the formula for $\cos(\alpha - \beta)$:

   $\cos\left(\frac{\pi}{2} - \Theta\right) = \sin\Theta$,

   $\sin\left(\frac{\pi}{2} - \Theta\right) = \cos\Theta$,

$\tan (\frac{\pi}{2} - \theta) = \cot \theta,$

$\cot (\frac{\pi}{2} - \theta) = \tan \theta,$

$\cos (-\theta) = \cos \theta,$

$\sin (-\theta) = - \sin \theta.$

4. To solve problems by means of the formula in Objective 3.
5. To derive the following addition and subtraction formulae from the formula for $\cos (\alpha - \beta)$:
$\cos (\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta,$
$\sin (\alpha + b) = \sin \alpha \cos \beta + \cos \alpha \sin \beta,$
$\sin (\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta,$

$$\tan (\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$\tan (\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

6. To solve problems by means of formulae in Objective 5.

Op.V. Permutations and Combinations (1 module)

The student should be able:

1. To use the Fundamental Counting Principle to solve problems.

2. To develop the formula:
   $$P(n,r) = \frac{n!}{(n-r)!},\ n > r.$$
3. To solve problems involving permutations of n things of which two or more are alike.
4. To solve problems involving circular permutations.
5. To develop the formula:
   $$C(n,r) = \frac{n!}{(n-r)!r!}.$$
6. To solve combination problems using the formula in Objective 5.

Op.VI. Conic Sections in Standard Position (1 module)

A. The Ellipse

The student should be able:

1. To define an ellipse and associated terms such as: foci, principal axis, vertices, centre, major axis and minor axis.
2. To derive an equation of the ellipse with foci at (c, 0) and (-c, 0) and sum of focal radii 2a.
3. To sketch the graph of an ellipse having an equation of the form
   $$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ with } a > b$$
4. To solve problems involving ellipses having equations of the form
   $$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

B. The Hyperbola

The student should be able:

1. To define an hyperbola and associated terms such as: foci, principal axis, vertices, centre, transverse axis, conjugate axis and asymptotes.
2. To derive an equation of the hyperbola with foci at (c, 0) and (-c, 0) and difference of focal radii 2a.
3. To sketch the graph of an hyperbola having an equation of the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.
4. To solve problems involving hyperbolas having equations of the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

C. The Parabola

The student should be able:

1. To define a parabola and associated terms such as: focus, directrix, principal axis and vertex.
2. To find an equation of a parabola given the focus and the directrix.
3. To derive an equation of the parabola whose vertex is at the origin and whose focus is at $(\frac{P}{2}, 0)$.
4. To sketch the graph of a parabola having an equation of the form $y^2 = 2px$.
5. To solve problems involving parabolas having equations of the form form $y^2 = 2px$.

Op.VII. Transformations of Conics from Standard Position (½ module)

The student should be able:

1. To solve problems involving parabolas having equations of the form $x^2 = 2py$.
2. To solve problems involving ellipses having equations of the form
$$\frac{y^2}{a^2} + \frac{x^2}{b^2} = 1, \text{ where } a^2 > b^2.$$
3. To solve problems involving hyperbolas having equations of the form
$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1.$$

4. To solve problems involving parabolas having equations of the form $(y-k)^2 = 2p(x-h)$ or $(x-h)^2 = 2p(y-k)$.
5. To solve problems involving ellipses having equations of the form
$$\frac{(x-h)^2}{m^2} + \frac{(y-k)^2}{n^2} = 1.$$
6. To solve problems involving hyperbolas having equations of the form
$$\frac{(x-h)^2}{m^2} - \frac{(y-k)^2}{n^2} = \pm 1.$$
7. To classify an equation of the form $Ax^2+Cy^2+2Dx+2Ey+F = 0$ where A and C are not both zero.

implementation

IMPLEMENTATION

RATIONALE

This guide contains the goals generally thought desirable for the cognitive development of grade 12 mathematics students. But the core objectives for each grade level are prescriptive, rather than exhaustive, and teachers are encouraged to consider these in relation to their particular classroom situations when formulating learning outcomes. Material in this guide takes into account the skills possessed by grade 11 graduates and the needs that they have and will develop. For this reason, it is necessary for grade 12 teachers to become familiar with the mathematics programs in grade 11.

Mathematics, as a formal structure, must be built upon a foundation of concrete experiences and applications. According to developmental learning theory, students progress through three stages of learning: concrete (objects), semi-concrete (pictures), and abstract (symbols). Care must be taken to articulate the grade 12 program with patterns of development established in the 7-9 and grades 10 and 11 guides. It may not be necessary to begin at the concrete level. If the material makes first-hand experience impractical, or if previous experiences have developed the underlying concepts, then initial activities should be at the semi-concrete or abstract levels. However, not all students in grade 12 can function totally at an abstract level. Teachers should introduce concepts with current practical applications.

Basic skill development in the grade 12 student should be balanced with an understanding of the structure and vocabulary of mathematics. To promote this balance, the guide stresses understanding of basic concepts with practical applications, and recommends appropriate practice. Further, it urges that topics, once developed, be reviewed and maintained.

Teachers are encouraged to employ a variety of teaching techniques incorporating visual aids, concrete materials, a number of different texts, puzzles, applications and activities. The activities and suggested materials in this guide can be supplemented with those that teachers consider appropriate to the needs of their students.

If the program is to be relevant to the present needs of students and, at the same time, provide the foundations for future learning, then mathematics should be correlated with other subjects. Wherever possible, examples and applications should be drawn from other subject areas.

GENERAL GOALS FOR 04, 01, 00 MATHEMATICS

In identifying the specific goals of the mathematics program, one must consider the present needs of individuals and society. The student goals upon which this guide is based are identified by course below.

04 Mathematics

1. To develop mathematical skills in the context of practical, everyday situations;

2. To realize the importance of mathematics for all consumers;
3. To develop facility with estimations;
4. To develop further understanding and facility with measurement;
5. To observe, to collect, to interpret, and to display data graphically;
6. To perform arithmetic calculations accurately using a calculator;
7. To become more proficient in problem solving related to work and leisure;
8. To participate in recreational or leisure mathematics activities;
9. To increase confidence in estimating answers and in using mathematical skills to solve everyday problems;
10. To develop further the skills of reading, listening and following directions.

01 Mathematics

1. To develop an awareness of the usefulness of mathematics;
2. To increase understanding of the concepts, and to develop mastery of basic arithmetic skills using whole numbers, integers and decimal fractions through the solution of practical problems in the consumer, business, trade and vocation areas;
3. To become more proficient in mental arithmetic and rapid calculation exercises;
4. To increase comprehension in the problem solving process;
5. To interpret graphs and to display data graphically;
6. To realize that mathematics is important for all consumers;
7. To develop positive attitudes towards, and curiosity about, mathematics;

8. To read mathematics with understanding;
9. To develop the capability for post-secondary training in mathematics.

00 Mathematics

1. To develop a proficiency in problem solving;
2. To understand and to master basic algebraic skills;
3. To observe and to identify geometric relationships involving two-dimensional figures;
4. To develop skills in deductive and inductive reasoning;
5. To interpret graphs and to display data graphically;
6. To realize that mathematics is important for all consumers;
7. To develop positive attitudes towards, and curiosity about, mathematics;
8. To read mathematics with understanding;
9. To develop the capability for post-secondary training in mathematics;

THE GUIDE

The program in this guide is a description of a set of basic skills for grade 12 mathematics students. Subsequent work relies on the mastery of these basic skills and an understanding of the concepts developed in this guide. Where necessary, enrichment activities should be included.

Terminology

Module - section of a mathematics course involving approximately 10% of the allotted time. A full-credit mathematics course would consist of 10 modules, some compulsory, others chosen from a prepared list of optional modules. For a non-semestered class, each module could be subdivided as follows:

14 periods for learning new concepts
4 periods for review, testing, remediation and/or enrichment

Any reference to periods in this guide refers to 40-minute class periods.

Core - compulsory modules which all students registered in a particular course are required to study. Core modules are designated "C".

Option - module(s) selected from a prepared list. A specified number of these are to be completed in order to fulfill course requirements. Option modules are designated "Op."

Sequencing

The grade 12 mathematics courses outlined in this guide consist of core and option modules in a topical arrangement. A preferred order has not been identified. Rather, the order to be followed should be determined by the teacher in accordance with available texts. Several options are possible:

a. Some teachers may prefer to complete all the core units before beginning option topics.
b. Other teachers may wish to integrate topics from core and option modules.
c. Others may wish to alternate core and option modules.

d. Still others may choose to intersperse particular option topics within core topics, using, for example, one period every week or two.

Time Allotment

As credit courses, Mathematics 304, 301, and 300 each require 110-120 hours of instruction time. All schools/school divisions are expected to ensure that sufficient time is allocated to provide for the development of the objectives in individual mathematics programs.

ROLE OF TEXTBOOKS

The guide allows for flexibility in scope and approach and is independent of any one textbook. It provides a framework, but encourages teachers to consult outside materials in order that programs be enriched or modified to meet student needs. Teachers need access to and familiarity with various resources in order to accommodate the different ability and interest levels of students, to select appropriate content and methodology, and to have at hand various exercises, activities, and games for remediation, enrichment and problem solving.

USE OF CALCULATORS

"The world is presently experiencing an explosion of electronic technology which is threatening to render obsolete much of what we teach in school mathematics. Electronic calculators, unknown in in the early 70's, are so prevalent that they can be found in nearly every home or office. Because of their low cost and wide spread use in society, calculators have the potential of reshaping the mathematics curriculum in our schools. School practices generally lag far behind societal changes and the utilization of calculators in school is no exception. While on-the-job and in-the-home use of calculators to perform complex computations is nearly standard, schools have yet to incorporate calculators into the mathematics and science curricula. This must change."[1]

In high school (9-12), the use of calculators is strongly recommended. They are useful for computations involving powers and roots, consumer mathematics and problem solving, and should replace mathematics tables for both logarithmic and trigonometric functions. As most computational skills have been introduced and developed in grades K-8, the emphasis in high school should be directed towards concept development and application.

In today's society, the use of calculators has increased the significance of two interrelated mathematics skills: checking reasonableness of answers and estimation. The many students who tend to believe that calculators inevitably give the correct answer should learn to realize that this confidence is unwarranted, since incorrect input operations produce wrong and unreasonable results. To develop the students' awareness of reasonable results, estimation skills should be an integral part of all mathematics programs. It is important that students be taught when and how to use the calculator properly. Total reliance on it is not the objective. Students must realize that at times it is more expedient to use paper and pencil calculations or mental arithmetic and that they still should use paper and pencil to record the steps in problem solving.

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[1] Charlotte L. Wheatley, "Calculator Use in the Middle Grades," School Science and Mathematics, 80, No. F (1980), p. 620.

It is also important to explore ways in which calculators may be used in the acquisition of mathematical concepts, in the study of numerical patterns, and in the acquisition of ideas.

"Calculators . . . should be used in imaginative ways for exploring, discovering and developing mathematical concepts and not merely for checking computational values or for drill and practices."[2]

Students should be interested in the types of calculators available for purchase, the functions that different types perform, and the functions necessary for each course level.

Students should be advised against the purchase of an expensive programmable calculator, since the technology is developing so rapidly that calculators which are purchased this year could be outdated and much less expensive next year.

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[2] National Council of Teachers of Mathematics, *An Agenda for Action: Recommendations for School Mathematics of the 1980's* (Reston, VA: MCTM, 1980), p. 9.

USE OF COMPUTERS

Wherever possible, mathematics teachers and students should use computers to full advantage. Students could be exposed to the computer as a tool for use in developing the concepts of logical thinking through algorithms, and in investigating mathematical concepts which can be made more meaningful through use of a computer. Topics could include powers and roots, wages and payroll, area and volume, compound interest, statistical values, solution of simultaneous equations, loans and mortgages, problem solving, etc.

Computer literacy should be an essential outcome of contemporary education. Students should acquire an understanding of the versatility and limitations of the computer by using it to solve mathematics problems. In particular, problem solving, simulations, data retrieval tasks, and drill/practice programs would best integrate the computer and mathematics.

EVALUATION

Evaluation of student achievement and program effectiveness must be an integral, ongoing part of the teaching-learning process. Evaluation should provide information regarding:

1. Diagnosis and remediation. The purpose of diagnosis is to identify a student's strengths and weaknesses. In the case of weaknesses, the cause of the problem can then be identified and appropriate remediation provided by the teacher.

2. Student growth in mathematical skills and understanding. This is an integral part of a developmental program. Because growth is continuous, evaluation must also be continuous. Teachers should assess a student's speed and accuracy, understanding of concepts and processes, and ability in applying skills, and should use evaluation to encourage further student attainment in mathematics.

3. Learning strategies. Results should be used to judge the effectiveness of the instructional process: program of experiences, materials of instruction and strategies of teaching.

4. Curriculum modification. Modifications should be ongoing. Therefore, continuing feedback from the schools regarding the appropriateness of the objectives and the degree of implementation of the current program is essential.

Testing is a vital component of evaluation. Tests may be structured to evaluate a specific concept, a series of interrelated concepts, or all the concepts found in a year's work. Classroom teachers should design and use cumulative tests so that students maintain proficiency in previously learned concepts and skills.

Students in grade 12 should be given some form of cumulative testing (be it cumulative unit tests or final examinations) involving the objectives outlined in this guide. Both oral and written tests could be used. Such tests should go beyond the lower level cognitive skills and examine higher level cognitive objectives.

Evaluation is far more than paper and pencil testing. Evaluation of student achievement may take several additional forms, such as daily observation, interviews or conference with individual students, class discussions, and student reports and projects related to definitive and open-ended problem solving situations.

SPECIAL NEEDS STUDENTS

Students with special needs are those with handicaps--whether physical, cultural, emotional or intellectual--those who have been identified as slower learners, and those who are gifted or talented. Students in each of these groups require programs that are adapted to their needs. Suggestions for modifying the program are discussed.

Handicapped Students

The educational philosophy currently favoured for dealing with handicapped students is normalization, a philosophy that, in many cases, has resulted in the mainstreaming of these individuals. However, a key concept of normalization is the "least restrictive placement" which implies that not all special students should be mainstreamed.

Integration into the regular program will depend upon thorough identification of and attention to the needs of special students. These needs can be assigned to one of the following four categories.

Physical - general health disability, deafness or hearing impairment, blindness or visual impairment, orthopedic handicap.

Cultural - inadequate language development or experiences for growth.

Emotional - problems with personal relationships, mental illness.

Intellectual - underdeveloped cognitive ability, especially in the areas of conceptualization and abstract thinking.

Students with needs in any of these categories may be mainstreamed, depending on the degree of the need, but there are at least three conditions which may be considered essential for successful mainstreaming.

1. Good communication must exist among the student, classmates and teacher.
2. The student must be socially acceptable to the class.
3. The student's intelligence and background must enable him/her to adapt to the program content and demands, to whatever extent it is modified.

Slower Learners

Not all special needs students will have significant intellectual or physical handicaps. Indeed, the majority of students labelled "slow learners" come from cultural or socio-economic environments that differ from the norm. Whether such students simply learn more slowly or whether their potential for learning is inherently limited is a subject of considerable academic debate. The progress of students who have been identified as slower learners in the regular program should be monitored closely. If adaptation of the regular program does not meet their needs, then these students should be transferred to another program.

Modifying the Program

Student background and experience--including lifestyle, interest, motivation--are the most important factors in determining the extent to which a handicapped student or slower learner may be accommodated in the regular program. Consideration of these factors, as well as of a student's cognitive ability, his or her learning strengths and weaknesses, should determine the selection of appropriate teaching strategies and materials. Some specific suggestions for modifying the program to suit the needs of these students are listed below.

1. Choose those options in this guide in which content is appropriate to student background/interest, or use new content commensurate with the overall objectives.
2. Select materials appropriate to the student's reading level. If there is a foreign language problem, use E.S.L. teaching materials developed by the Department of Education.
3. The student's intelligence and background must enable him/her to adapt to the program content and demands, to whatever extent it is modified.

4. Teach from the concrete to the abstract. Use a "life skills" approach.
5. Enhance the student's self-concept by providing activities which will ensure recognizable achievement.
6. Be sure the student knows exactly what is required. Give clear instructions to minimize misunderstanding. Teach one skill or objective at a time. Use only one sensory approach at a time to avoid student confusion.
7. Use community resources. Develop activities so that students see the practical value and application of what they learn in class.
8. Incorporate suggestions from the 04 course into your program.
9. Consider transferring the student into another program if the previous adjustments prove ineffective.

GIFTED OR TALENTED STUDENTS

Gifted students in mathematics may show one or more or the following characteristics:

- outstanding performance in most academic subjects;
- prolific reading and ability to read and understand mathematics textbooks and articles;
- preference for independent work with little teacher direction;
- curiosity towards abstract concepts;
- assumption of responsibility.

Since the Mathematics 300 outline identifies the minimum program for students, and since gifted students can attain the course objectives in less time than other students, teachers of mathematically gifted students are faced with the challenge of enriching the program with stimulating and worthwhile activities. Suggestions include the following.

1. Stimulating interest

   - Provide a wide variety of materials and resources, such as The Mathematics Teacher (NCTM) and other professional journals/magazines.
   - Emphasize reading in mathematics.
   - Encourage the pursuit of problems of interest to the student.
   - Use mathematics clubs to maximum advantage.
   - Organize seminars at which students can present papers and special projects for ultimate submission to journals and newspapers.
   - Encourage peer tutoring.

2. Meaningful Activity

   - Enrich the program rather than accelerate the students.

- Provide independent activities.
- Encourage entry in mathematics contests and competitions.
- Promote computer use in mathematics.
- Encourage mathematics research.
- Encourage students to apply their mathematics to fields such as engineering, finance, meteorology, science, urban planning, computer science, economics, statistics, etc. Applications of mathematics are difficult in that they require knowledge of the application area in addition to the ability to apply pure mathematics in unfamiliar situations.

3. Additional Skills

- Provide higher level problem solving, research, reading and writing skills activities than those required for regular programs.
- Emphasize the development of students' manipulative intellectual skills.

mathematics 304

INTRODUCTION

Basic skills in mathematics are necessary for functioning in today's society. The 04 program concentrates on the application of mathematical skills and concepts to everyday situations. Skills covered include information processing, reasoning, computation and problem solving in areas encountered by the student at home, at work and at leisure. Students intending to pursue post-secondary education should be advised against taking this program.

GENERAL GOALS OF THE 04 PROGRAM

The student goals upon which this program is based are as follows:

1. To develop mathematical skills in the context of practical, everyday situations;
2. To realize the importance of mathematics for all consumers;
3. To develop facility with estimation;
4. To develop further understanding and facility with measurement;
5. To observe, to collect, to interpret and to display data graphically;
6. To perform arithmetic calculations accurately using a calculator;
7. To become more proficient in problem solving related to work and leisure;
8. To participate in recreational or leisure mathematics activities;
9. To increase confidence in estimating answers and in using mathematical skills to solve everyday problems;
10. To develop further the skills of reading, listening and following directions.

04 MATHEMATICS STUDENTS

There are several categories of students taking the 04 sequence of mathematics courses:

1. Those choosing Mathematics 104, 204, 304 for credit towards regular high school graduation.
2. Those taking a complete 04 program in which mathematics is integrated with other subjects including Work Experience.

3. Those taking a complete 04 program in which mathematics is taken as a separate course.

For this reason, the competence of students in 04 courses varies greatly. Although it is intended that teachers follow the objectives of the course, it is also recognized that the difficulty of the examples and the complexity of the problems chosen will vary from class to class and from student to student. Each course is organized into modules which allow either a whole class or an individualized approach.

FEATURES OF THE 04 COURSES

1. The 04 courses approach mathematics from the point of view of "everyday life" situations.

2. Estimation and the use of calculators are considered to be particularly important to the students of these courses. Therefore, they are included at the beginning of the 104 course. It is intended that calculators be employed wherever appropriate. Note that the need to concentrate on the use of calculators may be greatly decreased in future years if students are taught this process in previous grades.

3. Major course themes/topics include: accommodation, business management, clothing, employment, food, leisure activities, money management, recreational mathematics and transportation. Specific mathematical topics include measurement and geometry, statistics and graphing, and algebraic applications.

4. The teaching of basic arithmetic skills (i.e., the use of whole numbers, decimals, simple fractions, metric units and percentages) is to be incorporated into the courses as needed. All students should tackle problems and applications to be found in everyday life and should become aware of situations that require them to apply mathematical skills with accuracy. It is not expected that students will have reached a predetermined skill level before doing applications.

5. The individual needs of students vary greatly. For some students, teachers may need to provide remediation with simple applications or, alternatively, may find that some students can solve more complex problems.

STRUCTURE OF THE 04 COURSES

The following table gives an outline of the time and emphasis allotted to the topics identified for Mathematics 104, 204 and 304. The numbers represent the number of modules.

TOPICS	MATH 104		MATH 204		MATH 304	
	Core	Options	Core	Options	Core	Options
Accommodation			1½	1	1	½
Algebraic Applications					½	½
Business Management			1			½
Clothing		½				
Computer Awareness						1
Employment	1	1				
Estimation/Mental Arithmetic	1		½		½	
Food			½	½		
Graphing/Statistics	½	½	½	½		1
Insurance					1	
Leisure Activities		½				½
Measurement/Geometry	½	1	1½	1		
Money Management	1	1	½	1½	1	½
Probability						½
Ratio, Rate and Percent					2	
Recreational Mathematics		½		½		
Taxation				1		1
Transportation	1½	1				
Use of Calculators	½					
	6	6	6	6	6	6

MATHEMATICS 304 PROGRAM

CORE TOPICS (60%)

(Do each of the following modules.)

			Page References
C.I.	Accommodation	(1 module)	61
C.II.	Algebraic Applications - Formulae	(½ module)	75
C.III.	Mental Arithmetic	(½ module)	83
C.IV.	Insurance	(1 module)	87
C.V.	Money Management - Consumer Credit	(1 module)	97
C.VI.	Ratio, Rate and Percent	(2 modules)	107

OPTION TOPICS (40%)

(Do the equivalent of 4 modules from the following.)

Op.I.	Accommodation - Improving Your Residence	(½ module)	125
Op.II.	Algebraic Applications - Coordinate Graphing	(½ module)	129
Op.III.	Business Management	(½ module)	135
Op.IV.	Computer Awareness	(1 module)	139
Op.V.	Graphing	(1 module)	151
Op.VI.	Leisure/Sports Activities	(½ module)	167
Op.VII.	Money Management - Foreign Exchange	(½ module)	171
Op.VIII.	Probability	(½ module)	177
Op.IX.	Taxation	(1 module)	183

C.I. ACCOMMODATION (1 MODULE)

The intention of this module is to discuss some of the skills required to operate and maintain one's own dwelling place. Some of these topics have been introduced already in Math 104 and 204. Therefore, teachers are to review these skills as necessary and continue from there. Teachers should keep in mind that some of the students in Math 304 may not have taken Math 104 and 204.

This module is divided into two parts:

A. Utility Costs: Students are to be made aware of the recurring costs in maintaining their own places.

B. Decorating/Redecorating: This section will provide students with certain skills needed to calculate the material required to keep up an apartment or a house.

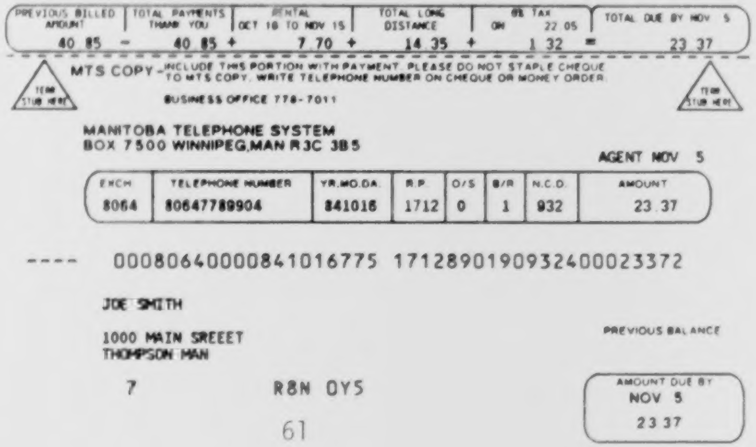
OBJECTIVES	NOTES TO TEACHERS

A. UTILITY COSTS

The student should be able:

1. to interpret telephone bills;

Samples of telephone bills may be supplied by the teacher or students.



PREVIOUS BILLED AMOUNT		TOTAL PAYMENTS THANK YOU		RENTAL OCT 16 TO NOV 15		TOTAL LONG DISTANCE		6% TAX ON 22.05		TOTAL DUE BY NOV 5
40.85	−	40.85	+	7.70	+	14.35	+	1.32	=	23.37

MTS COPY - INCLUDE THIS PORTION WITH PAYMENT. PLEASE DO NOT STAPLE CHEQUE TO MTS COPY. WRITE TELEPHONE NUMBER ON CHEQUE OR MONEY ORDER.

TEAR STUB HERE

TEAR STUB HERE

BUSINESS OFFICE 778-7011

MANITOBA TELEPHONE SYSTEM
BOX 7500 WINNIPEG,MAN R3C 3B5

AGENT NOV 5

EXCH	TELEPHONE NUMBER	YR.MO.DA.	R.P.	O/S	B/R	N.C.D.	AMOUNT
8064	80647789904	841016	1712	0	1	932	23.37

---- 00080640000841016775 17128901909324000023372

JOE SMITH

1000 MAIN SREEET
THOMPSON MAN

7 R8N 0Y5

PREVIOUS BALANCE

AMOUNT DUE BY
NOV 5
23.37

BUISNESS OFFICE 778-7011

PAGE		RES	TELEPHONE NUMBER	STATEMENT DATE
1	JOE SMITH	832	8064 778 8804	OCT 18 88

LONG DISTANCE

	DATE	FROM PLACE OR CONNECT TIME	MINUTES	TYPE	NUMBER CALLED	PLACE CALLED	AMOUNT
1	SEP 27	WINNIPEG MB	22	4	778 5008	THOMPSON MB	7 67
2	SEP 28	4 58 PM	1	*	453 3218	WINNIPEG MB	54
3	SEP 29	8 52 AM	8	*	885 3902	WINNIPEG MB	3 90
4	OCT 1	11 14 AM	7	*	885 3902	WINNIPEG MB	2 24

EXPLANATION OF LONG DISTANCE CALLS, ABBREVIATIONS AND SURCHARGE

TYPES OF LONG DISTANCE CALLS

* DIRECT DIALED
1 OPERATOR HANDLED
2 CALL BILLED TO A 3RD NUMBER
3 CALLING CARD
4 COLLECT
5 ZENITH

ABBREVIATIONS

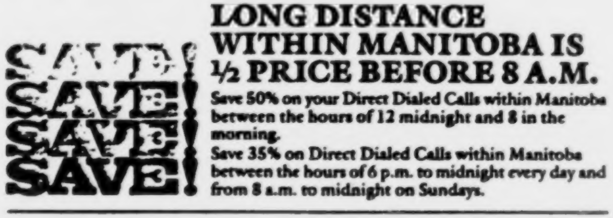
E – TAX EXEMPT
CR – CREDIT
DR – DEBIT
MIN – MINUTES

LONG DISTANCE AMOUNT SHOWN ON THE BILL IS THE DISCOUNTED AMOUNT (WHEN APPLICABLE)

FOR LONG DISTANCE DISCOUNT RATES AND TIMES PLEASE CHECK YOUR LOCAL DIRECTORY OR DIAL "0" FOR OPERATOR

SURCHARGE

IF PAYMENT FOR THE PREVIOUS MONTH'S BILL HAS NOT BEEN RECEIVED AT THE TIME YOUR NEW BILL IS PROCESSED, AN OUTSTANDING BALANCE WILL APPEAR. A SURCHARGE OF 1½%, 19.56% ANNUALLY (OR SUCH OTHER RATE AS MAY BE APPROVED FROM TIME TO TIME) WILL BE APPLIED WHEN THE OUTSTANDING BALANCE EXCEEDS $20.00


SAVE!
SAVE!
SAVE!
SAVE!

LONG DISTANCE WITHIN MANITOBA IS ½ PRICE BEFORE 8 A.M.

Save 50% on your Direct Dialed Calls within Manitoba between the hours of 12 midnight and 8 in the morning.

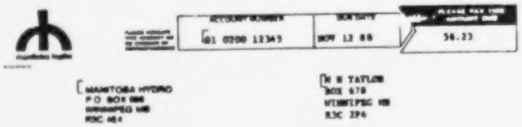
Save 35% on Direct Dialed Calls within Manitoba between the hours of 6 p.m. to midnight every day and from 8 a.m. to midnight on Sundays.

OBJECTIVES	NOTES TO TEACHERS
	Sample questions (based on examination of both sides of the bill):

a) What are the different types of long distance calls?
b) Explain how each type of call is made.
c) Determine the connection time for each call.
d) What is a directory assistance call?
e) What is the total long distance charge for this month?
f) What is the due date of this bill?
g) What is the sales tax?
h) What is the monthly rental charge for a phone?
i) What kind of discount does a customer get if he makes a direct call within Manitoba between midnight and 8:00 a.m.?

2. to interpret hydro bills;

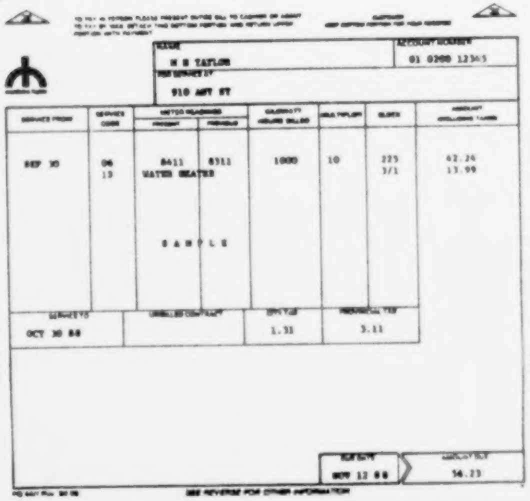
Samples of hydro bills may be supplied by the teacher or student.



01 0200 12345
NOV 12 88
56.23

MANITOBA HYDRO
P O BOX
WINNIPEG MB
R3C

H E TAYLOR
BOX 678
WINNIPEG MB
R3C 2P6



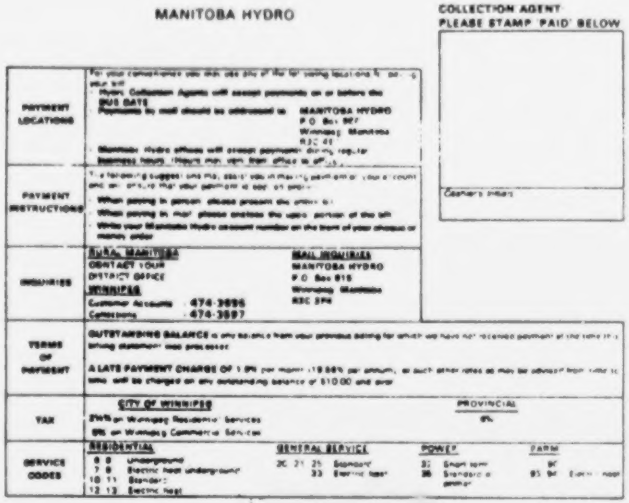
H E TAYLOR
01 0200 12345
910 ANY ST

SEP 30	06 13	8411 WATER HEATER	8311	1000	10	225 3/1	42.24 13.99

S A M P L E

OCT 30 88
1.31
3.11

NOV 12 88
56.23



MANITOBA HYDRO

COLLECTION AGENT
PLEASE STAMP 'PAID' BELOW

PAYMENT LOCATIONS

DUE DATE

MANITOBA HYDRO

PAYMENT INSTRUCTIONS

INQUIRIES

RURAL MANITOBA
CONTACT YOUR
DISTRICT OFFICE
WINNIPEG
Customer Accounts 474-3896
Collections 474-3897

MAIL INQUIRIES
MANITOBA HYDRO

TERMS OF PAYMENT

OUTSTANDING BALANCE

A LATE PAYMENT CHARGE OF

TAX

CITY OF WINNIPEG

PROVINCIAL

SERVICE CODES

RESIDENTIAL

GENERAL SERVICE

POWER

FARM

Sample questions (based on examination of both sides of the bill):

a) What is the consumption?
b) Is the consumption estimated or actual?
c) How is the consumption figure determined?
(The difference between the present month's bill and the previous month's bill is multiplied by 10.)
d) Identify the part of the bill which should be returned with the payment.
e) What is the account number?
f) What is meant by "Agency Date"?
g) What are the two kinds of taxes charged on a hydro bill?
h) What period of time does this bill cover?
i) What will Manitoba Hydro charge for a late payment?
j) Where can one call with an inquiry regarding this bill?

Note: The same questions could be asked regarding the Winnipeg Hydro bill or statement pictured on the following page.

READING YOUR BILL

Winnipeg Hydro billings today include, basically, the same information they have always shown. The average, single-month billing will include the consumer's name and address, account number, agency date, meter readings and billing dates (these will be shown as fixed dates each month due to computer procedure) and ultimately the charges based on the resulting consumption. Charges for other services where applicable, such as flat-rate water heating, water tank rentals, or other meters, will appear as separate items as shown on the sample billing overleaf. The sample billing is for one metered service, a flat-rate water heater, plus arrears and a late payment charge. Charges for appliance repairs, insulation loan payments, etc., would be identified separately.

Occasionally an unfamiliar item may appear on a billing ... "Estimated Consumption", for example. Estimating a customer's consumption is necessary when access cannot be gained to read an inside meter. Receiving a bill calculated from an estimated reading can be avoided by quickly returning a self-reading card to the utility, or by phoning your reading to us at 946-0344—Winnipeg Hydro's self-reading, "phone- in" line. At times, however, when for various reasons there are insufficient working days in a month to read all meters, meter readings for meters in some areas may have to be estimated. For these and in a few other instances Winnipeg Hydro simply projects what the consumption probably was (based on historical use) and processes a billing. With the exception of some customers who have erratic consumption patterns, most estimations are very close to what was actually used. This method of determining consumption provides customers with a regular monthly billing rather than a two or three month bill.

While billing information has not changed, the form on which it appears has undergone several changes over the years, mainly due to computer application. In order to assist our customers in understanding billing details, Winnipeg Hydro has produced the sample billing (below) which highlights key areas with an explanation of each.

KEY TO CODE (at "O")

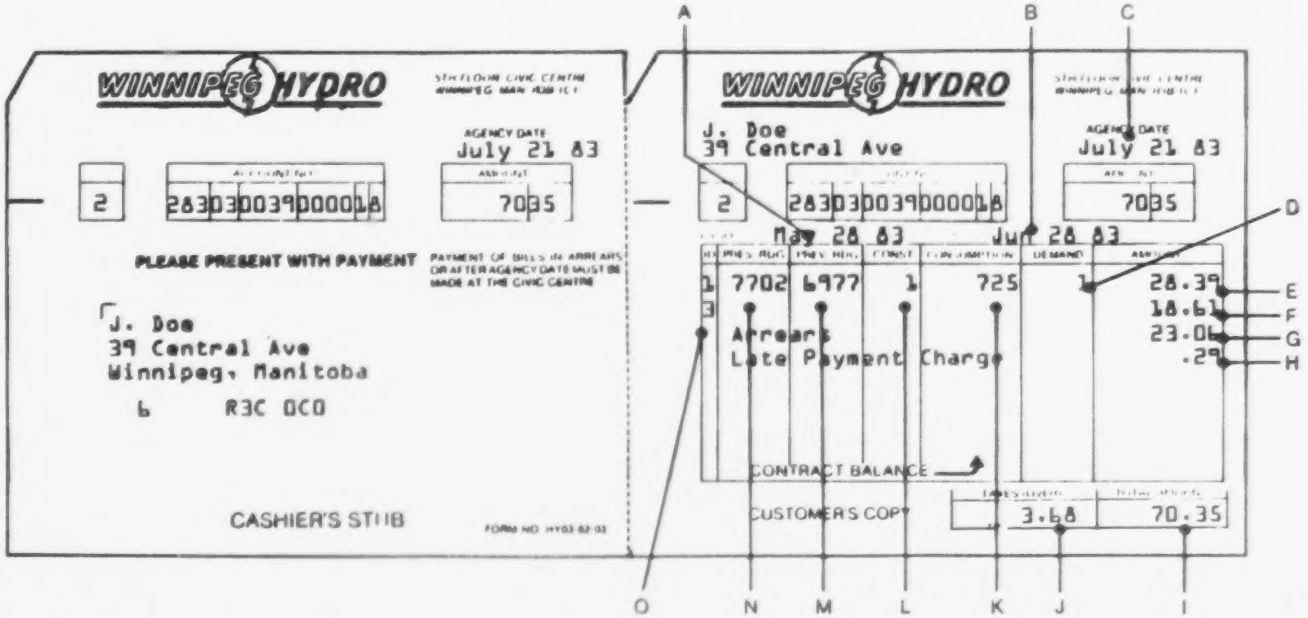
DOMESTIC

1	Metered
2	Metered (all electric)
3	Flat Rate Water Heating

COMMERCIAL

5	General Service
6	General Service (all electric)
7	Heating & Cooking
8	Flat Rate Water Heating
9	Power

A	**Previous month's billing date.**
B	**Current month's billing date.**
C	**Last day bill can be paid at agency. (At least 12 days are allowed for payment.)**
D	**A computer code to determine either commercial minimum charges; number of units to be charged; or billing demand.**
E	**Charge for current month's consumption calculated in accordance with the existing rate schedule, including taxes.**
F	**Total charges for water heating, including taxes.**
G	**Amount owing from previous bill(s).**
H	**Late payment charge of 1.25% per month on arrears.**
I	**Total amount to be paid.**
J	**Total taxes. (NOTE: Variations in tax charges will occur with Commercial and All-Electric accounts.)**
K	**Subject to the constant (see "L"), CONSUMPTION is the difference between "N" and "M" (meter readings).**
L	**CONST. stands for Constant or multiplier. The difference between "N" and "M" is multiplied by the factor shown in this column. In the sample the constant is "1". Some billings will indicate a higher constant.**
M	**Previous month's meter reading.**
N	**Current month's meter reading.**
O	**Type of electrical service as shown on rear of billing. (See key above).**



WINNIPEG HYDRO

AGENCY DATE July 21 83

2

283030039000018

7035

PLEASE PRESENT WITH PAYMENT

PAYMENT OF BILLS IN ARREARS OR AFTER AGENCY DATE MUST BE MADE AT THE CIVIC CENTRE

J. Doe
39 Central Ave
Winnipeg, Manitoba
6 R3C 0C0

CASHIER'S STUB

WINNIPEG HYDRO

J. Doe
39 Central Ave

AGENCY DATE July 21 83

2

283030039000018

7035

May 28 83 Jun 28 83

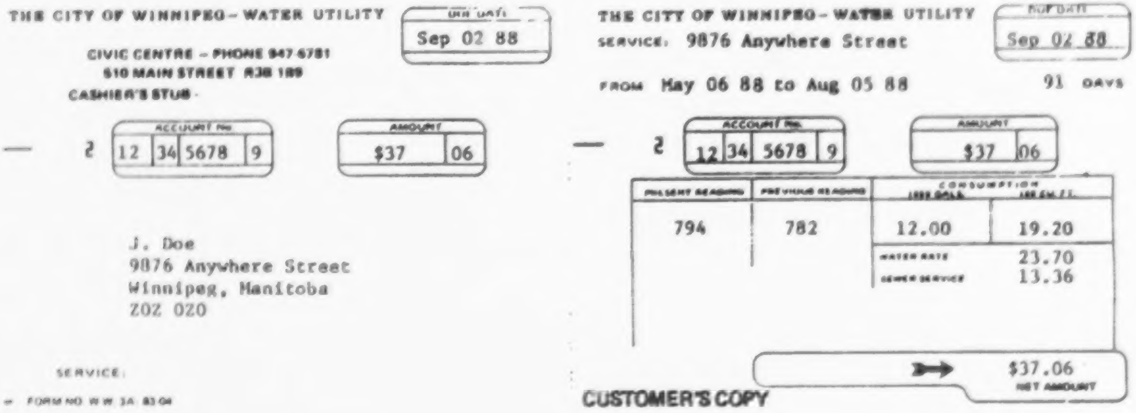
	PRES. RDG.	PREV. RDG.	CONST.	CONSUMPTION	DEMAND	AMOUNT
1	7702	6977	1	725	1	28.39
3						18.61
	Arrears					23.06
	Late Payment Charge					.29

CONTRACT BALANCE

CUSTOMER'S COPY

3.68	70.35

OBJECTIVES	NOTES TO TEACHERS
3. to interpret water bills;	Water bills will differ depending on where a person lives. In a city or town, where people are charged for the water they use, they may use the water bill sent to them by their municipality. If a local water bill is not available, a sample is supplied.



THE CITY OF WINNIPEG – WATER UTILITY

Sep 02 88

CIVIC CENTRE – PHONE 947 6781
510 MAIN STREET R3B 1B9
CASHIER'S STUB ·

	ACCOUNT No.				AMOUNT	
2	12	34	5678	9	$37	06

J. Doe
9876 Anywhere Street
Winnipeg, Manitoba
Z0Z 0Z0

SERVICE:

FORM NO. W.W. 3A 83 04

THE CITY OF WINNIPEG – WATER UTILITY

SERVICE: 9876 Anywhere Street

Sep 02 88

FROM May 06 88 to Aug 05 88 91 DAYS

	ACCOUNT No.				AMOUNT	
2	12	34	5678	9	$37	06

PRESENT READING	PREVIOUS READING	CONSUMPTION 1000 GALS.	100 CU. FT.
794	782	12.00	19.20
		WATER RATE	23.70
		SEWER SERVICE	13.36

$37.06
NET AMOUNT

CUSTOMER'S COPY

OBJECTIVES	NOTES TO TEACHERS

Sample questions:

a) What units are used to measure the amount of water used?
b) What period of time is covered by this bill?
c) Where can this bill be paid?
d) What percent of the total bill is for water used?

Example (figures taken from sample bill):

$$\frac{\text{Water used}}{\text{Total bill}} = \frac{23.70}{37.06} = 0.64 = 64\%$$

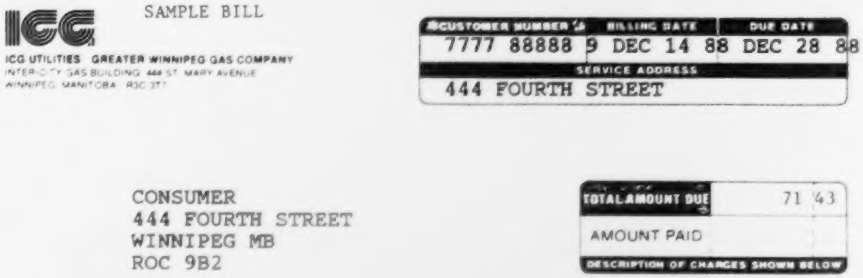
e) What percent of the total bill is for sewer service?

Example (figures taken from sample bill):

$$\frac{\text{Sewer service}}{\text{Total bill}} = \frac{13.36}{37.06} = 0.36 = 36\%$$

4. to interpret heating bills.

Bills will differ depending on the fuel used. Use the type of billing which is common to your area. The following sample is from the ICG Utilities-Greater Winnipeg Gas Company.



ICG

ICG UTILITIES GREATER WINNIPEG GAS COMPANY
INTER-CITY GAS BUILDING 444 ST. MARY AVENUE
WINNIPEG, MANITOBA R3C 3T7

SAMPLE BILL

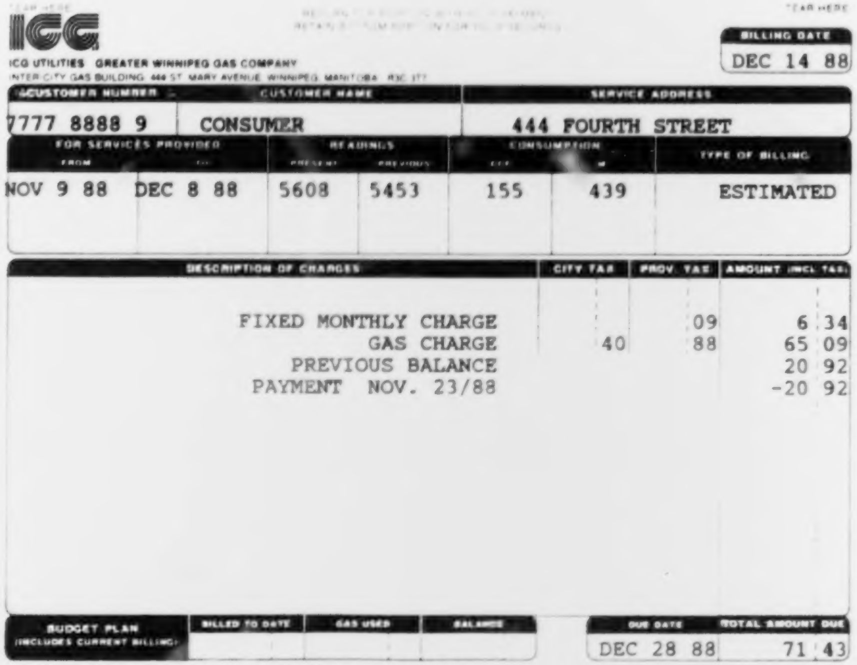
CUSTOMER NUMBER	BILLING DATE	DUE DATE
7777 88888 9	DEC 14 88	DEC 28 88

SERVICE ADDRESS
444 FOURTH STREET

CONSUMER
444 FOURTH STREET
WINNIPEG MB
ROC 9B2

TOTAL AMOUNT DUE	71	43
AMOUNT PAID		

DESCRIPTION OF CHARGES SHOWN BELOW



TEAR HERE TEAR HERE

ICG

ICG UTILITIES GREATER WINNIPEG GAS COMPANY
INTER-CITY GAS BUILDING 444 ST. MARY AVENUE WINNIPEG, MANITOBA R3C 3T7

BILLING DATE
DEC 14 88

CUSTOMER NUMBER	CUSTOMER NAME	SERVICE ADDRESS
7777 8888 9	CONSUMER	444 FOURTH STREET

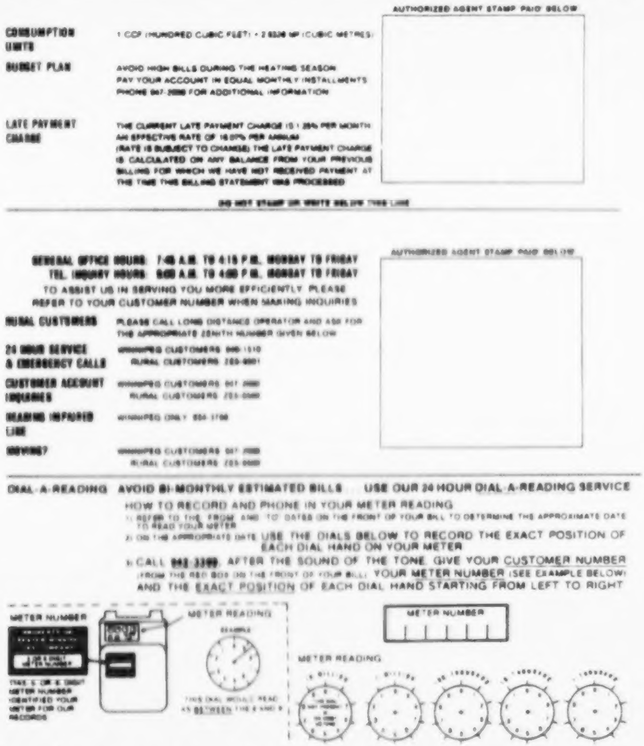
FOR SERVICES PROVIDED		READINGS		CONSUMPTION		TYPE OF BILLING
FROM	TO	PRESENT	PREVIOUS	CCF	M³	
NOV 9 88	DEC 8 88	5608	5453	155	439	ESTIMATED

DESCRIPTION OF CHARGES	CITY TAX	PROV. TAX	AMOUNT (INCL. TAX)
FIXED MONTHLY CHARGE		09	6 34
GAS CHARGE	40	88	65 09
PREVIOUS BALANCE			20 92
PAYMENT NOV. 23/88			-20 92

BUDGET PLAN (INCLUDES CURRENT BILLING)	BILLED TO DATE	GAS USED	BALANCE

DUE DATE	TOTAL AMOUNT DUE
DEC 28 88	71 43

OBJECTIVES	NOTES TO TEACHERS



AUTHORIZED AGENT STAMP PAID BELOW

CONSUMPTION UNITS	1 CCF (HUNDRED CUBIC FEET) = 2.8328 M³ (CUBIC METRES)
BUDGET PLAN	AVOID HIGH BILLS DURING THE HEATING SEASON PAY YOUR ACCOUNT IN EQUAL MONTHLY INSTALLMENTS PHONE 947-2000 FOR ADDITIONAL INFORMATION
LATE PAYMENT CHARGE	THE CURRENT LATE PAYMENT CHARGE IS 1.25% PER MONTH AN EFFECTIVE RATE OF 16.07% PER ANNUM (RATE IS SUBJECT TO CHANGE) THE LATE PAYMENT CHARGE IS CALCULATED ON ANY BALANCE FROM YOUR PREVIOUS BILLING FOR WHICH WE HAVE NOT RECEIVED PAYMENT AT THE TIME THIS BILLING STATEMENT WAS PROCESSED

DO NOT STAMP OR WRITE BELOW THIS LINE

GENERAL OFFICE HOURS: 7:45 A.M. TO 4:15 P.M., MONDAY TO FRIDAY
TEL. INQUIRY HOURS: 8:00 A.M. TO 4:00 P.M., MONDAY TO FRIDAY
TO ASSIST US IN SERVING YOU MORE EFFICIENTLY, PLEASE REFER TO YOUR CUSTOMER NUMBER WHEN MAKING INQUIRIES

AUTHORIZED AGENT STAMP PAID BELOW

RURAL CUSTOMERS	PLEASE CALL LONG DISTANCE OPERATOR AND ASK FOR THE APPROPRIATE ZENITH NUMBER GIVEN BELOW
24 HOUR SERVICE & EMERGENCY CALLS	WINNIPEG CUSTOMERS [illegible] RURAL CUSTOMERS [illegible]
CUSTOMER ACCOUNT INQUIRIES	WINNIPEG CUSTOMERS 947-2000 RURAL CUSTOMERS [illegible]
HEARING IMPAIRED LINE	WINNIPEG ONLY [illegible]
MOVING?	WINNIPEG CUSTOMERS 947-2000 RURAL CUSTOMERS [illegible]

DIAL-A-READING AVOID BI-MONTHLY ESTIMATED BILLS ... USE OUR 24 HOUR DIAL-A-READING SERVICE

HOW TO RECORD AND PHONE IN YOUR METER READING

1) REFER TO THE "FROM" AND "TO" DATES ON THE FRONT OF YOUR BILL TO DETERMINE THE APPROXIMATE DATE TO READ YOUR METER
2) ON THE APPROPRIATE DATE USE THE DIALS BELOW TO RECORD THE EXACT POSITION OF EACH DIAL HAND ON YOUR METER
3) CALL [illegible]. AFTER THE SOUND OF THE TONE, GIVE YOUR CUSTOMER NUMBER (FROM THE RED BOX ON THE FRONT OF YOUR BILL), YOUR METER NUMBER (SEE EXAMPLE BELOW) AND THE EXACT POSITION OF EACH DIAL HAND STARTING FROM LEFT TO RIGHT

METER NUMBER
THIS 5 OR 6 DIGIT METER NUMBER IDENTIFIES YOUR METER FOR OUR RECORDS
METER READING
EXAMPLE
THIS DIAL WOULD READ AS BETWEEN THE 2 AND 3
METER NUMBER
METER READING

Sample questions (based on examination of both sides of the bill):

a) What period of time is covered by this bill?
b) What part of the bill should be returned with the payment?
c) Where can this bill be paid?
d) How is the total amount of fuel used calculated?

Note: The agency fee charged when a payment is made should be discussed.

OBJECTIVES — NOTES TO TEACHERS

B. DECORATING/RE-DECORATING

The student should be able:

1. to calculate the amount of paint needed and the total cost of painting a room, an apartment or a home;

The following steps could be used:

Step 1: Students are to calculate the area of the 4 walls and the ceiling. Students should be made aware that the areas of the doors and windows are included. This allows for a safe margin of error. However, if a large area of a wall is not to be painted (example: picture window), this area should be calculated and subtracted from the total.

Step 2: Divide the total area by the number of square metres which can be covered by 1 litre of paint.

Step 3: Estimate the total cost of the paint needed.
(Note: Paint can be bought in containers of 1 litre or 4 litres).

Possible topics to be discussed:

1) Consider quality of paint (refer to Consumer Reports).
2) Accent walls - effect on a room, extra cost possible.
3) Types of paint to be used (latex, oil, satin finish, etc.).
4) Use of "stains."
5) Skills needed to paint (where to start, follow wood grain, etc.).
6) Compare cost of "do it yourself" and hiring a painter.

2. to calculate the amount of wallpaper needed to cover the walls of a certain room;

The following steps could be used:

Step 1: Calculate the area of all walls, including all openings.

Step 2: Divide the area by 3 to determine the number of rolls needed.
(There are approximately 3 square metres in one single roll.)

Step 3: Calculate the total cost of the paper needed.

Possible topics to be discussed:

1) Compare the quality, price and appropriateness of certain kinds of wallpaper (example: vinyl, foil, paper).
2) Tools needed to hang wallpaper.
3) Procedure used to hang wallpaper.
4) Compare cost of "do it yourself" and hiring someone.

Student Project: Calculate the wallpaper needed to cover the walls of your living room.

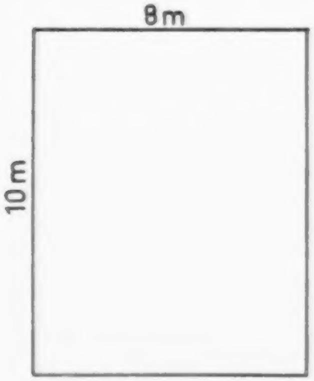
3. to calculate the amount and the total cost of carpet needed to cover the floor of a certain room;

Students are to be made aware that:

a) carpets are sold only in widths of 3.66 m (12 ft.), 2.74 m (9 ft.) and 1.83 m (6 ft.);
b) the consumer is charged by the m^2.

Suggested method to calculate the amount of carpet needed:

a) Examine the area to be carpeted.



b) Decide in what direction to run the strips of carpet, keeping in mind the widths available. Make students aware that the grain direction of the carpet must be considered when two strips are placed side by side.

c) Calculate the number of strips needed by using the following formula:

$$\text{Number of widths required} = \frac{\text{Width of room}}{\text{Width of carpet}}.$$

Example (using figures from diagram):

If the strips are to be placed lengthwise, the number of widths

$$\text{required} = \frac{8\text{ m}}{3.66\text{ m}} = 2.185\text{ widths.}$$

Make students aware that they cannot buy parts of widths. Therefore, they will have to buy 3 widths of 10 metres, which will result in some waste.

d) Calculate how many square metres for which they will be charged.

Example:

Area = L x W

Area = 10 m x 10.98 m

= 109.8 m^2

L = 10 m
W = 3 widths of 3.66 m = 10.98 m

e) Calculate the cost if the carpet is sold at $18/m^2$.

Example: 109.8 m^2 x $18 = $1976.40.

f) Have students repeat steps c, d and e, placing the strips in the other direction.

g) Compare the cost and the amount of waste when using the two methods of installing the carpet.

Possible topics to be discussed:

a) Types of carpets available (shag, loop, sculptured).
b) Cost of installation.
c) Area rug vs. wall-to-wall covering.

4. to calculate the number of tiles needed to cover a floor or ceiling;

Suggested method:

Step 1: Measure the length of the room in centimetres.

Step 2: Divide the length of the room by the length of a tile. This will provide the number of tiles needed for the length.

Step 3: Measure the width of the room in centimetres.

Step 4: Divide the width of the room by the width of one tile. This will provide the number of the tiles needed to go across the room.

Step 5: To find the number of tiles, multiply the number of tiles needed for the length (Step 2) by the number of tiles needed for the width (Step 4).

Using graph paper, students can do their own designs by using tiles of different colors (different patterns can be used, such as an alternating pattern).

OBJECTIVES	NOTES TO TEACHERS
	Possible topics to be discussed: 1) Types of tiles used. 2) Advantages/disadvantages of using tiles instead of carpets. 3) Procedure used to install tiles. (Note: A professional installer could be asked to demonstrate the procedure used to lay tiles.)
5. to construct a scale drawing of a room, and of the furniture to be bought.	Using a scale drawing, a student can determine possible arrangements of furniture and determine the maximum size of the furniture to be bought. (Note: This objective can be covered at this time or deferred to C.VI.) Resources: 1. Bolster, L.C. et al., *Consumer and Career Mathematics* (Toronto, ON: Gage Publishing Company, 1980), pp 204-211. 2. Cutting, Robert J., *Math You Need, Book 1* (Scarborough, ON: Nelson Canada, Ltd., 1981), Unit 10. 3. -------, *Math You Need, Book 2* (Scarborough, Ont.: Nelson Canada Ltd., 1982), pp 35-39. 4. Itzkovitz, M. and Richard, T., *Mastering Essential Mathematics Skills* (River Falls, IL: Laidlaw Brothers, 1980), pp 209-216. 5. Consumer Reports: "Woodstoves," February, 1982 "Painting," August, 1982.

C.II. ALGEBRAIC APPLICATIONS - FORMULAE (½ MODULE)

Algebra (i.e., evaluation of formulae) is introduced for the first time in this program. This module is not intended to be difficult. Therefore, only familiar formulae are used and simple numerical substitutions are made. Development proceeds from one type of evaluation to another. In the final type the student determines the formulae from word statements.

OBJECTIVES

NOTES TO TEACHERS

The student should be able:

1. to evaluate formulae:

   a) by substituting numerical values for letters in a formula,

Examples:

1. Given: $d = 2r$

   Find d when $r = 8$

   Solution: $d = 2 \times 8 = 16$

2. Given: $A = \ell w$

   Find A when $\ell = 14$ and $w = 8$

   Solution: $A = 14 \times 8 = 112$

3. Given: $i = prt$

   Find i when $p = \$3600$, $r = 18\%$, $t = 3$

   Solution: $i = \$3600\ (.18)(3)$

   $= \$1944$

4. Given: $V = \frac{1}{3}bh$

   Find V when $b = 12$ and $h = 7$

   Solution: $V = \frac{1}{3}(12)(7) = 28$

5. Given: $A = \pi r^2$

   Find A when $r = 5$ and $\pi = 3.14$

   Solution: $A = 3.14(5)^2 = 3.14(5)(5)$

   $= 78.5$

6. Given: $P = 2\ell + 2w$

   Find P when $\ell = 13$ units and $w = 8$ units

   Solution: $P = 2(13) + 2(8)$

   $= 26 + 16$

   $= 42$ units

b) by substituting numerical values for letters where letters are identified by words;

Examples:

1. Given: $C = rs$
   where C = commission
   r = rate
   s = sales

   Find commission when sales = \$8450 and rate is 7%.

   Solution: $C = .07(\$8450) = \$591.50$

2. Given: $m = n + d$
   where m = marked price
   n = net price
   d = discount

   Find the marked price when the net price is \$4.78 and discount is \$0.87.

   Solution: $m = \$4.78 + \$0.87 = \$5.65$.

3. Given: $c = m - d$
   where c = cost
   m = marked price
   d = discount

   Find cost when marked price is \$400.96 and discount is \$22.70.

   Solution: $c = \$400.96 - \$22.70 = \$378.26$

4. Given: $d = rt$
where d = distance
r = rate
t = time

Find distance when the rate is 90 km/h and the time is $3\frac{1}{2}$ hours.

Solution: $d = 90 \times 3\frac{1}{2} = 315$ km.

5. Given: $p = s + c$
where p = total pay
s = salary
c = commission

Find the total pay when salary is \$225 and commission is \$85.

Solution: $p = \$225 + \$85 = \$310$.

6. Given: $G = 2.1M + 3.7$
where G = green time for traffic signal light in seconds
M = average number of vehicles travelling in each lane per light cycle

Find the green time for a traffic signal in a street that averages 18 vehicles in each lane per cycle.

Solution: $G = 2.1(18) + 3.7$

$= 37.8 + 3.7 = 41.5$ seconds

Note: Other possible formulae include:

i) $V = \ell wh$
ii) $C = 2\pi r$
iii) $r = \frac{1}{2}d$

iv) $i = \frac{E}{R}$

v) S.P. = C.P + P where S.P is selling price, C.P is cost price and P is profit

vi) $n = \ell - d$
where n = net price
ℓ = list price
d = discount

vii) $n = p - d$
where n = net price
p = gross pay
d = deduction

viii) $ch = a - c$
where ch = change
a = amount tendered
c = cost

2. to translate from a word statement to an algebraic formula.

Example: In each of the following, have students write the algebraic formula and then solve the problem.

1) The perimeter of a square is equal to four times the length of a side. Find the perimeter when the length of a side is 8 cm.

Solution: $P = 4(\ell)$
where P = perimeter
ℓ = length

$P = 4 \times 8 = 32$ cm

2) The area of a triangle is equal to one half times the length of the base times the height. Find the area when the base is 7 cm and the height is 6 cm.

Solution: $A = \frac{1}{2}bh$.
where A = area,
b = length of base
h = height of triangle

$A = \frac{1}{2}(7)(6) = 21 \text{ cm}^2$

3) The par rating of a golf course is equal to the length of the course in metres divided by 183 plus 38.25. Find the par rating when the length of the course is 5539 m.

Solution: $P = \dfrac{\ell}{183} + 38.25$

where P = par rating
ℓ = length in metres

$P = \dfrac{5539}{183} + 38.25 = 30.268 + 38.25$

$= 68.518 \approx 69$

It is a par 69 course.

Resources:

1. Department of Education, Exercises to Accompany Mathematics 201 (Winnipeg, MB: The Author, 1982). Available at the Manitoba Text Book Bureau.
2. Ebos, F. and Zolis, M., Math Matters Book 1 (Scarborough, ON: Nelson Canada Ltd. 1982), pp 428-429.
3. Graham, R.H., Math Applications For Technical Trades (Toronto, ON: Copp Clark, 1983), pp 25-27, 84-87, 144-146, 200-202, 367-387, 438-440.
4. Keedy, M. et al., Applying Mathematics (Menlo Park, CA: Addison-Wesley Publishing Co., 1983). See "Formula" in Index.
5. Price, J. et al. Mathematics For The Real World: Canadian Edition (Weston, ON: Merrill Publishing Company, 1980). See "Formula" in Index.

C.III. MENTAL ARITHMETIC (½ MODULE)

Although the calculator has become a very successful tool in solving mathematical problems, it is still necessary for the student to maintain his/her arithmetic skills. Mental arithmetic questions are to be answered without the aid of pencil and paper. These will be used only for putting the answer on paper, as all questions can be answered through mental thought processes. This section should not be done at one time, but integrated throughout the whole course. Perhaps teachers could use a few minutes at the beginning/end of each class.

Note: Teachers are encouraged to use short speed tests throughout the course. These can be used for a student to better his/her own speed and accuracy. Graphs could be used for a student to keep a record of scoring.

OBJECTIVES	NOTES TO TEACHERS
The student should be able:	
1. to perform mental calculations using whole numbers in addition, subtraction, multiplication and division;	Some items to be considered: 1) horizontal and vertical calculations $\begin{array}{r}9\\+2\\\hline\end{array}$ 2 + 9 = $\begin{array}{r}9\\-2\\\hline\end{array}$ 9 - 2 = $\begin{array}{r}9\\\times 2\\\hline\end{array}$ 9 x 2 = $2\overline{)9}$ 9 ÷ 2 = 2) sequences of addition 9 + 4 + 3 + 6 + 5 =

OBJECTIVES	NOTES TO TEACHERS
	3) questions presented in different ways $3\overline{)9}$; $9 \div 3$; $\frac{9}{3}$ 2 subtracted from 7; 7 minus 2 the product of 6 and 3; 6 times 3. 4) questions with different formats $4 \times \_\_ = 28$ $\_ \times 3 = 21$ $57 - \_\_ = 48$ Note: Addition and subtraction questions should be limited to numbers with 2 digits. Multiplication questions should be limited to a single digit multiplier. Division questions should be limited to a single digit divisor.
2. to multiply and divide by powers of ten (limit to 1, 10, 100, 1000);	Metric conversions and changing percentages to decimals are practical uses of this process.
3. to perform mental calculations using decimal fractions in addition, subtraction, multiplication and division;	
4. to round off and estimate cost calculations;	Estimate the cost of the following: 1) 4 items if each is \$19.95 (answer \$80) 2) 1 item if 4 cost \$38.95 (answer \$10) 3) total bill if the amount is \$9.75 and 5% sales tax (answer \$10.50)

OBJECTIVES

NOTES TO TEACHERS

5. to complete a given sequence;

Complete the following sequences:

1) 1, 3, 5, 7, __, __, __
2) 4, 8, 12, 16, __, __, __
3) 75, 71, 67, 63, __, __, __
4) __, __, 9, 12, 15, __, __, __
5) 1, 3, 6, 10, __, __, __

6. to respond to oral arithmetic questions;

Some items to be considered:

1) reading numbers orally by student
2) teacher reads a number, student writes the number on paper
3) stating place values

7. to perform calculations involving order of operations;

Examples:

1) 6 + (2 x 4) =
2) 5 + 6 ÷ 3 =
3) Determine a question for which the answer is 21. Use at least 2 operations.

8. to calculate the amount of change for a sales transaction.

Students should not be subtracting to find the amount of change. The aim is to have students use the least amount of coinage.

Have students fill in the spaces to show how much change would be given. Notice some amounts presented include coinage. Sheets can be made up as follows:

Amount Presented	Amount Owed	1¢	5¢	10¢	25¢	$1	$2	$5	$1	$10
$20.00	$9.13									
$ 7.51	$7.41									
$ 5.25	$3.23									

Note: Puzzles are an interesting and motivating way of doing arithmetic questions.

1		7	14
8			11
		16	5
15	6		

Complete the magic square so that sum of each row, column and diagonal is 34.

Resources:

1. Bolster, L. and Woodburn, H., Mathematics In Life (Toronto, ON: Gage Publishing Co., 1978), Chapters 1, 2, 3.
2. Department of Education, Exercises to Accompany Mathematics 101 (Winnipeg, MB: The Author, 1981). Available at Manitoba Text Book Bureau.
3. "Rapid Calculation Drills". Distributed to all schools.
4. Price, J., et al., Mathematics For The Real World, Canadian Edition (Weston, ON: Merrill Publishing Co., 1979), pp 292-293, 374-376.

C.IV. INSURANCE (1 MODULE)

Insurance is a contract binding a company to compensation of an insured party against specified loss. The premium is the amount assessed for this protection. Common types include: automobile, health and accident, homeowner's and renter's, life and unemployment insurance.

Some insurance must be purchased individually (such as automobile and homeowner's or renter's insurance), while other insurance may be purchased by individuals belonging to a group (such as health, life and accident insurance). The latter may result in lower rates and subsequent savings to the consumer.

OBJECTIVES	NOTES TO TEACHERS
A. AUTOMOBILE INSURANCE	
The student should be able:	
1. to identify the main parts of the basic Autopac plan;	These parts are: 1. No fault accident benefits for injury or death 2. All perils coverage of the vehicle (subject to the deductible) 3. Third party liability insurance 4. Underinsured motorist coverage
2. to identify the main types of insurance use.	These uses are: 1. Preferred - covers vehicles used only for pleasure driving and can only be driven to and from work or school up to 1609 (1000 miles) a year or four days a month. 2. School Car - used for pleasure driving and to and from or part way to and from school, but cannot be driven to and from or part way to and from work for more than 1069 km (1000 miles) a year, or more than four days a month. 3. All Purpose - covers vehicles used for driving to and from or part way to and from work, but cannot be driven more than 1609 km (1000 miles) a year for business purposes.

4. Business - covers vehicles: (i) registered in company name (ii) owned by Government of Manitoba or a municipality, or (iii) otherwise registered and used for 1609 km (1000 miles) or more driving a year during the registration year in connection with business, trade or occupation.

5. Farm - covers vehicles owned by a person who lives on a farm and is primarily occupied in farming or retired from farming.

Note: Other types of insurance are available including Farm Car-All Uses, U-Drive, Delivery Car, and Public Bus. Teachers may wish to continue with a discussion of the options available.

3. to calculate, given the appropriate rates or tables, the cost of insuring a given vehicle in a given situation.

Using appropriate Autopac rates for Territory 1, 2, 3, or 4, consider the following questions:

1) A vehicle is in rating group 6 and is used for pleasure driving. For $250 deductible and $200 000 third party liability, calculate the Autopac premium.

2) If the above vehicle is used for all purpose driving, calculate the new premium.

3) State the maximum third party liability insurance and the premium assessment.

4) A vehicle is in rating group 3 and is registered for business. If the deductible is $150 and the third party liability is $500 000, calculate the premium.

5) A vehicle is in rating group 8 and is used for all purpose driving. With a third party liability of $1 000 000, calculate the premiums for:

   a) $350 deductible
   b) $250 deductible
   c) $150 deductible

6) A vehicle in rating group 9 is used for pleasure driving and has the minimum third party liability coverage. Calculate the difference between the premium for $250 deductible and $150 deductible.

Note: Territory 1: Includes City of Winnipeg and surrounding municipalities, including West St. Paul, East St. Paul, Headingly, Fort Whyte, St. Norbert and others.

Territory 2: All Manitoba south of 53rd parallel, excluding Territory 1.

Territory 3: All Manitoba north of 55th parallel. Some towns here are: Thompson, Lynn Lake, Churchill, Gillam.

Territory 4: All Manitoba between 53rd and 55th parallels. Some towns here are: Easterville, Grand Rapids, Flin Flon, The Pas.

Resources:

1) Your 1989 Autopac Policy Guide booklet available from your local Autopac agent or Manitoba Public Insurance Corporation, Box 6300, Winnipeg, Manitoba R3C 4A4. This publication is also available in French.

2) Local Autopac agent.

B. HOMEOWNER'S OR RENTAL INSURANCE

The student should be able:

1. to distinguish between physical damage to property (Section 1) and legal liability of the insured (Section II);

Section 1 coverage includes:

a) dwelling building
b) private structures (garages, etc.)
c) personal property
d) additional living expenses (extra expense if the residents have to move out while repairs are made)

OBJECTIVES

NOTES TO TEACHERS

Section II coverage includes:

e) personal liability
f) medical payments (for anyone injured on or about premises)
g) property damage to the property of others

2. to define "replacement cost coverage";

This coverage pays on a "replacement" or non-depreciating basis.

3. to identify "personal articles floater";

This covers special items such as jewelry, furs, silverware, stamp & coin collections, photographic equipment, etc. and insures them both at home and away from home.

Note: Insurance policies normally cover loss by theft of jewelry/furs to a maximum value of $1000 - $2000. Special floaters on an "all-risk" basiscan be obtained to cover items appraised at a higher value.

4. to calculate a total annual premium given particular rates.

Example:

Consider the following Rental or Tenant's Package:
Personal Property $10 000
Floater on Camera worth $500
Floater on Jewelry worth $800

	Company A	Company B
Basic Premium	$49.00	$64.00
Replacement Cost Coverage	$15.00	$25.00
Camera ($500.)	$1.25 per $100. value	$1.50 per $100. value
Jewelry ($800.)	$1.10 per $100. value	$1.25 per $100. value

Calculate the total premium for each company. Which company offers the lower premium? By how much?

OBJECTIVES	NOTES TO TEACHERS
	Notes: 1) When paying a mortgage, a person should carry homeowner's insurance at least to the value of the mortgage. 2) A person should make an inventory of all items and update it as necessary. This could be done by room. Photographs or videos of valuable items could be included. This would make any claim easier to sort out. 3) Smoke detectors, properly installed and maintained, can be used to gain a 5% discount on some insurance premiums. 4) Non-smokers are rewarded with a 9% discount by some insurance companies. Statistics for 1980 show that careless smokers caused approximately 30% of home fires in Canada. 5) Inflation protection coverage, which automatically increases protection about 2% every four months during the life of the policy, is another way to peg coverage to inflation. Resource: Local insurance agent.
C. UNEMPLOYMENT INSURANCE The student should be able:	
1. to define "unemployment insurance";	Unemployment insurance premiums are paid into a government fund so that benefits may be drawn by an eligible employee who is out of work. Benefits vary according to current regulations, according to earnings and according to the period of contributions.
2. to calculate unemployment insurance deductions given a weekly, semi-monthly or monthly salary.	Teachers are encouraged to obtain tables for such deductions from: Revenue Canada, 391 York Avenue, Winnipeg, Manitoba, R3C 0P5.

OBJECTIVES	NOTES TO TEACHERS
D. LIFE INSURANCE	
The student should be able:	
1. to identify the main types of life insurance;	These include: 1) Ordinary Life (Whole or Straight Life) - has one of the lowest premiums. It stays in effect until the insurer dies. 2) Limited Payment Life Insurance - is more expensive. It is like ordinary life insurance, but premiums are paid for a certain time (eg., 20 years). It stays in effect until the insurer dies. 3) Endowment Insurance - is the most expensive. Premiums are paid for a definite time period (eg., 20 years). At the end of the time, the insured can collect the "face value" of the policy. Then, the insurance is no longer in effect. 4) Term insurance - is the least expensive. Unless the insured dies during the time of the term, no money is collected.
2. to list some advantages/disadvantages of life insurance policies;	It is suggested that a life insurance agent be brought in to discuss this topic.
3. to calculate annual premiums.	Use appropriate tables.

OBJECTIVES	NOTES TO TEACHERS
E. LONG-TERM SALARY CONTINUANCE	
The student should be able:	
1. to define "long-term salary continuance";	This insurance provides a person with a regular income to replace salary which is lost as a result of a lengthy disability due to accident or sickness. Students should be aware that salary continuance is a condition of employment in some cases.
2. to calculate the weekly or monthly deduction, given the annual premium.	Given a yearly premium of $221.20, have the students calculate: a) weekly deduction b) monthly deduction Note: This coverage is more affordable if done as part of a group of employees. It is suggested that an agent be used to discuss features and benefits of various salary continuance plans.
F. SUPPLEMENTARY HEALTH BENEFIT PLAN	
The student should be able:	
1. to identify some of the features of a supplementary health benefit plan;	Main features include: ambulance service, semi-private hospital room, prescription drug costs (minus Pharmacare), private duty nursing by a R.N., and mechanical devices such as wheelchairs, crutches, etc.
2. to calculate the annual premium for a single person and a family, given the monthly rates;	Use current rates to find the annual premiums.

OBJECTIVES

NOTES TO TEACHERS

3. to identify and to calculate the premium for additional travel health insurance.

Travel insurance includes: hospital expenses, medical bills, ambulance service expenses, additional return air fare if illness or accident interrupts a charter flight, accidental dental injury expenses, special nursing care costs, and prescription drug bills.

Problems should be posed using the following rates (or current rates).

Days Inclusive	Single	Family
1-4	$ 6.50	$ 13.00
1-8	9.00	18.00
1-16	16.00	32.00
1-25	22.00	44.00
1-32	30.00	60.00
1-45	39.00	78.00
1-62	55.00	110.00
1-93	77.00	154.00
1-120	105.00	210.00
1-150	132.00	264.00
1-183	150.00	300.00
1-214	180.00	360.00

G. OTHER TYPES OF INSURANCE

The student should be able to identify some other types of insurance.

Other types of insurance include: mortgage insurance, accident/disability insurance, crop insurance, and trip cancellation insurance, etc.

OBJECTIVES	NOTES TO TEACHERS

Resources:

1. Bolster, L.C. et al., Consumer and Career Mathematics (Toronto, ON: Gage Publishing Co., 1980), pp 160-163, 226-227.
2. Cutting, Robert J., Math You Need Book 2 (Scarborough, ON: Nelson Canada Ltd., 1981), Unit 9.
3. Department of Education, Exercises to Accompany Mathematics 301 (Winnipeg, MB: The Author, 1984). Available at the Manitoba Text Book Bureau.
4. Keedy, M., et al., Applying Mathematics (Menlo Park, CA: Addison-Wesley Publishing Co., 1983), Chapter 7.
5. Manitoba Public Insurance Corporation (Autopac) class sets of materials available.
6. Other insurance companies.
7. Local insurance agents as speakers.
8. Information Centre, 20 Queen St. W., Suite 2500, Toronto, Ont., M5H 3S2. Toll Free Number: 1-800-268-8099.

C.V. MONEY MANAGEMENT: CONSUMER CREDIT (1 MODULE)

The aim of this module is to provide students with skills in the use of consumer credit.

OBJECTIVES	NOTES TO TEACHERS
The student should be able:	
1. to identify the role of debit cards;	Refer to Math 204 (Op.V) to review credit cards as necessary. When the debit card is presented for a purchase, the owner's bank account is automatically debited. A debit card, such as Master Card II, is expected to replace credit cards in the future.
2. to identify basic consumer credit agreements:	
a) promissory note,	A promissory note is a promise to repay a specified amount of money at a stated interest rate over a given period of time.
b) chattel mortgage,	A chattel mortgage is usually required for people financing movable goods such as a car, boat or trailer.
c) conditional agreement (time),	A conditional sales contract is usually required for people financing non-movable goods.
d) overdraft privileges;	With overdraft privileges, a customer is allowed to write a cheque for more than he/she has in his/her account. The overdraft is treated as a personal loan. The customer must pay interest until the overdraft is repaid.
3. to identify types of personal loans available from banks:	Each type of personal loan should be discussed, keeping in mind the following:
a) installment loans,	1) credit charges
b) demand loans,	2) agreement signed
c) single payment loans,	3) disability and life insurance
d) student loans,	4) different repayment plans
e) home improvement loans;	5) qualification criteria

OBJECTIVES	NOTES TO TEACHERS
4. to calculate the true cost of borrowing money;	Examples: (Current rates of interest should be used.) 1) Loan of \$10 200 at 23½% repayable over 24 months. Monthly payments are \$536.72. \$536.72 x 24 = \$12 881.52 Total interest is: \$12 881.52 - \$10 200 = \$2 681.52 True % rate = $\frac{2\ 681.52}{10\ 200}$ = 26.3% 2) Loan of \$10 000 at 13% for 36 months. Monthly payments are \$333.45. \$333.45 x 36 = \$12 004.20 Total interest is \$12 004.20 - \$10 000 = \$2 004.20 True % rate = $\frac{2\ 004.20}{10\ 000}$ = 20%
5. to identify some positive and detrimental factors involved in establishing a credit standing;	Students should be made aware that a good credit standing is very important. Possible factors to be discussed are: a) Positive factors - good character - reliability - trustworthiness - sufficient earning capacity - good credit history - permanency of employment - permanency of residence

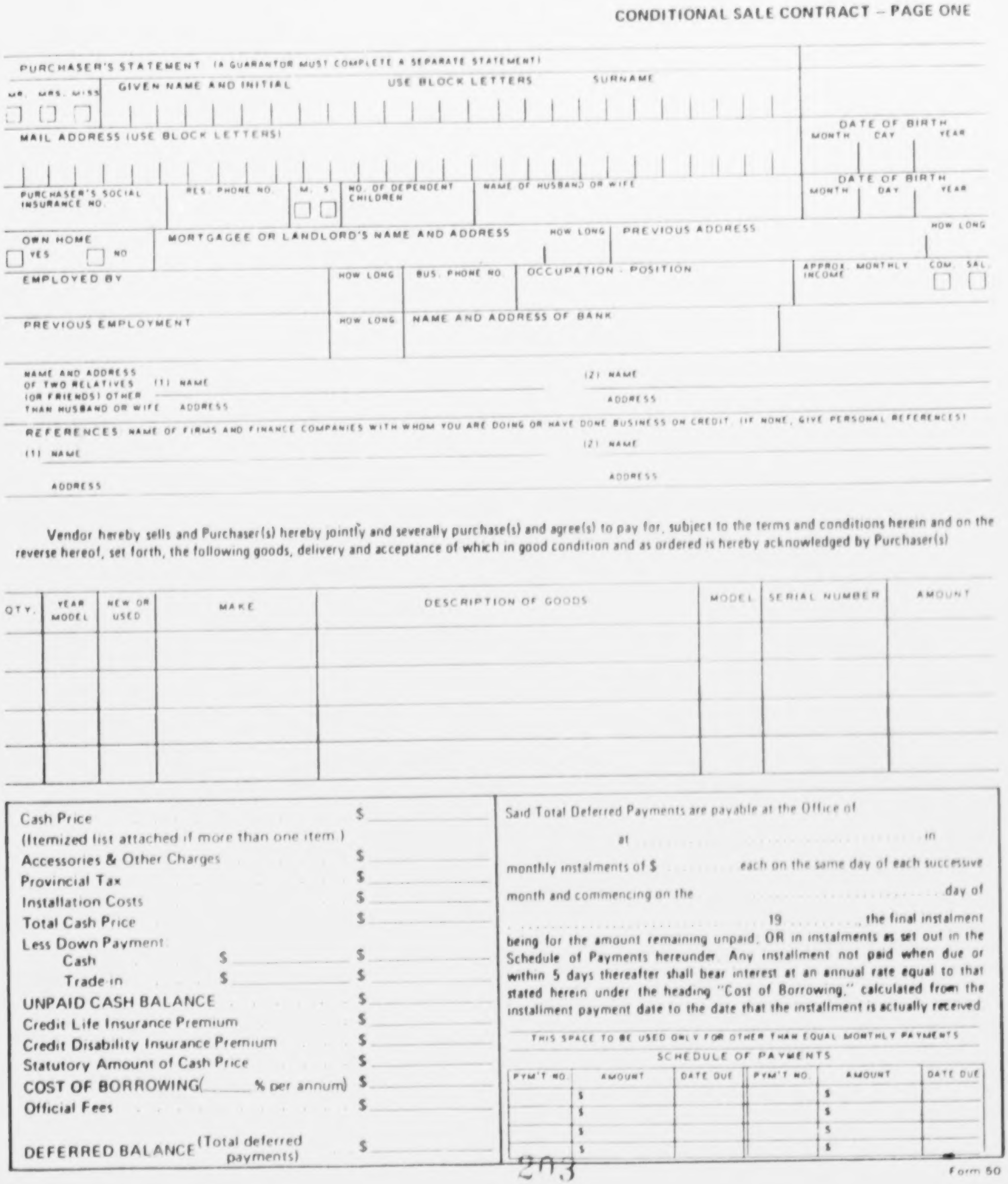
OBJECTIVES	NOTES TO TEACHERS
	b) Detrimental factors - frequent changes of jobs or residences - slow payment of obligations - delinquent obligations - charge accounts in arrears - repossessions - "N.S.F." cheques - too much credit already
6. to identify methods used to establish a good credit standing;	Make students aware that it may be difficult for a young adult to obtain credit without previous credit experience. Some ways to establish a good credit standing are: a) open an account with a retail store b) open a chequing and/or saving account c) apply for a small loan and repay it on schedule d) obtain bank, gas and department store credit cards and promptly repay the full amount e) lay away items and pay for them on a regular schedule f) use payroll deductions for savings and loan repayments, if available
7. to identify the responsibilities of the borrower in a credit transaction;	Some of the responsibilities to be discussed are: a) read and understand everything before signing on the dotted line b) never sign a contract with blank spaces c) supply accurate information to the lender d) get a copy of the written agreement e) adhere to the terms of the agreement f) contact the lender immediately if the borrower cannot meet his/her obligations g) borrow only the amount which is necessary h) compare costs of credit i) know the total cost of the credit j) accept the consequences for failure to adhere to terms of agreement

OBJECTIVES	NOTES TO TEACHERS
8. to have an understanding of the "Manitoba Personal Investigation Act";	Make students aware of the existence of this Act. Some points to be discussed with students are: a) What are Personal Reporting Agencies? b) What can be in your file? c) What cannot be in your file? d) Can you find out what is in your file? e) What are your rights to protest if some information is not accurate? f) Where do they get their information? Note: The pamphlet "You and Your Shadow" is available from: Coordinator of Consumer Communications Manitoba Consumer Affairs Division 10th Floor, Woodsworth Bldg. 405 Broadway Avenue Winnipeg, Manitoba R3C 3L6
9. to identify some of the Manitoba laws regarding credit cards;	Some of the laws are: 1) Sending unsolicited credit cards is against the law and should be reported to the Manitoba Consumer's Bureau. 2) Lost or stolen cards should be reported immediately in person or registered mail. 3) You have no obligation to pay for any unauthorized amounts charged after you have notified the company. 4) If you did not notify the company, the maximum amount of unauthorized charges you are responsible for is $50.00. 5) In case of a dispute, it is up to the issuing company to prove charges were authorized by you. Note: A kit called "On the Dotted Line" is available from: Manitoba Consumer's Bureau, 307 Kennedy Street, Winnipeg, Manitoba R3C 0V8, Telephone: Winnipeg 956-2040 or toll free 1-800-262-8844.

OBJECTIVES	NOTES TO TEACHERS
10. to calculate the cost of buying on credit.	Have students compare the cash purchase price of an object with the final cost of the object if it was financed for a certain length of time. Example: The cash price of a car is $5000. Calculate the final price of the same car if a down payment of $1000 was made and $4000 was financed at 20% for 36 months. Compare the cash price and the final cost of the car after 36 months.

TRANSCRIPT OF TRANSPARENCY #2

Example of a Conditional Sale Contract



CONDITIONAL SALE CONTRACT – PAGE ONE

PURCHASER'S STATEMENT (A GUARANTOR MUST COMPLETE A SEPARATE STATEMENT)

MR. ☐ MRS. ☐ MISS ☐ GIVEN NAME AND INITIAL USE BLOCK LETTERS SURNAME

MAIL ADDRESS (USE BLOCK LETTERS)

DATE OF BIRTH MONTH DAY YEAR

PURCHASER'S SOCIAL INSURANCE NO. | RES. PHONE NO. | M. ☐ S. ☐ | NO. OF DEPENDENT CHILDREN | NAME OF HUSBAND OR WIFE | DATE OF BIRTH MONTH DAY YEAR

OWN HOME ☐ YES ☐ NO | MORTGAGEE OR LANDLORD'S NAME AND ADDRESS | HOW LONG | PREVIOUS ADDRESS | HOW LONG

EMPLOYED BY | HOW LONG | BUS. PHONE NO. | OCCUPATION - POSITION | APPROX. MONTHLY INCOME | COM. ☐ SAL. ☐

PREVIOUS EMPLOYMENT | HOW LONG | NAME AND ADDRESS OF BANK

NAME AND ADDRESS OF TWO RELATIVES (OR FRIENDS) OTHER THAN HUSBAND OR WIFE (1) NAME ______ ADDRESS ______ (2) NAME ______ ADDRESS ______

REFERENCES NAME OF FIRMS AND FINANCE COMPANIES WITH WHOM YOU ARE DOING OR HAVE DONE BUSINESS ON CREDIT. (IF NONE, GIVE PERSONAL REFERENCES)

(1) NAME ______ ADDRESS ______ (2) NAME ______ ADDRESS ______

Vendor hereby sells and Purchaser(s) hereby jointly and severally purchase(s) and agree(s) to pay for, subject to the terms and conditions herein and on the reverse hereof, set forth, the following goods, delivery and acceptance of which in good condition and as ordered is hereby acknowledged by Purchaser(s)

QTY.	YEAR MODEL	NEW OR USED	MAKE	DESCRIPTION OF GOODS	MODEL	SERIAL NUMBER	AMOUNT

Cash Price		$ ______
(Itemized list attached if more than one item.)		
Accessories & Other Charges		$ ______
Provincial Tax		$ ______
Installation Costs		$ ______
Total Cash Price		$ ______
Less Down Payment		
Cash	$ ______	$ ______
Trade-in	$ ______	$ ______
UNPAID CASH BALANCE		$ ______
Credit Life Insurance Premium		$ ______
Credit Disability Insurance Premium		$ ______
Statutory Amount of Cash Price		$ ______
COST OF BORROWING(______% per annum)		$ ______
Official Fees		$ ______
DEFERRED BALANCE (Total deferred payments)		$ ______

Said Total Deferred Payments are payable at the Office of .... at .... in .... monthly instalments of $ .... each on the same day of each successive month and commencing on the .... day of .... 19 ...., the final instalment being for the amount remaining unpaid, OR in instalments as set out in the Schedule of Payments hereunder. Any installment not paid when due or within 5 days thereafter shall bear interest at an annual rate equal to that stated herein under the heading "Cost of Borrowing," calculated from the installment payment date to the date that the installment is actually received.

THIS SPACE TO BE USED ONLY FOR OTHER THAN EQUAL MONTHLY PAYMENTS

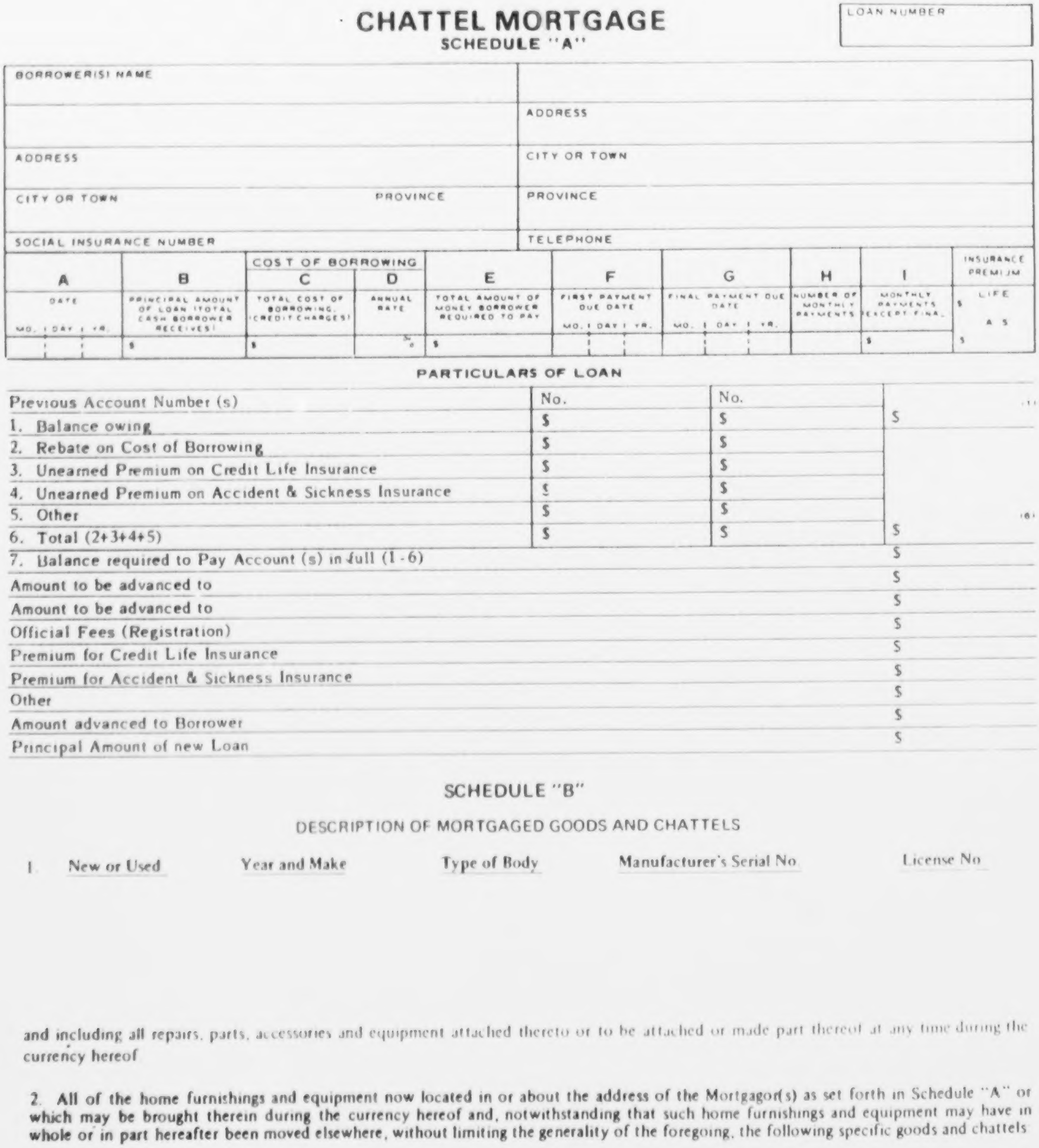
SCHEDULE OF PAYMENTS

PYM'T NO.	AMOUNT	DATE DUE	PYM'T NO.	AMOUNT	DATE DUE
	$			$	
	$			$	
	$			$	
	$			$	

Form 50

TRANSCRIPT OF TRANSPARENCY #3

Example of a Chattel Mortgage



CHATTEL MORTGAGE

SCHEDULE "A"

LOAN NUMBER

BORROWER(S) NAME	
	ADDRESS
ADDRESS	CITY OR TOWN
CITY OR TOWN PROVINCE	PROVINCE
SOCIAL INSURANCE NUMBER	TELEPHONE

A	B	COST OF BORROWING C	D	E	F	G	H	I	INSURANCE PREMIUM
DATE MO. | DAY | YR.	PRINCIPAL AMOUNT OF LOAN (TOTAL CASH BORROWER RECEIVES)	TOTAL COST OF BORROWING (CREDIT CHARGES)	ANNUAL RATE	TOTAL AMOUNT OF MONEY BORROWER REQUIRED TO PAY	FIRST PAYMENT DUE DATE MO. | DAY | YR.	FINAL PAYMENT DUE DATE MO. | DAY | YR.	NUMBER OF MONTHLY PAYMENTS	MONTHLY PAYMENTS (EXCEPT FINAL)	$ LIFE A S
	$	$	%	$				$	$

PARTICULARS OF LOAN

Previous Account Number (s)	No.	No.	
1. Balance owing	$	$	$
2. Rebate on Cost of Borrowing	$	$	
3. Unearned Premium on Credit Life Insurance	$	$	
4. Unearned Premium on Accident & Sickness Insurance	$	$	
5. Other	$	$	
6. Total (2+3+4+5)	$	$	$
7. Balance required to Pay Account (s) in full (1-6)			$
Amount to be advanced to			$
Amount to be advanced to			$
Official Fees (Registration)			$
Premium for Credit Life Insurance			$
Premium for Accident & Sickness Insurance			$
Other			$
Amount advanced to Borrower			$
Principal Amount of new Loan			$

SCHEDULE "B"

DESCRIPTION OF MORTGAGED GOODS AND CHATTELS

1.	New or Used	Year and Make	Type of Body	Manufacturer's Serial No	License No

and including all repairs, parts, accessories and equipment attached thereto or to be attached or made part thereof at any time during the currency hereof

2. All of the home furnishings and equipment now located in or about the address of the Mortgagor(s) as set forth in Schedule "A" or which may be brought therein during the currency hereof and, notwithstanding that such home furnishings and equipment may have in whole or in part hereafter been moved elsewhere, without limiting the generality of the foregoing, the following specific goods and chattels

TRANSCRIPT OF TRANSPARENCY #1

Examples of Promissory Notes

PROMISSORY NOTE ON DEMAND

______________________________ 19 ___

On demand for value received,

__________ promise to pay, at the office of the ⑥, here,

to ______________________________ or order, the sum

of ______________________________ /100 Dollars ($ ______ . ______)

with interest, payable on the last day of each month, at the rate of __________ per cent per annum until maturity, and __________ per cent per annum after maturity.

____

PROMISSORY NOTE ON TIME (payments)

PROMISSORY NOTE - PERSONAL MONTHLY-INSTALMENT LOAN

Number __________ $ __________

__________ (Branch) __________ (Date)

For value received, I hereby promise to pay to the order of here, the amount of ______________________________ /100 Dollars as follows: __________ /100 Dollars on the __________ day of __________ 19___, and on the __________ day of each and every following month, up to the __________ day of __________ 19___ inclusive, and __________ /100 Dollars on the __________ day of __________ 19___.
Should I fail to make any of the payments herein mentioned, I hereby undertake to pay interest at the rate of per cent per annum on each such defaulted payment from the date of default until complete payment thereof. In case of non-payment, at maturity, of the said instalments, or any one of them, the whole of the balance of this note shall forthwith become due and payable, at the option of said with interest at the rate of per cent per annum on the whole balance, capital and interest, in arrears.

Amount of loan $ __________
Borrowing cost
(Interest at the rate of % per annum) $ __________
Amount of note $ __________
F. 822-A-1 (Rev. 9-73)

____
BORROWER'S SIGNATURE

____
ADDRESS TELEPHONE

Resources:

Films: Consumer Fraud Series. Library, Manitoba Department of Education.
"Your Credit Is Good" - 3446
"I'll Only Charge You For The Parts" - 3448
"Bait, Bite, and Switch" - 0768
"The $6 000 000.00 Sell"

Kit: Consumer at Large. Coronet Instructional Media Ltd., 200 Steelcase Road East, Markham, Ontario L3R 1G2
"Budget Blues"
"Everyday Shopping Savvy"
"Buying The Big Ones"
"Shopping For Services"
"Cash or Credit"
"Nurturing a Nest Egg"

Booklets:

1. "A Working Guide To Money Management," Department of Health, Home Economics Directorate, 1980.
2. "Credit-Wise - Guide to Consumer Credit". From: Canadian Bankers Association, P.O. Box 282, Toronto Dominion Centre, Toronto, Ontario M5K 1K2

Books:

1. Bolster, L.C. et al., Consumer and Career Mathematics (Toronto, ON: Gage Publishing Co. 1980), Unit 2.
2. Cutting, Robert J., Math You Need Book 2 (Scarborough, ON: Nelson Canada Ltd., 1981), Unit 8.
3. Department of Education, Exercises to Accompany Mathematics 201 (Winnipeg, MB: The Author, 1982), pp 30-35. Available at Manitoba Text Book Bureau.
4. Kravitz, W., and Brant, V., Consumer Related Mathematics (Toronto, ON: Holt, Rinehart, Winston, 1976), Chapter 8.

OBJECTIVES	NOTES TO TEACHERS
	5. Itzkowitz, M. and Richard, T., Mastering Essential Mathematics Skills (River Falls, IL: Laidlaw Brothers, 1980), pp 97-100.
	6. Price, J. et al., Mathematics For The Real World, Canadian Edition (Weston, ON: Merrill Publishing, 1979), pp 67-68, 74-84.

C.VI. RATIO, RATE AND PERCENT (2 MODULES)

This is a new topic for many students. Research indicates that students are unable to deal successfully with ratios and rates until they reach the stage of formal operations. For these reasons, teachers are advised to introduce this module using examples from the students' environments. Encourage students to find examples of ratios/rates, such as orange juice mixtures, anti-freeze solutions, map scales, scale drawings/blueprints, gear ratio, compression ratio, automobile fuel consumption, concrete mixtures, pitch of roof, batting averages, etc.

OBJECTIVES	NOTES TO TEACHERS
A. RATIOS	
The student should be able:	
1. to recognize and to express ratios;	A ratio is a comparison of two like quantities. Students should be familiar with the various techniques or notations for expressing a ratio. For example, the ratio of 3 to 5 can be expressed as: 3 to 5, 3:5 or $\frac{3}{5}$. Expose students to many practical applications involving ratios. Examples could include: gear ratios, pulley ratios, probabilities and odds, the use of ratios in newspapers and magazines, grade or slope, etc. Depending upon student ability, teachers may wish to compare the perimeter or area of different size snapshots in order to create ratios.
2. to interpret three-term ratios;	Applications could include fertilizer ingredients, win:loss:tie comparisons in sports, goals:assists:points for hockey players, etc. Note: The chemical amounts in fertilizer are given by a 3-term ratio indicating the amount of nitrogen, phosphate (phosphorus) and potash (potassium). If a fourth number is included, it represents sulphur and is identified last in the sequence.

Examples:

1) Translate 18:4:16 into its component parts.

Answer: 18 parts nitrogen, 4 parts phosphate, 16 parts potash.

2) Indicate the correct 4-term ratio for a fertilizer containing 16 parts nitrogen, 14 parts sulphur, 0 parts potash and 20 parts phosphate.

Answer: 16:20:0:14

3. to reduce ratios;

Students need considerable practice in reducing ratios and in identifying those ratios which are in lowest terms from a given list of ratios.

Examples:

1) The amount of gold in jewelry is measured in karats. Pure gold is represented by 24 karats. Express the gold content as a ratio reduced to lowest terms for a 14-karat gold chain.

Solution: $\frac{14}{24} = \frac{7}{12}$

2) From the following list of ratios, identify those which have not been reduced to lowest terms:

$\frac{2}{3}, \frac{5}{10}, \frac{4}{30}, \frac{10}{8}, \frac{5}{8}, \frac{5}{2}.$

Answer: $\frac{5}{10}, \frac{4}{30}, \frac{10}{8}.$

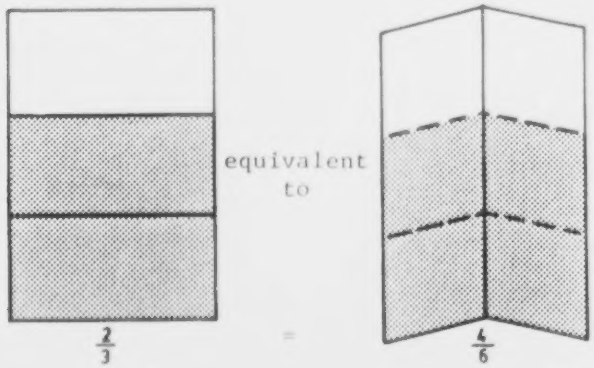
Note: It is suggested that students learn to distinguish between $\frac{5}{2}$, which is a reduced ratio in improper form, and 2½ or 2.5, which is its mixed number equivalent.

4. to determine equivalent ratios;

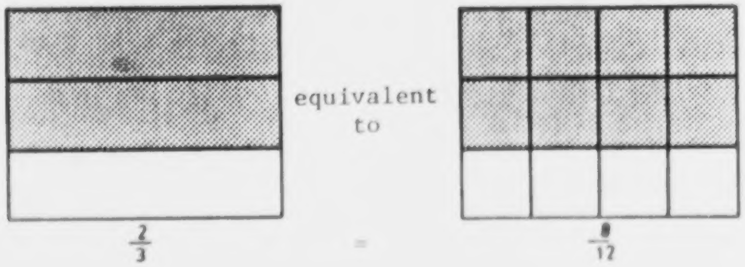
This is a most important concept, necessary for future development in percent and proportion. The concept of equivalent ratios parallels that of equivalent fractions and should be developed with concrete materials prior to using LCM (lowest common multiple), Property of One, or other abstract formal methods.

Material representations involving equivalent ratios could include:

1. Paper folding



2. Geometric figures drawn on graph paper or on a geoboard



5. to use equivalent ratios to solve problems involving:

   a) scale drawings,

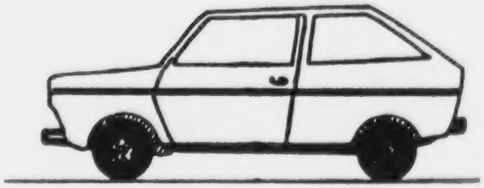
Note that the idea is to have students working with proportion without terminology.

In a scale drawing, the first term corresponds to the measure on the scale diagram while the second term corresponds to the measure on the actual object.

Examples should proceed from those in which the scale diagram is smaller than the actual object, to those in which the scale diagram is larger than the actual object, to those in which the scale is given in mixed units (i.e., 1 cm represents 50 m).

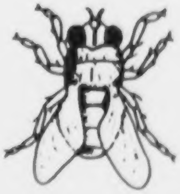
Examples:

1) If the scale is 1:60 in the accompanying sketch, what is the "wheelbase" for the car?



2. Scale 2:1

Determine the overall length for the housefly pictured on the right.

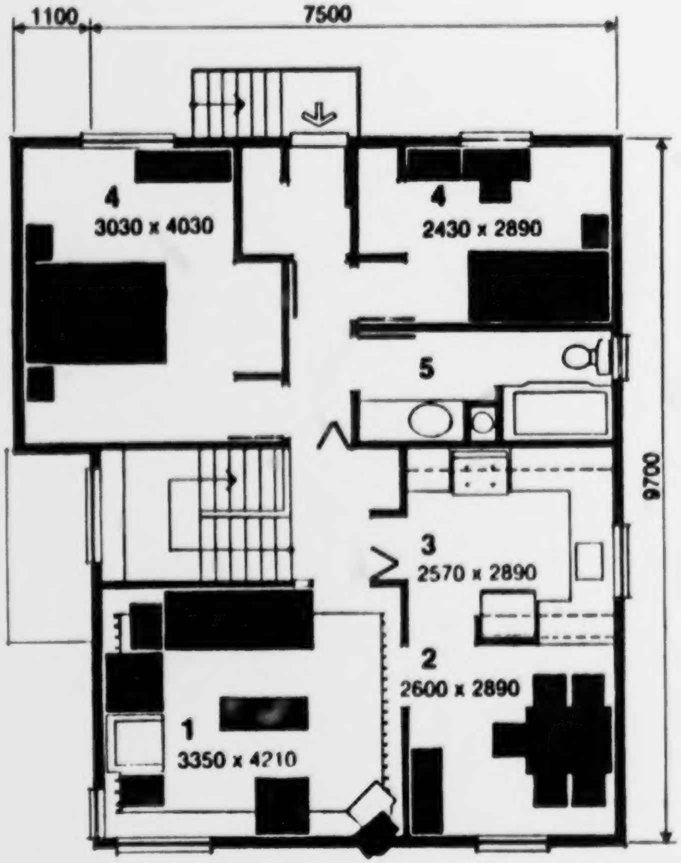


3) Floor plan with 1 cm = 1000 mm
Courtesy: Central Mortgage and Housing Corp.

All dimensions are given in millimetres unless otherwise indicated.

Identification of Rooms and Spaces

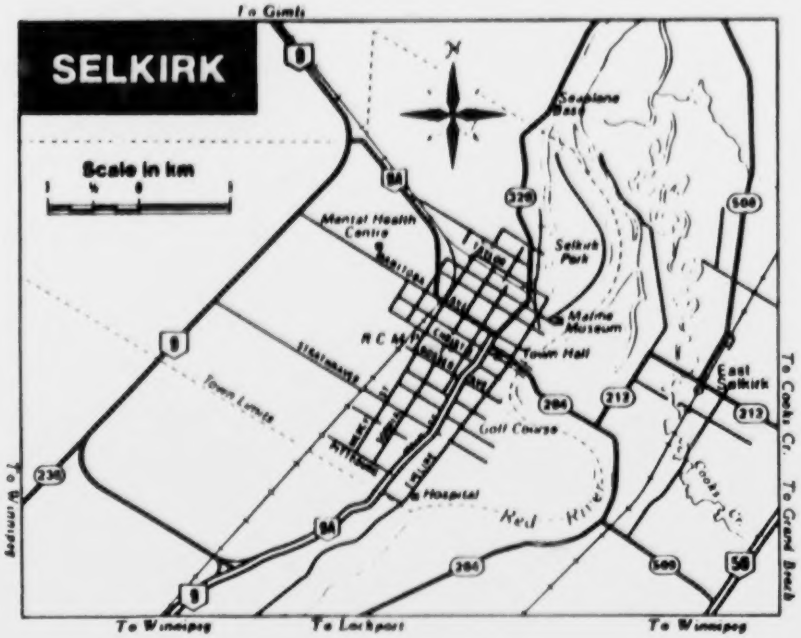
1. Living
2. Dining
3. Kitchen
4. Bedroom
5. Bathroom
6. Washroom
7. Future Bedroom
8. Storage
9. Utility
10. Carport


1100
7500
9700
4
3030 x 4030
4
2430 x 2890
5
3
2570 x 2890
2
2600 x 2890
1
3350 x 4210

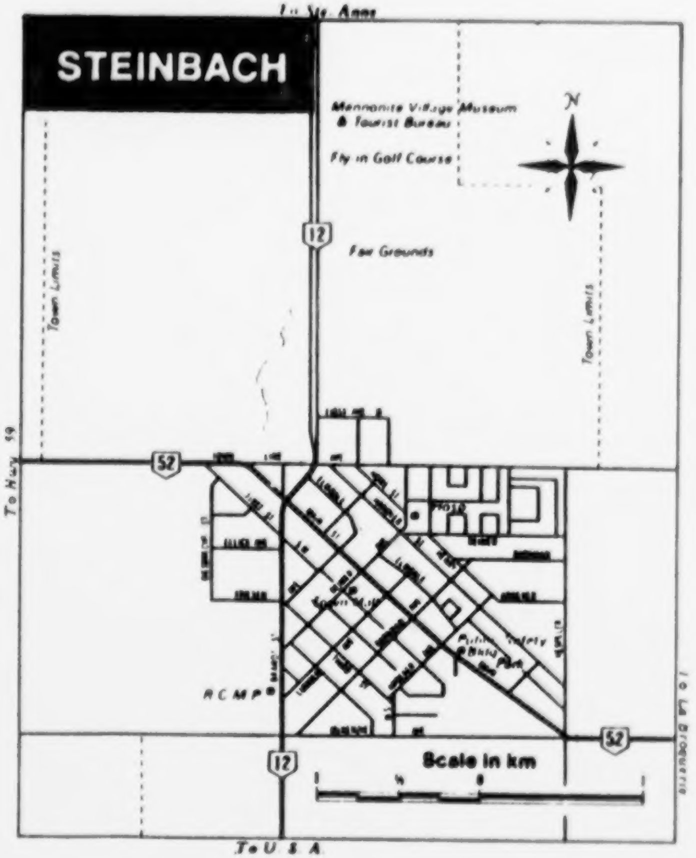
b) map scales.

On a map, the scale may be given as a ratio in which the first term of the ratio represents distance on the map and the second term represents distance on the earth's surface, or as a legend such as:

1) Use the scale on the following map to find the approximate distance from the corner of Pittsburg and Mercy to the corner of Taylor and Sophia.


To Gimli
SELKIRK
Scale in km
½
9
9A
Seaplane Base
320
Mental Health Centre
Selkirk Park
508
Marine Museum
R C M P
Town Hall
Town Limits
East Selkirk
204
212
Golf Course
230
To Winnipeg
Hospital
Red River
Cooks Cr.
509
59
To Cooks Cr.
To Grand Beach
To Winnipeg
To Lockport
To Winnipeg

2. Using the scale on the accompanying map of Steinbach, have students determine the length of Hanover Street.


To Ste. Anne
STEINBACH
Mennonite Village Museum & Tourist Bureau
Fly in Golf Course
N
12
Fair Grounds
Town Limits
Town Limits
To Hwy 59
52
RCMP
Public Safety Build.
Park
12
52
Scale in km
½
0
To La Broquerie
To U.S.A.

OBJECTIVES	NOTES TO TEACHERS

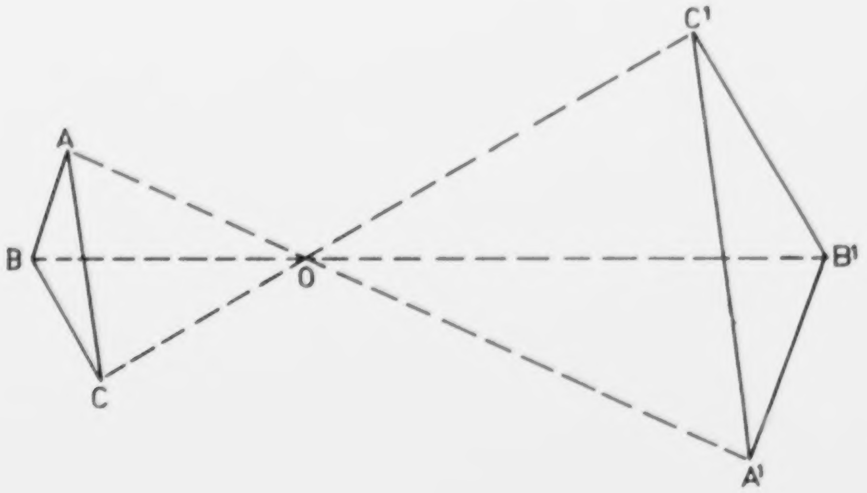
ENRICHMENT

Centres of Enlargement/Reduction

Copying, using a centre of enlargement/reduction, can be used to teach students about measurement, scale, ratio, proportion and similarity.

Examples:

1) Make a copy of a drawing of the following figure using a scale ratio of $\frac{2}{1}$ (i.e., double the linear dimensions of $\triangle ABC$).



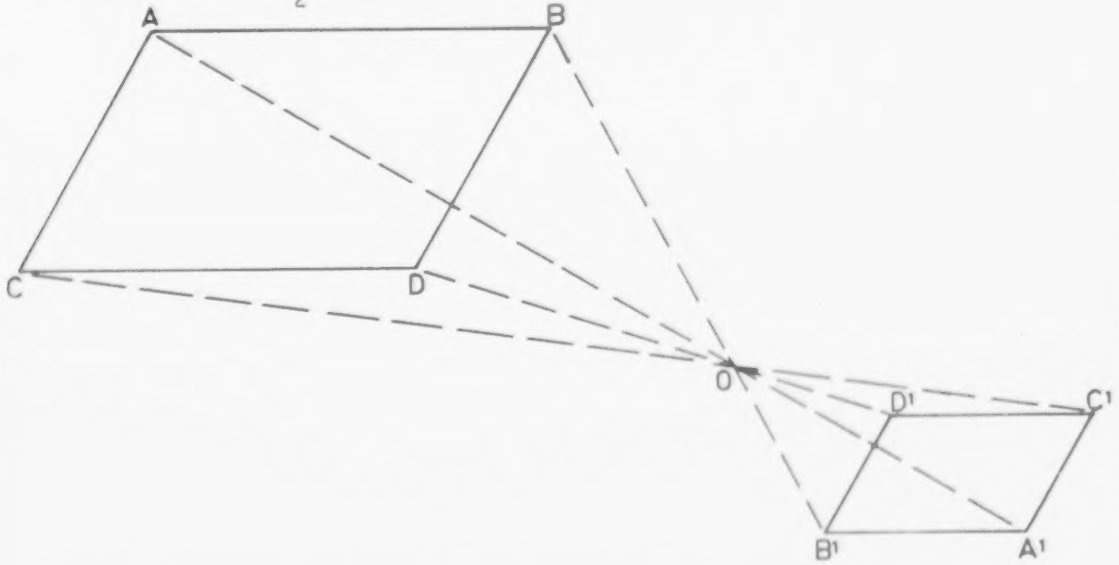
OBJECTIVES

NOTES TO TEACHERS

Procedure:

Select O as the centre of enlargement. Note that O can be any point but it is suggested that O be outside $\triangle ABC$. Measure the distance from A to O and, since the scale ratio is $\frac{2}{1}$, extend AO to A^1 such that the length of A^1O is twice that of AO. Similarly, measure lines from vertices B and C through the centre of enlargement and extend them until they are double in length. Join $A^1 - B^1 - C^1$ to obtain a new triangle whose sides are 2 times as long as in the original triangle.

2. Scale ratio of $\frac{1}{2}$ (reduction).



Note: Teachers may wish to show students the grid approach to enlarging or reducing a given figure.

OBJECTIVES	NOTES TO TEACHERS
B. RATES	
The student should be able:	
1. to recognize and to express rates;	A rate is a comparison of two unlike quantities. Examples could include automobile fuel consumption (L/100 km), heartbeats, rates of speed, medicine dose/body mass, density (mass/volume), batting averages, unit pricing, reading rates, etc. Examples: Consider rates of speed. Encourage students to obtain some world record from the Guiness Book of World Records. For example, track results, skipping, sit-ups or push-ups, reading aloud, or winking could be used. Convert the record to its equivalent rate per second or rate per minute and then have each student conduct an experiment to determine his/her rate for the same activity. Various comparisons can be made.
2. to calculate equivalent rates;	Students should realize that rates can be worked with in the same manner as ratios.

OBJECTIVES	NOTES TO TEACHERS
3. to use equivalent rates to solve problems relating to:	
a) comparison shopping,	Unit price is the usual method used to determine a "better buy" in comparison shopping. Example: Which is the better buy, 62 L of gasoline for \$25.92 or 24 L for \$9.67? Solution: 62 L costs \$25.92 — 24 L costs \$9.67 1 L costs $\frac{25.92}{62}$ = \$.418 or 41.8¢ — 1 L costs $\frac{9.67}{24}$ = \$.4029 or 40.3¢ 24 L for \$9.67 is a better buy in terms of the price per litre.
b) fuel economy.	Fuel economy is expressed in litres per 100 kilometres or L/100 km. Many students will have difficulty with this and it is suggested that students first determine L/km and then convert this to L/100 km.

Example:

On a particular trip, a half-tonne truck drives 350 km on 42 L of fuel. Express the fuel economy of the truck in L/100 km.

Answer: 350 km for 42 L

1 km takes $\frac{42}{350} = 0.12$ L

100 km takes 100 x 0.12 = 12 L

The truck's fuel economy is 12 L/100 km.

Note: Students should be given required formula such as:

i) distance travelled = speed x time taken

ii) speed = $\frac{\text{distance travelled}}{\text{time taken}}$

iii) time taken = $\frac{\text{distance travelled}}{\text{speed}}$

Other examples could include car rental rates, heartbeat rates, precipitation record rates, production rates, crop yields, etc.

OBJECTIVES	NOTES TO TEACHERS

C. PERCENT

The student should be able:

1. to determine the percent of a diagram which is/is not shaded;

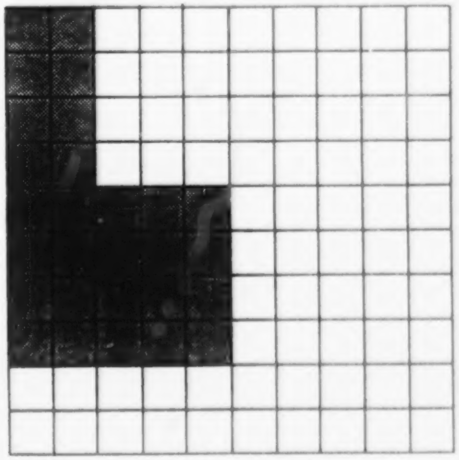
"Percent" means per hundred or parts of one hundred.

Considerable time may be needed on this task in order for students to conceptualize the definition of percent.

Begin with diagrams containing 100 sections, some of which are shaded, and proceed to those with other than 100 sections/divisions after objective #2 has been dealt with.

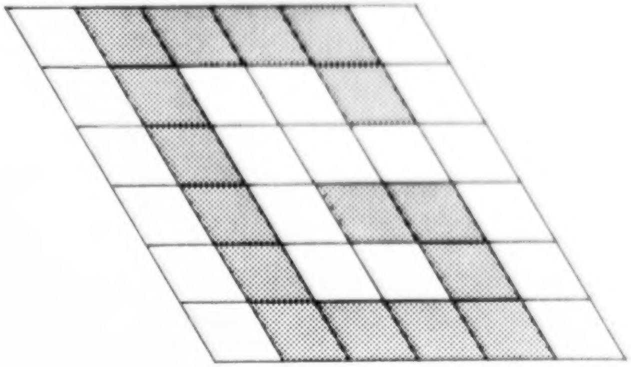
Examples:

1. Give the percent of the following figure which is shaded or is not shaded.



$\frac{28}{100}$ = 28% is shaded.

2) Give a percent to indicate how much of the following figure is or is not shaded:



Answer: Shaded $\frac{16}{36} = 0.44\overline{4}$ or 44.4%

The use of calculators is assumed.

2. to convert a percent to a decimal fraction and vice-versa;

Note: Teachers may wish to have students use the % key on a calculator, in which case it is not necessary to convert percents to their decimal fraction equivalents.

OBJECTIVES

NOTES TO TEACHERS

3. to solve percent problems involving finding the percent of a number.

Emphasize applications of percent. Spend considerable time on practical problems of salary increase or decrease, discount, sales tax, interest, commission, etc.

Examples:

1) Find 40% of 60

Answer: 40% x 60 = 0.40 x 60 = 24

2) Find 38% of 12

Answer: 38% x 12 = 0.38 x 12 = 4.56

3) A student is to receive a 10% wage increase after 6 months on the job. If the starting wage is $4.80 per hour, what is the new wage after 6 months?

Solution: 10% of $4.80 = 10% x 4.80
= 0.10 x 4.80
= 0.48 or 48¢ increase

The new wage is $4.80 + $0.48 = $5.28.

4) A record album regularly sells for $9.95. The store advertises a sale with a discount of 25%. How much is saved during the sale? What is the sale price?

Solution: 25% of $9.95 = 0.25 x 9.95
= $2.49 saving

The sale price is $9.95 - $2.49 = $7.46.

OBJECTIVES | NOTES TO TEACHERS

For enrichment, teachers may wish to cover percent problems of the following type: "4000 is what % of 5000?"

Such examples can lead to discussion/problems relating to cost of living, percent rate of increase in salaries, etc.

Resources:

1. Cutting, Robert J., Math You Need, Book 1 (Scarborough, ON: Nelson Canada Ltd., 1981), Unit 6.
2. -------, Math You Need, Book 2 (Scarborough, ON.: Nelson Canada Ltd., 1982), pp 63-67.
3. Department of Education, Exercises to Accompany Mathematics 101 (Winnipeg, MB: The Author, 1981), Option V. Available from the Manitoba Text Book Bureau.
4. -------, Exercises to Accompany Mathematics 201 (Winnipeg, MB: The Author, 1982), pp 77-87. Available from the Manitoba Text Book Bureau.
5. Ebos, F. and Zolis, P., Math Matters Book 1 (Scarborough, ON: Nelson Canada Ltd., 1981), Unit 6.
6. Price, J. et al., Mathematics For The Real World, Canadian Edition (Weston, ON: Merrill Publishing, 1979). See Index.

Op.I. IMPROVING YOUR RESIDENCE (½ MODULE)

This module should be done in project form. Students are required to complete two projects similar to those shown in this unit.

OBJECTIVES	NOTES TO TEACHERS
The student should be able to complete at least two of the proposed projects.	Project 1 - Students should be able to estimate the cost of installing a driveway. Note: See - Consumer and Career Mathematics, p. 246 (Math 204). Project 2 - Students should be able to determine the size of an air conditioner necessary to cool a particular area, using factors related to heat sources. Note: See - Consumer and Career Mathematics, p. 251. Project 3 - Students should be able to estimate the cost of lumber and other materials needed for building a certain project (example: tool shed). Project 4 - Students should be able to estimate the total cost of installing a swimming pool. Note: See - Consumer and Career Mathematics, pp. 352-353. Project 5 - Students should be able to determine the material needed and the cost to install a solar heating system for a house or a swimming pool. Project 6 - Students should be able to determine what is necessary to build a super-insulated house.

Points to be considered:

a) Attic insulation of R-60
b) Basement insulation of R-20
c) Walls = 30 cm thick with 15 cm spaces between studs allowing for R-44 insulation
d) Triple-paned windows
e) Magnetic weather-stripped steel doors, rated R-12
f) Air-to-air heat exchanger which brings in warmed-up outside air
g) No windows on north side and large picture windows on south side.
h) Continuous hole-free vapour barrier

Project 7 - Students should be able to compare the cost of heating a home using oil, wood, natural gas or electricity.

The table provided below can be used to compare these costs. To use the table, locate the price of the fuel you are now using. The numbers which line up horizontally with the price in the other columns give the break-even price for the same amount of heat from other fuels.

Example:

For an oil furnace burning oil at 29¢ per litre, the break-even price for wood would be $130 per cord, 4.5¢ per kwh for a resistance electrical furnace and 25.6¢ per cubic metre for a regular natural gas furnace. Combinations of these methods of heating should be discussed.

COSTS FOR THE SAME AMOUNT OF HEAT FROM VARIOUS SOURCES					
NATURAL GAS		ELECTRIC ENERGY		HEATING OIL	WOOD
Standard Furnace Per m^3	High Efficiency Furnace Per m^3	Resistance Heaters Per Kwh	Heat Pump Per Kwh	Per Litre	Per Cord
23.0¢	37.9¢	4.0¢	5.8¢	27.0¢	$116.50
24.7	40.3	4.4	6.2	29.0	125.50
26.5	43.6	4.7	6.6	31.0	134.50
28.3	46.3	5.0	7.1	33.0	143.50
30.0	49.1	5.3	7.5	35.0	152.50
31.7	52.4	5.6	8.0	37.0	161.50
33.6	55.2	5.9	8.4	39.0	170.50
35.3	57.9	6.2	8.9	41.0	179.50
37.1	60.7	6.5	9.3	43.0	188.50
38.8	63.5	6.8	9.7	45.0	197.50

Figures based on the following seasonal efficiencies: oil furnace, 60%; conventional gas furnace, 55%; high-efficiency condensing gas heat furnace, 90%; electrical resistance heat, 100%; heat pump, 143%; wood stove, 45%.

OBJECTIVES	NOTES TO TEACHERS
	Project 8 - Teachers may select another project which would be appropriate to the topic "Improving Your Residence".

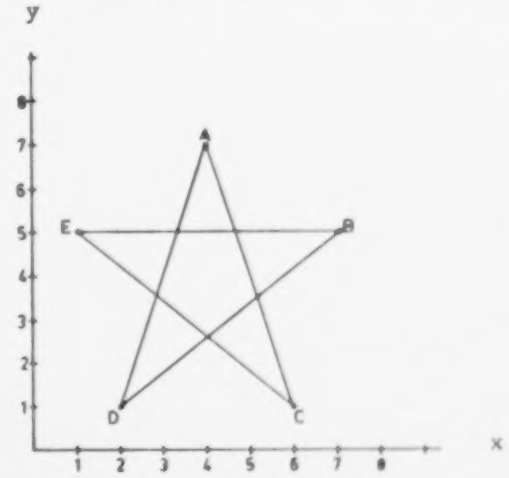
Resources:

1. Bolster, L.C. et al., Consumer and Career Mathematics (Toronto, ON: Gage Publishing Co, 1978).
2. Energy Information Centre, Department of Energy and Mines, Colony Square, 500 Portage Avenue, Winnipeg, MB R3C 0V8.
3. Keedy, M. et al., Applying Mathematics - A Consumer/Career Approach (Menlo Park, CA: Addison-Wesley Publishing Company, 1983), pp. 192-194 and 202.

Op.II. ALGEBRAIC APPLICATIONS - COORDINATE GRAPHING (½ MODULE)

The study of coordinate graphing in this module will be limited to quadrant 1 and will be done through experiments rather than algebraic applications.

OBJECTIVES	NOTES TO TEACHERS
The student should be able:	
1. to demonstrate an understanding of terminology involved in a coordinate system;	Terms that students should learn: 1) coordinate axes labelled x and y 2) coordinates or ordered pairs 3) origin
2. to describe the location of a given point;	Write the ordered pair where x represents the coordinate found on the horizontal axis and y represents the coordinate found on the vertical axis.
3. to plot points given the ordered pair;	Note: Coordinates can be given so that a picture is formed when the points are connected. Have students graph and label the following points. A(4,7); B(7,5); C(6,1); D(2,1); E(1,5). Join AC, AD, EB, EC, and BD. What is pictured?

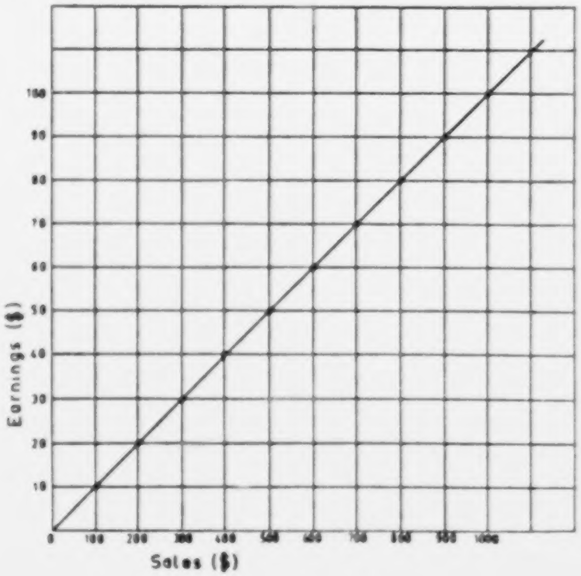


4. to construct, using data, graphs of direct relationships involving a straight line passing through the origin;

Examples:

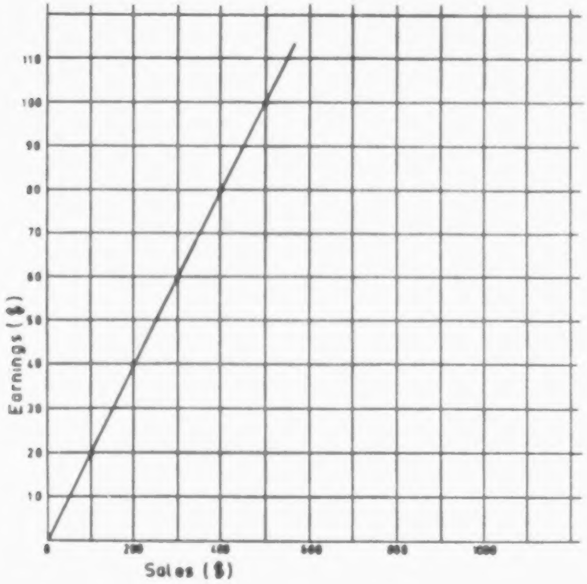
1) Straight commission earnings. A carpet salesman receives $10 for every $100 worth of carpet he sells. Plot his earnings against the dollar amount of sales after completing the following table.

Sales	$0.00	$100.00	$200.00	$300.00	$400.00	$700.00
Earnings	$0.00	$ 10.00	$ 20.00	$ 30.00	$ 40.00	$ 70.00



What effect does changing the rate of commission have on the graph, i.e., if the salesman's commission rate is changed to $20 per $100 he sells? Have students complete a table and graph the results.

Sales	0	100	200	300	400	700
Earnings	0	20	40	60	80	140

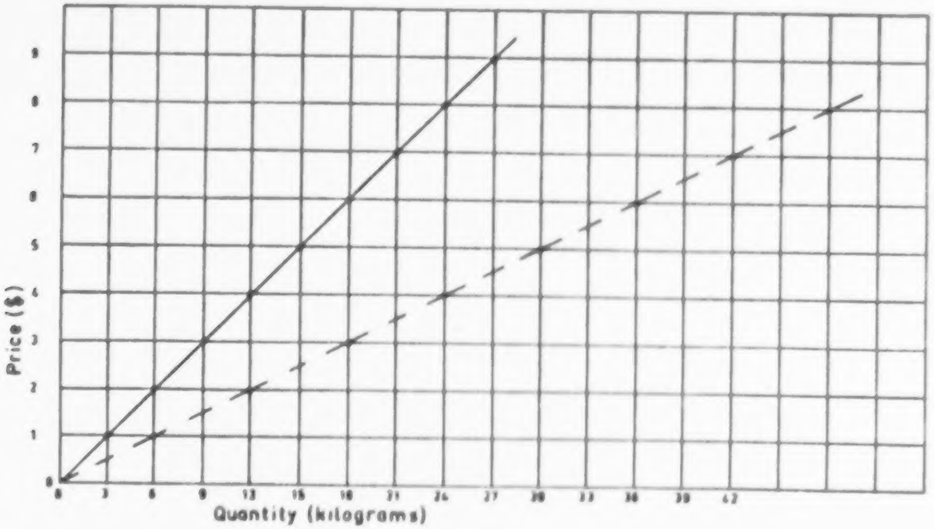


Increasing the rate of commission has what effect on the graph?
Decreasing the rate of commission has what effect on the graph?

2) Price vs. Quantity. At the grocery store you can buy 3 kg of potatoes for $1.00. Plot the amount bought against the price. (—) What effect does having a sale, 6 kg for $1.00, have on the graph? (- - -)

Quantity	0	3	6	9	12	18
Price $	0	1.	2.	3.	4.	6.

Quantity	0	6	12	18	24	36
Price $	0	1.	2.	3.	4.	6.



Note: If the rate increases, the line becomes steeper and if the rate decreases, the slope of the line becomes more gradual.

We have equations of the form $y = mx$. However, we do not expect students to write equations of lines.

OBJECTIVES	NOTES TO TEACHERS

5. to define slope.

Steepness of a straight line is given by the rise over run. Other names for slope include gradient (roads), pitch (roofs), steepness (stairways and ramps) and rate of change (business and vocational examples).

$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$

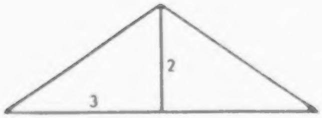
Examples:

1) Road Gradient Gradient = slope = $\frac{\text{rise}}{\text{run}}$

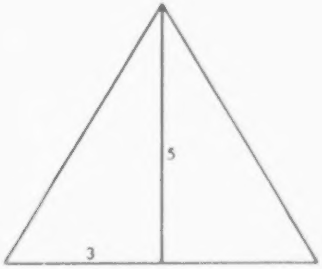
level road (no slope)

road with a slope (rise)

2) Roof Pitch Pitch = slope = $\frac{\text{rise}}{\text{run}}$



slope = $\frac{2}{3}$



slope = $\frac{5}{3}$

Resources:

1. Ebos, F. and Zolis, P., Math Matters Book 2 (Scarborough, ON: Nelson Canada Ltd., 1982).
2. Keedy, M. et al., Applying Mathematics - A Consumer/Career Approach (Menlo Park, CA: Addison-Wesley Publishing Company, 1983), pp 192-194 and 202.
3. Price, J., et al., Mathematics For The Real World: Canadian Edition, (Weston, ON: Merrill Publishing, 1979), pp 297-300, 301-303.
4. Rand, Ken, Point - Counter Point. Available from: Setsco Educational Ltd., 567 Clarke Road, Coquitlam, B.C. V3J 3X4.

Op.III. BUSINESS MANAGEMENT (½ MODULE)

A knowledge of how goods are marketed is important to people in the shipping/receiving and clerical departments of a company. This unit is an introduction to the mathematics of invoices and discounts.

OBJECTIVES	NOTES TO TEACHERS
The student should be able:	
1. to identify methods of marketing goods;	Methods are: 1) producer - consumer (farmer selling produce in his roadside stand) 2) producer - retailer - consumer (farmer sells to local stores who sell to consumer) 3) producer - wholesaler - retailer - consumer (blue jeans are manufactured, shipped to wholesalers, retailer buys them and sells to consumer) 4) consignment (owner of car gets dealer to sell it for him)
2. to explain the method by which a business buys its products;	Steps are: 1) requisition to purchasing department 2) purchase order to supplier 3) supplier ships goods and invoice
3. to interpret payment date on an invoice;	Examples: 1) If invoice terms are 2/10, n/45 and invoice date is April 1, then a 2% discount is allowed if paid by April 11 (April 1 + 10 days). The invoice must be paid by May 16 (April 1 + 45 days). 2) If invoice terms are 1/10, n/30 E.O.M. (E.O.M. is End of Month), and the invoice date is May 12, a 1% discount is allowed if the invoice is paid by June 10 (10 days from end of May). The invoice must be paid by June 30 (30 days from end of May).

4. to calculate discount and discount series on an invoice;

Example:

Calculate net amount and total discount on an invoice of $700 less discounts of 10% and 5%.

$700 x 10% = $70

$700 - $70 = $630

$630 x 5% = $31.50

$630 - $31.50 = $598.50

Total discount = $70 + $31.50

= $101.50

This answer differs from taking straight 15% ($700 x 15% = $105).

Note: Discount series can be calculated in any order.

ENRICHMENT

A more difficult problem would be the following:

If the consignor wants $1500 for his car and the dealer wants a 10% commission, what is the lowest price for which the car can sell?

The $1500 represents 90% so $\frac{\$1500}{0.90}$ = $1666.67 is the lowest price for which the car can be sold.

OBJECTIVES | NOTES TO TEACHERS

5. to define F.O.B. terms;

F.O.B. means "free on board". When the seller agrees to pay the freight charges, the shipping terms on the invoices are designated as F.O.B. destination.

When the buyer agrees to pay the freight charges, the shipping terms on the invoice are designated as F.O.B. shipping point.

6. to calculate total cost given an invoice form.

Example: Find the total cost to the buyer if the following invoice is paid June 9. The freight charges from Winnipeg to Dauphin are $16.50. The rates of trade discount are 10% and 5%.

SWEETS UNLIMITED WHOLESALE CANDY DISTRIBUTORS
WINNIPEG, MANITOBA

Sold To: Black's Candy Shop Dauphin, Manitoba

F.O.B. Winnipeg

Date: June 1, 19__
Terms 2/10, n/30

QUANTITY	DESCRIPTION	UNIT PRICE	AMOUNT
48 boxes	Black Magic Chocolates	3.20	153.60
24 boxes	Dairy Mix Chocolates	2.75	66.00
96 boxes	Turtles Chocolates	2.90	278.40
			498.00

OBJECTIVES	NOTES TO TEACHERS

Have students use the following three steps to calculate the solution.

1. Net Price

   $498 x 10% = $49.80
   $498 - $49.80 = $448.20
   $448.20 x 5% = $22.41
   $448.20 - $22.41 = $425.79

2. Less discount of 2%
   $425.79 x 2% = $8.52
   $425.79 - $8.52 = $417.27

3. Plus transportation charge
   $417.27 + $16.50 = $433.77
   This is the total cost to the buyer.

Resources:

1. Ebos, F., and Zolis, P., Math Matters Book 1 (Scarborough, ON: Nelson Canada Ltd., 1983), Unit 4.
2. -------, Math Matters Book 2 (Scarborough, ON: Nelson Canada Ltd., 1983), pp 139, 261.
3. Manitoba Department of Education, Exercises to Accompany Mathematics 101 (Winnipeg, MB: The Author, 1981), Option II. Available from the Manitoba Textbook Bureau.

OP.IV. COMPUTER AWARENESS (1 MODULE)

It is the intention of this unit to familiarize the students with the history, parts, advantages, limitations, role and impact of computers on society, some computer vocabulary and some career opportunities. It is not to be a programming course.

The sections in this unit are not in a specific order. However, where computers are available, the student should have some "hands on" experience with it; e.g., computer games, etc. It is a good idea to have such "hands on" experience near the beginning of this module for motivation purposes. This module lends itself to field trips and projects.

OBJECTIVES — NOTES TO TEACHERS

A. HISTORY

The student should be able:

1. to be aware of the historical development of the electronic computer;

With the students, discuss the following historical development of the computer:

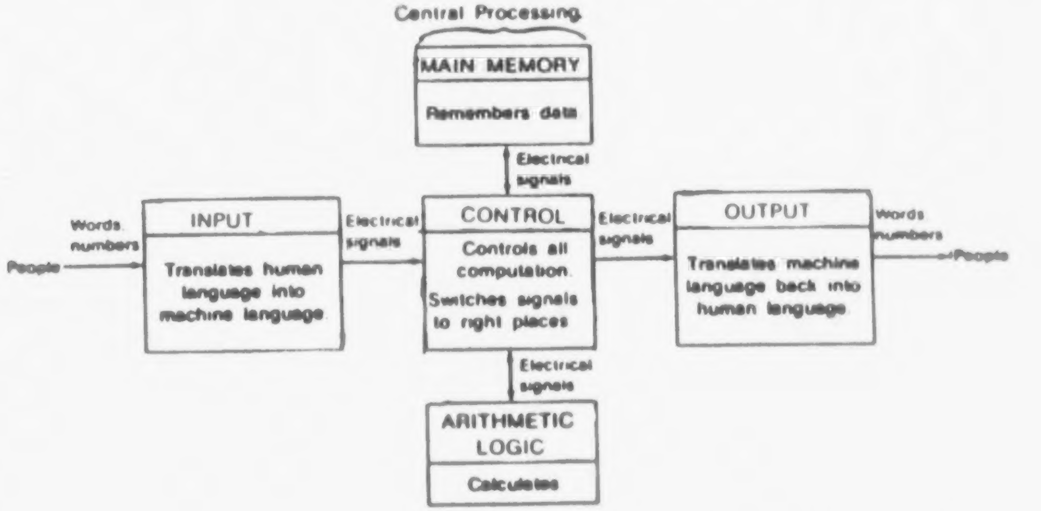
- . abacus - early counting device
- . Napier's bones - a tool for multiplication and division
- . slide rule
- . Pascal's mechanical calculating machine 1642 - could add and subtract
- . Leibniz' calculating machine 1671 - could add, subtract, multiply and divide
- . Babbage - in the 1800s, worked on a machine with many principles of modern computer: arithmetic unit, storage unit, control unit
- . Hollerith - invented electromechanical machine activated by punched cards, used in 1890 census
- . analog computer - could measure pressure, voltage, temperature and velocity by using gauges and meters - 1931
- . digital computers - operate on numbers, letters of the alphabet and other symbols

Examples:

- . Mark I - early electromechanical calculating machine - 1944
- . ENIAC - early electronic automatic computer - 1946
- . EDVAC - electronic discrete variable automatic computer - 1952
- . UNIVAC I - first commercial electronic data processing computer - 1951
- . IBM computers-1959 on

OBJECTIVES	NOTES TO TEACHERS
2. to identify four generations of digital computers.	With students, discuss the four generations of digital computers. 1. used vacuum tubes, 1942-58 2. used transistors, 1959-65 3. used integrated circuit, 1965 early 1970s 4. microminiature circuit, silicon chip, early 1970s to present
B. PARTS OF A COMPUTER The student should be able:	
1. to describe the parts of the computer and how they work.	Note: Explain that the human brain was the first computer. Relate the computer to the brain.

PARTS OF A COMPUTER



Central Processing
MAIN MEMORY
Remembers data
Electrical signals
People
Words numbers
INPUT
Translates human language into machine language
Electrical signals
CONTROL
Controls all computation.
Switches signals to right places
Electrical signals
OUTPUT
Translates machine language back into human language
Words numbers
People
Electrical signals
ARITHMETIC LOGIC
Calculates

Input

. keyboard
. cassette tape player
. card reader - punched cards are put in the card reader which reads them
. floppy disk - thin milar plate, easily bent, softer than phono record, permanently put in a cardboard jacket - data and programs stored on it
. paper tape reader - reads perforated paper tape
. hard disk made of rigid plastic and metallic oxide - permanently sealed - holds many times more information than floppy disk

Central processing unit

. main memory - receives and stores data and instructions from input device - like one's brain
. control unit - directs and coordinates the operations of the computer - like one's brain
. arithmetic/logic unit - performs arithmetic operations

Output

. speaking, writing
. video
. cassette tape recorder
. printer - prints information from the computer

C. VOCABULARY

The student should be able:

1. to understand common computer terms.

Terminology could include:

. Binary number system: a number system with a base 2 that uses two symbols (0 and 1) to represent values.

OBJECTIVES | NOTES TO TEACHERS

- ROM (Read Only Memory): data recorded in the memory when it is manufactured. This data can be read and used but cannot be altered. The data is not erased when the machine is turned off.
- RAM (Random Access Memory): used for writing, reading, and then rewriting data in storage. Data can be quickly and easily changed. It is temporary; it loses its programs, instructions or information when power is removed.
- Data: any kind of information, facts, concepts or instructions.
- Flowchart: a diagram that shows the major steps or operations that take place in a computer program.
- Program: a set of step-by-step instructions that tells a computer to do a sequence of tasks.
- Software: programs written for computer systems.
- Bug: a problem with a program or a mistake in software.
- Debug: to correct a mistake in a program or software.
- Hardware: the physical components of a computer system (tape recorder, printer, keyboard, video screen, etc.).
- Hard copy: any printed information produced by a computer and used for permanent storage.
- Modem: a device which connects computer and terminals over a telephone line.
- Terminal: a computer input/output device. It usually contains a video screen and keyboard. It connects to a modem or computer for processing data.

Kinds of computers:

- mainframe - large computer system
- minicomputer - has higher performance than microcomputer, higher price
- microcomputer - used extensively in schools

Languages:

- FORTRAN (Formula Translation): a mathematical and scientific language used to solve algebraic equations.

- COBOL (Common Business Oriented Language): the most popular business - oriented language, used to process large quantities of data in a repetitive routine.

- BASIC (Beginner's All-Purpose Symbolic Instruction Code): Most microcomputers use some version of Basic. It is easier to learn than Fortran and can be used to solve scientific, mathematical, and engineering problems. It can be used in commercial applications, in medicine, economics, student records and personal records in the home. It is the most common and popular English-like computer language.

- PASCAL: a highly structured language, becoming very popular as an introductory programming language on microcomputers.

- LOGO: a symbol-oriented, interpretive, list-processing language, used especially with younger children.

OBJECTIVES	NOTES TO TEACHERS
D. ADVANTAGES, LIMITATIONS, ROLE AND IMPACT; CAREER OPPORTUNITIES	Project and field trips would be relevant in this section.
The student should be able:	
1. to understand the main advantages of the computer;	Review the following advantages of the computer: . has speed and accuracy . has versatility . can handle repetitive tasks . has large data-storage capacity . can access information to and from a long distance by means of a modem
2. to understand its limitations;	Review the following limitations of a computer: . cannot think . only as good as its program, not to blame for errors . cannot make value judgements . cost . software availability . limit on storage capacity
3. to recognize specific uses of computers in some fields;	Discuss some specific uses of computers in the following fields: . business and industry . education - computer assisted instruction . home . government . law enforcement . medicine . engineering . military . recreation . transportation It is suggested that students do a project in this section and/or go on field trips.

OBJECTIVES	NOTES TO TEACHERS
4. to have some understanding of the impact the computer has on the individual and society;	Discuss the impact that computers have on individuals and society. For example: . automation - possible job threat, also job creation . invasion of a person's right to privacy - credit checks . cashless-chequeless society - possibility of fraud vs. convenience . public surveillance more feasible Projects would be suitable here.
5. to learn about computer-career opportunities.	Discuss some of the following computer-career opportunities: . computer programmers . systems analysts . computer operators . computer service personnel . data entry personnel . computer engineer . computer salesperson . key punch operator Project: Using newspaper, have students find ads for a number of different computer jobs and prepare a brief description of the job(s).
E. THE COMPUTER	
The student should be able:	
1. to have "hands on" experience with the microcomputer;	Have students locate keys - letters, numbers, shift, return, backspace, etc.
2. to use the microcomputer in recreational and educational activities;	Use "canned games" and "simulation programs" to familiarize the student with the computer.

OBJECTIVES

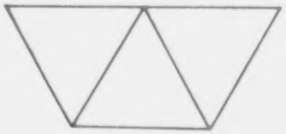
3. to create simple programs in either Logo or BASIC;

NOTES TO TEACHERS

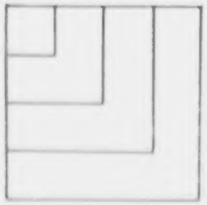
Teachers will need to select the computer language of instruction. Only Logo or BASIC seem viable at this time, but other suitable languages may be available in the future. Logo, because of its structure and the figures or pictures created, will be easier for students than BASIC which will appear more abstract in its examples and approach.

Possible Logo Assignments:

1. Using immediate mode, create the following figures.
    a) Concepts involved: Drawing a triangle by setting a heading and drawing a straight line.

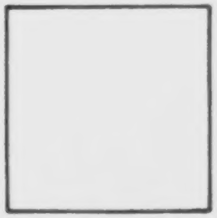


    b) Concepts involved: Drawing a square using the REPEAT function.



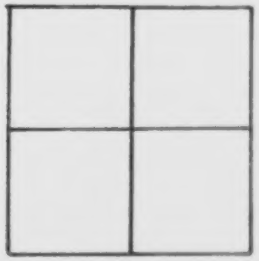
2. Procedures

a) Concepts involved: Definition of a simple procedure.



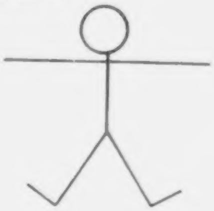
```
TO BOX
   REPEAT 4 [FD 20 LT 90]
END
```

b) Concepts involved: Definition of a super-procedure using simple procedures.



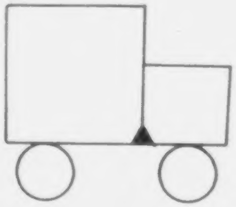
```
TO WINDOW
   REPEAT 4 [BOX RT 90]
END
```

c) Concepts involved: Definition of a super-procedure using several simple procedures.

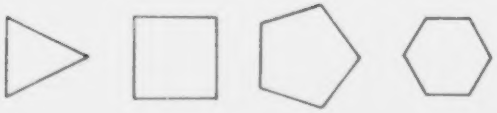


3. Procedures with variables

a) Concepts involved: Definition of procedures using variable input.



b) Concepts involved: Procedure using a calculated variable input.



```
TO POLYGON :SIZE :SIDES
   REPEAT :SIDES [FD :SIZE RT 360/:SIDES]
END
```

OBJECTIVES | NOTES TO TEACHERS

Possible BASIC Assignments:

1. Have students do arithmetic exercises using +, -, *, / and ().

   Example: PRINT (19+32)*2/10+8 <RETURN>

2. Introduce fundamental BASIC statements or commands such as LOAD, RUN, LIST, NEW, SAVE, PRINT, LET and REM.

3. Introduce simple three- or four-step programs such as:

```
10 REM Average of Three Numbers
20 LET S=34+76+217
30 LET A=S/3
40 PRINT A
RUN
```

Resources: available from Manitoba Education Library, Main Floor, 1181 Portage Avenue, Winnipeg, MB R3G 0T3

1. "Caught in the Middle" (Videotape)
2. Applications and Implications of Computer Use in Manitoba
3. Computer Careers - Jobs For the 80's (Kit)
4. "Computers in the Classroom" (Videotape)

Op.V. GRAPHING (1 MODULE)

Students should be able to read and interpret information from tables as well as from bar graphs, line graphs, pictographs and circle graphs. Review as necessary. Refer to C.III Reading and Interpreting Graphs and Tables in the Math 104 outline, and C.IV Graphing in the Math 204 outline.

OBJECTIVES NOTES TO TEACHERS

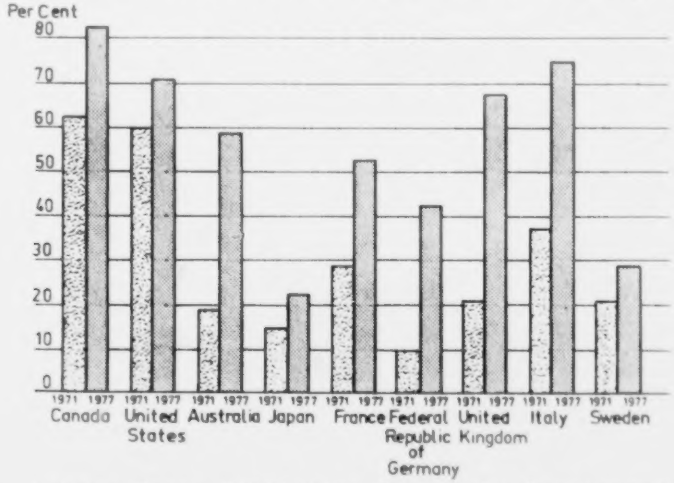
The student should be able:

1. to read and interpret multiple bar graphs;

A multiple bar graph illustrates two or more sets of data for each time category.

Graph A

UNEMPLOYMENT RATES FOR SELECTED INDUSTRIAL NATIONS, 1971 AND 1977



Source: Labour Force Statistics Quarterly Supplement to the Yearbook OECD Paris 1978.

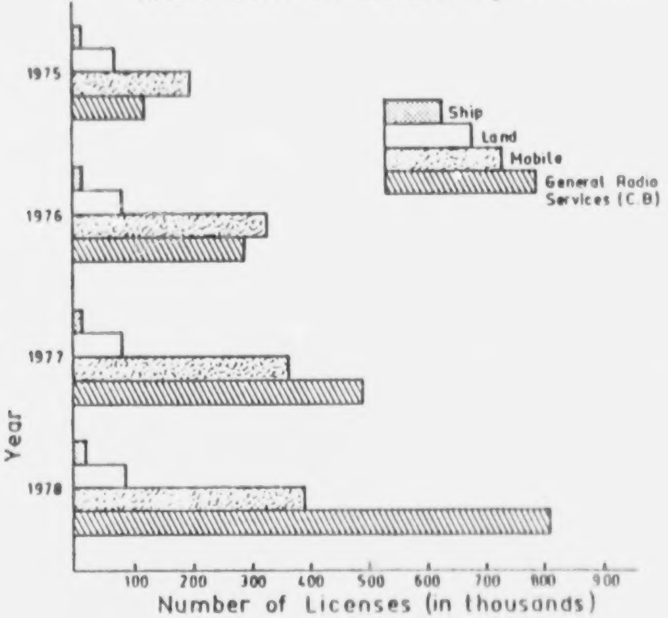
Sample Questions (for Graph A):

1) Which country had the largest and/or smallest increase in unemployment from 1971 to 1977? How much was this increase? Note that Sweden experienced an actual decrease.

2) What is the difference between the highest and lowest unemployment rate in 1971? in 1977?

Graph B

RADIO STATION LICENSES, 1975-1978



Source: Courtesy of Statistics Canada

Sample Question (For Graph B):

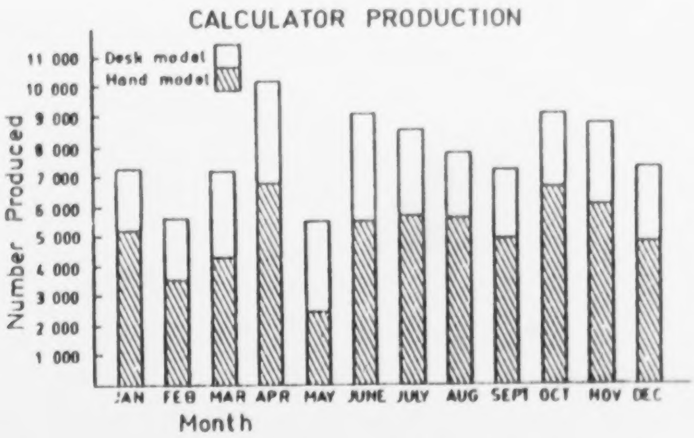
Compare the total number of licenses issued in 1975 to the total number issued in 1978.

2. to read and interpret component bar graphs;

A component bar graph stacks the various sets of data so that the top of the bar is the total of the sets.

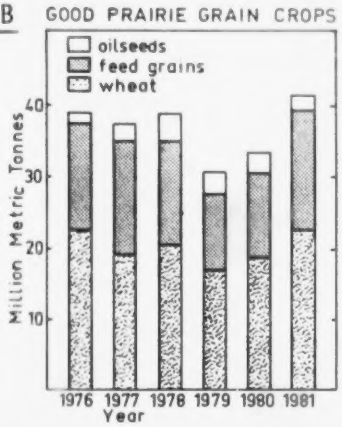
Graph A

For the month of June the total number of calculators produced was approximately 9 000. Of this total, approximately 5 600 were hand models and 3 400 were desk models.



Source: Courtesy of Statistics Canada.

Graph B



Sample Questions:

1. What was the total number of metric tonnes of grain crops harvested in 1980?

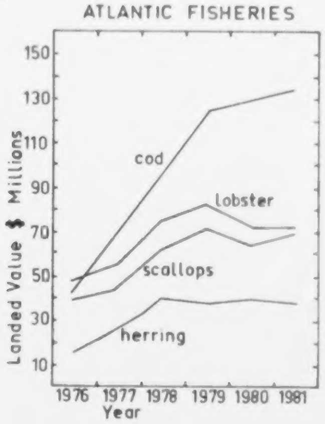
2. How many tonnes of feed grains were harvested in 1980?

3. What percentage of the total did oilseed crops represent for 1980?

Source: Courtesy of Statistics Canada.

3. to read and interpret multiple line graphs;

The multiple line graph shows two or more sets of data over the same period of time.



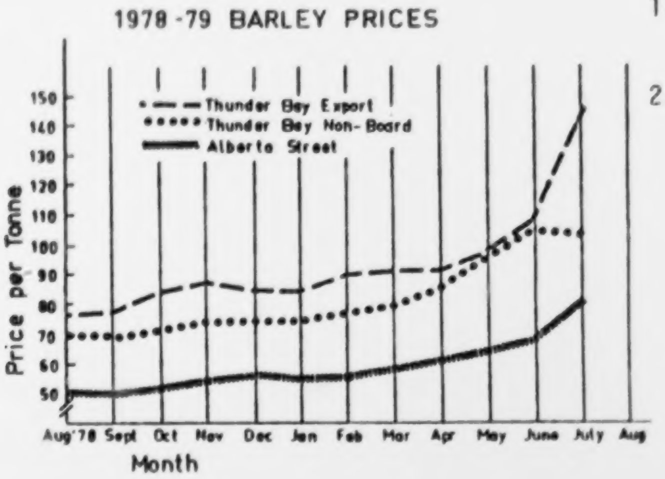
Sample Questions:

1. For the year 1979, give the dollar value for each type of fish.

2. What is the dollar value increase for each type of fish over the five-year period?

Source: Courtesy of Statistics Canada.

Graph B



Sample Questions:

1. Compare the three prices per tonne of barley for April 1979.
2. What is the difference in price per tonne between Thunder Bay Export, Thunder Bay Non-Board and Alberta Street for April 1979?

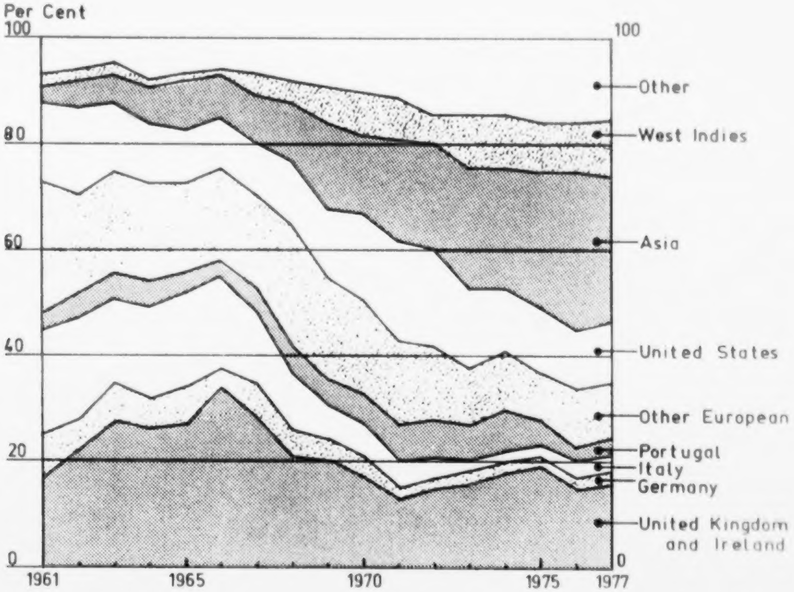
Source: Courtesy of Statistics Canada.

4. to read and interpret component line graphs;

In a component line graph, each component part makes up part of the whole. The top line represents the total of all the components. To determine the size or percent of the component, one must subtract.

Graph A

IMMIGRATION TO CANADA BY LAST COUNTRY OF RESIDENCE, 1961-77



Source: Immigration Statistics, Employment and Immigration Canada, 1966-77

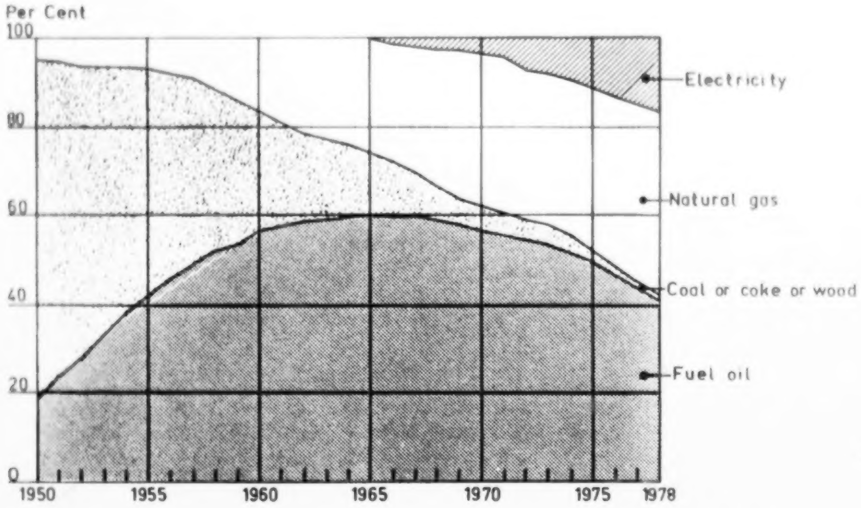
Sample Questions (for Graph A):

1. What percent of immigrants came from the United States in 1977?
   Answer: 11.2% (46.7 - 35.5)

2. Give the percent of immigrants from the countries listed in 1967 and compared them to the percent for 1977.

Graph B

DISTRIBUTION OF FUELS USED FOR HEATING DWELLING UNITS, 1950-78



Source: Household Facilities and Equipment Catalogue 64-202, 1950-78

Sample Questions (for Graph B):

In 1955 what percent of coal or coke or wood was used?
Answer: Approximately 52% (94%-42%)

In 1975 what percent of coal or coke or wood was used?
Answer: Approximately 2% (53%-51%)

OBJECTIVES

NOTES TO TEACHERS

ENRICHMENT:

Ask students to construct multiple and/or component bar and line graphs.

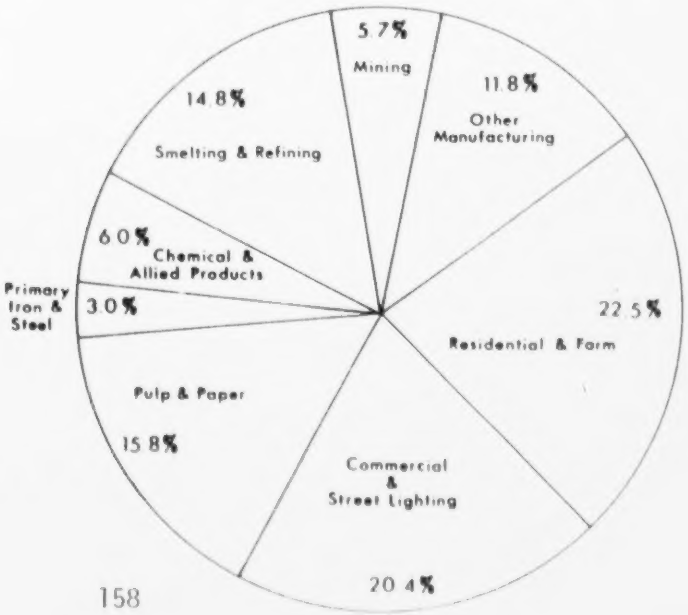
5. to interpret and to construct a circle graph;

Review as necessary the terminology of the circle and measuring angles. Refer to C.V Measurement and Geometry Math 204 outline.

Depending on the information used in constructing a circle graph, students may have to measure angles greater that 180°. Review as necessary.

Example:

CONSUMPTION OF ELECTRICITY 1966



The process of determining the angle representing a part of the circle graph may require considerable time. In the preceding example, residential and farm service represents 22.5% of consumption of electricity. To determine the size of angle, calculate 360° x 22.5% = 81°. Students must realize that all angles must add up to 360°.

Have students graph the following information for 1976 and compare to 1966.

Consumption of Electricity 1976

Residential and farm service	27.8%
Commercial and street lighting	28.0%
Pulp and paper	12.2%
Primary iron and steel	2.4%
Chemical and allied products	4.5%
Smelting and refining	7.8%
Mining	6.7%
Other manufacturing	10.6%

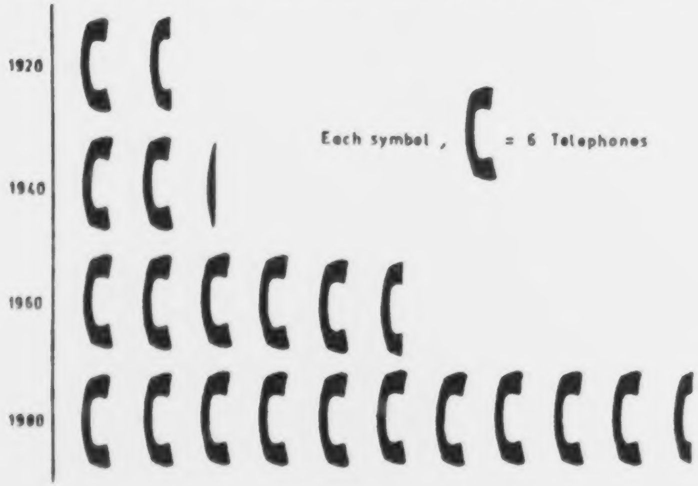
6. to recognize distortion in graphs.

Graphs may be drawn so as to distort information. For example, by removing the bottom of a graph, by stretching the vertical axis, or by drawing the symbols of a pictograph disproportionately, the visual impression can be changed dramatically. Even the wording and location of the title of a graph can affect the impression.

Examples:

1) Telephones in Canada

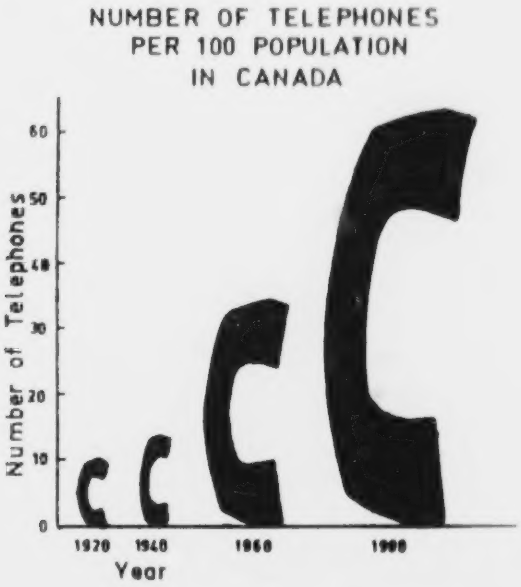
Graph A - This graph illustrates a normal pictograph.



NUMBER OF TELEPHONES PER 100 POPULATION IN CANADA

Source: Courtesy Statistics Canada.

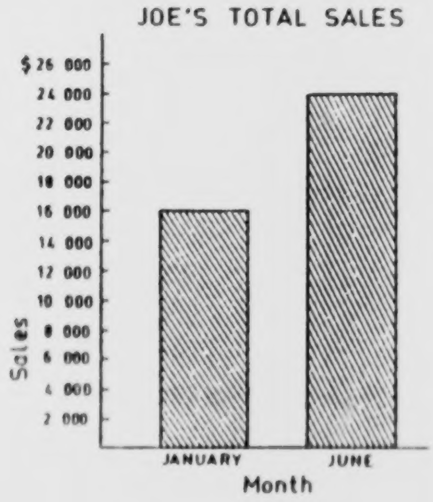
Graph B - Example of distortion.



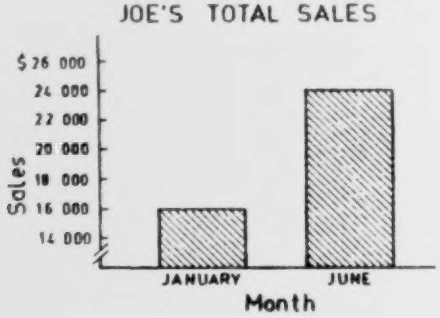
Source: Courtesy Statistics Canada

OBJECTIVES	NOTES TO TEACHERS

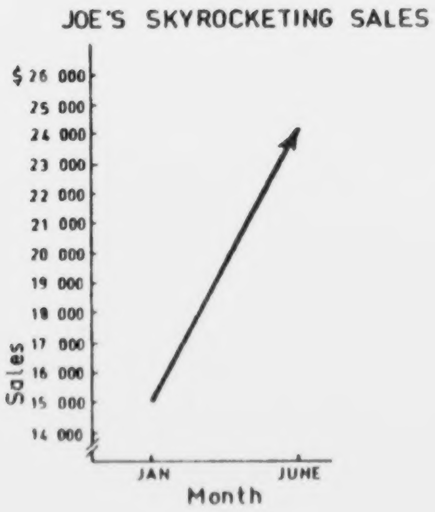
2. Graph A: Salesman Joe graphed his total sales for January and June the way he had been taught in Math 304.



Graph B: In order to impress his boss, Joe removed the bottom of the graph. The scale did not begin at zero but it certainly showed his increased sales volume in a better light.



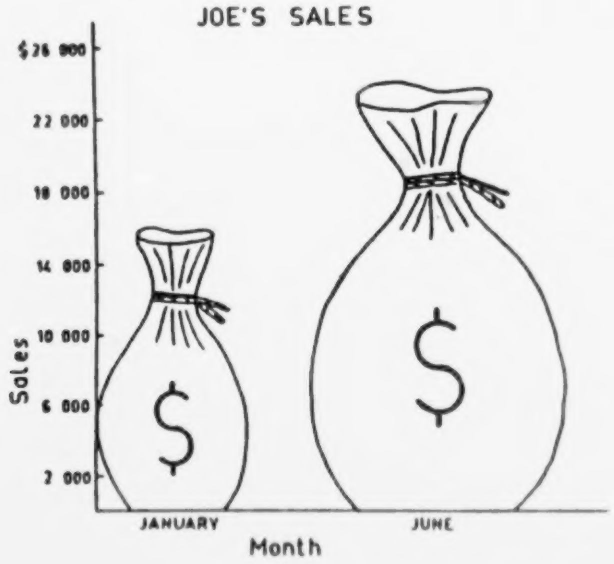
Graph C: Still, the effect was not pronounced enough so Joe stretched the vertical axis and left off the bottom of the graph. A slight title change did not hurt, either.



313

313

Graph D: Joe's final effort was a pictograph. Since the ratio of June sales to January sales was 3:2, he had to draw the second money bag 1½ times as tall as the first. To keep the proportions accurate, he also increased the width of the second bag by 1½ times. The numbers still say 3:2 but the area says 9:4. However, since a money bag is three-dimensional, it actually leaves the impression that this sales have increased $(3/2)^3$ or approximately 340% when in fact his sales had increased only $8000 over his $16 000 in January for an actual increase of only 50%.



OBJECTIVES NOTES TO TEACHERS

Resources:

1. Holmes, Nigel. Time Charts as found in Time Magazine. Many are multiple line graphs, bar graphs and pictographs all in one.
2. Kestell, M. et al., Math Matters Plus - Book 1 (Scarborough, ON: Nelson Canada Ltd., 1982), Chapters 11-3, 11-4, 11-5.
3. Price, J. et al., Mathematics For The Real World, Canadian Edition (Weston, ON: Merrill Publishing, 1979), Chapter 11.
4. Manitoba Departmentof Education, Exercises to Accompany Mathematics 101 (Winnipeg, MB: The Author, 1981), Option VII. Available from the Manitoba Textbook Bureau.

Op.VI. LEISURE/SPORTS ACTIVITIES (½ MODULE)

This module provides numerous opportunities for the development of mathematical concepts/skills. Arithmetic operations often are involved in various leisure and sport activities.

OBJECTIVES	NOTES TO TEACHERS
A. DIRECT INDIVIDUAL/TEAM PARTICIPATION	
	It is suggested that students do one individual and one team sport. Physical education departments may have information for use in this section. Assistance may be obtained from leadership courses offered in the school.
The student should be able:	
1. to develop diagrams or tables comparing various leisure/sports activities;	Assign students projects of drawing to scale any of the following: racquetball, squash or tennis court; baseball diamond; curling ice; soccer field; football field; ping-pong table; pool table; bowling alley; etc. Tables could show the duration of participation (consider age factor) and the skills involved in participating in various leisure/sport activities.
2. to calculate the cost of participating in various leisure/sports activities.	Students can determine cost of equipment, maintenance costs and fees (membership fees/dues may or may not be required).
B. HOBBIES	
	Suggested hobbies: pet care, stamp and coin collecting, customizing cars, ceramics, model-building, gardening, games (chess, checkers, backgammon, cards, dice), horse racing, bingo, lotteries.

OBJECTIVES	NOTES TO TEACHERS
The student should be able to identify costs in hobby involvement and to perform calculations for a given hobby.	Costs could include equipment, facilities, initial costs, maintenance costs, etc. Students could compare costs of do-it-yourself projects vs purchasing finished products. Customizing cars, model-building and ceramics lend themselves to this type of comparative study. Students or persons in the school or community involved with a particular hobby may act as resource persons.
C. SPECTATOR ACTIVITIES	
	Spectator activities may include: football, baseball, hockey, boxing, wrestling, etc.
The student should be able to compile and to interpret statistical tables.	Statistical tables may give the following information: standings, averages, individual achievements, etc. Students could compute: a) batting average $\frac{\text{hits}}{\text{times at bat}}$ = batting average (to nearest thousandth) b) standings in football using point system two points for a win, and one point for a tie c) percentage of completed passes passing percentage = $\frac{\text{completions}}{\text{attempts}}$ d) average salary per hockey player Budgetary items such as salaries, pensions, facilities and travel could be discussed when determining operating expenses for a commercial sports organization as these items are considered in determining the cost of tickets.

OBJECTIVES NOTES TO TEACHERS

Resources:

1. Cutting, Robert J., Math You Need Book 2 (Scarborough, ON: Nelson Canada Ltd., 1981), Unit 1.
2. Fraser, Don, Power Play 2 (Don Mills, ON: Collier-MacMillan Canada Ltd., 1983), Chapter 9.
3. Keedy, M. et al., Applying Mathematics (Menlo Park, CA: Addison-Wesley Publishing Co., 1983, Chapter 9.
4. Itzkowitz, M. and Richards, J., Mastering Essential Mathematics Skills (River Falls, OK: Laidlaw Brothers, 1980), Unit 1.
5. Sentlowitz, M. and Thelen, J., Baseball (Menlo Park, CA: Addison-Wesley Publishing Co., 1977).

Op.VII. MONEY MANAGEMENT - FOREIGN EXCHANGE (½ MODULE)

This module deals with foreign currency and conversion from one currency to another.

OBJECTIVES	NOTES TO TEACHERS
The student should be able:	
1. to recognize situations when foreign exchange transactions are necessary;	When a Canadian visits a foreign country, he/she must use the currency of that country. When a firm in Canada purchases an article from another country, the article must be paid for in the currency of that country. Today many people travel from country to country and international trade is essential to the well being of each country's economy. Therefore, it is necessary that a person or firm be able to convert money from one currency to another.
2. to identify the unit of currency of several countries;	Students could start with the currencies of countries they have visited or wish to visit or ones with which they are familiar. They may be asked to name the coins or fractions of units used in each country. This exercise may be assigned as a project. Refer to quotation sheets for information.

Place		Unit	Canadian	United States
Afghanistan	AFA	Afghani	.02187	.01794
Albania	ALL	Lek	.2152	.1765
Algeria	DZD	Alg. Dinar	.2930	.2403
Antigua	XCD	EC $	.4511	.3700
Argentina	ARP	Peso	.0001024	.000084
Australia	AUD	Aus. Dollar	1.2948	1.0620
Austria	ATS	Schilling	.07455	.06115
Bahamas	BSD	Bahamian $	1.2192	1.0000
Bahrain	BHD	Dinar	3.2461	2.6625
Bangladesh	BDT	Taka	.05657	.04640
Barbados	BBD	BDS $	.6096	.5000
Belgium	BEC	B. Franc	.02822	.02315

Place		Unit	Canadian	United States
Belgium	BEL	B. Financial Franc	.02560	.02100
Belize	BZD	BZE $	.6096	.5000
Bermuda	BMD	BD $	1.2192	1.0000
Bolivia	BOP	Bolivian Peso	.02771	.02273
Botswana	BWP	PULA	1.3045	1.0700
Brazil	BRC	Cruzere	.008004	.006565
British Virgin Is.	USD	US $	1.2192	1.0000
Brunel	BND	Dollar	.5757	.4722
Bulgaria	BGL	Lev	1.2424	1.0190
Burma	BUK	Kyat	.1902	.1560
Cambodia		Riel	n/a	n/a
Cameroons	XAF	CFA franc	.003942	.003233
Canada	CAD	Dollar	1.0000	.8202
Cayman Islands	KYD	Cayman Dollar	1.4630	1.2000
Chile	CLP	C. Peso	.03135	.02572
China	CNY	Renminbi	.6718	.5510
Columbia	COP	Peso	.01918	.01625
Congo (Brazzaville)	XAF	CFA Franc	.003942	.003233
Costa Rica	CRC	Colon	n/a	n/a
Cuba	CUP	Peso	1.5069	1.2360
Cyprus	CYP	Cyprus Pound	2.6329	2.1595
Czechoslovakia	CSK	Crown	.1969	.1615
Dahomey Benin	XOF	CFA Franc	.003942	.003233
Denmark	DKK	Krone	.1542	.1265
Djibouti (Fr. Somali)	AIF	Djibouti Franc	.007145	.005860
Dominica	XCD	EC $	.4511	.3700
Dominican Republic	DOP	Peso	1.2192	1.0000

Source: The Banks.

OBJECTIVES	NOTES TO TEACHERS
3. to list the current value of foreign currencies in terms of Canadian dollars;	Some newspapers show the value of foreign currencies in terms of Canadian dollars in the financial section. Current information can be obtained from banks.
4. to complete calculations comparing Canadian dollars to currencies of other countries.	Point out to students that there is a difference in rate when one buys foreign currency at a bank compared to selling the currency to the bank. Current buying rates and selling rates can be obtained from banks and should be used in calculations.

Sample questions use information from the following exchange rates dated June 8, 1983 and shown on page 175.

1) Before travelling to Denmark, Tim converted \$300 Canadian to krones. How many krones did the bank sell him?

Solution:

$$\begin{aligned} \$0.1375 \text{ (Can)} &= 1 \text{ krone} \\ \$1 &= \frac{1}{0.1375} \\ \$300 &= \frac{1}{0.1375} \times 300 \\ &= 2182.82 \text{ krone} \end{aligned}$$

2) The cost of an average hotel room for various countries is given below. Which are higher than the Canadian average of $32?

   a) China, 60 renminbi
   b) Austria, 600 schillings
   c) Italy, 45 000 lire
   d) Japan, 9600 yen
   e) Australia, 29.00 dollars

3) After returning from their world trip, Susan and Chris sold the following currency to the bank. Calculate the amount they received in Canadian currency for each sale.

   a) 89 French francs
   b) 23.50 New Zealand dollars
   c) 52 Australian dollars
   d) 73 South African rands
   e) 124 German deutschemarks
   f) 15 Swiss francs

THE ROYAL BANK OF CANADA

INTERNATIONAL MONEY MARKETS
TORONTO, ONT.
(June 8, 1983)

GENERAL EXCHANGE RATES

COUNTRY	CURRENCY	BUYING RATE		SELLING RATE	
		Pay Client	Debit I.M.M.	Charge Client	Credit I.M.M.
AUSTRALIA	Dollar	1.0745	1.0765	1.0985	1.096.5
AUSTRIA	Schilling	.06755	.06765	.06925	.06915
BELGIUM	Franc (Convertible)	.02365	.02375	.02445	.02435
BELGIUM	Franc (Financial)	.02365	.02375	.02445	.02435
CHINA, People Rep. of	Renminbi	.6140	.6155	.6300	.6285
DENMARK	Frone	.1295	.1310	.1375	.1360
FRANCE	Franc	.1560	.1575	.1630	.1615
GERMANY	Deutschemark	.4735	.4750	.4865	.4850
HONG KONG	Dollar	.1575	.1590	.1655	.1640
INDIA	Rupee	.1215	.1230	.1305	.1290
ITALY	Lire	.000750	.000765	.000840	.000825
JAPAN	Yen	.005065	.005080	.005195	.005180
NETHERLANDS	Guilder	.4210	.4325	.4340	.4325
NEW ZEALAND	Dollar	.7995	.8010	.8225	.8210
NORWAY	Krone	.1655	.1670	.1745	.1730
PORTUGAL	Excudo	.01185	.01200	.01315	.01300
SOUTH AFRICA	Rand	1.1125	1.1160	1.1495	1.1460
SPAIN	Peseta	.00800	.00815	.00900	.00885
SWEDEN	Krone	.1575	.1590	.1655	.1640
SWITZERLAND	Franc	.5725	.5740	.5875	.5860

Resources:

1. Department of Education, Exercises to Accompany Mathematics 301 (Winnipeg, MB: The Author, 1984). Available from the Manitoba Textbook Bureau.
2. Financial institutions: banks, trust companies, credit unions.

OP.VIII. PROBABILITY (½ MODULE)

Finding probability and predicting outcomes should be clear to the students after they have had many opportunities to participate in various experiments/activities.

It is suggested that several examples of the types given be done by all the students.

OBJECTIVES

NOTES TO TEACHERS

The student should be able:

1. to calculate the probability of an event as the ratio of desired outcomes to possible or total outcomes;

Examples:

1) A jar contains 4 marbles: 1 red, 1 blue, 1 black, 1 yellow. The probability of selecting a blue marble at random is 1/4 (one desired outcome, four possible outcomes).

2) The probability of producing a head when flipping a coin is 1/2 (one desired outcome, 2 possible outcomes).

3) The probability of rolling a 5 on a die is 1/6 (one desired outcome, 6 possible outcomes).

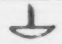
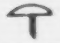
4) The probability of rolling a 3 or a 6 on a die is 2/6 or 1/3 (2 desired outcomes, 6 possible outcomes).

5) A box contains the following letters: 3-M's, 1-X, 4-D's, 2-T's. What is the probability of selecting at random: a T? 2/10 or 1/5; an M? 3/10; a D? 4/10 or 2/5; an X or a D? 5/10 or 1/2; an L? 0/10 or 0?

2. to determine relative frequency using data obtained experimentally;

Experiment 1

Students toss a tack 20 times and record the number of times it lands point up and point down.

Outcome		Tally
	(Point Up)	~~1111~~ 11
	(Point Down)	~~1111~~ ~~1111~~ 111

From the preceding table, one could predict that the probability the tack would land point up on the next toss would be 7/20, or point down 13/20.

Have students determine the relative frequency when the number of experimental tosses are increased (for example, doubled). The frequency of either outcome occurring should approach one-half as the number of tosses are increased.

Experiment 2

Have students toss 1 die 24 times and record the number of times each face appears. From their tally, the frequency of any number appearing can be determined. Compare this to the frequency when the number of tosses are increased. As the tosses increase, each face should appear about one-sixth of the time.

Experiment 3

Shuffle a deck of 52 cards. If one were to draw a card (replace the card each time, and reshuffle) the probability of drawing a spade is 13/52 or 1/4; a ten is 4/52 or 1/13; and ace of diamonds is 1/52.

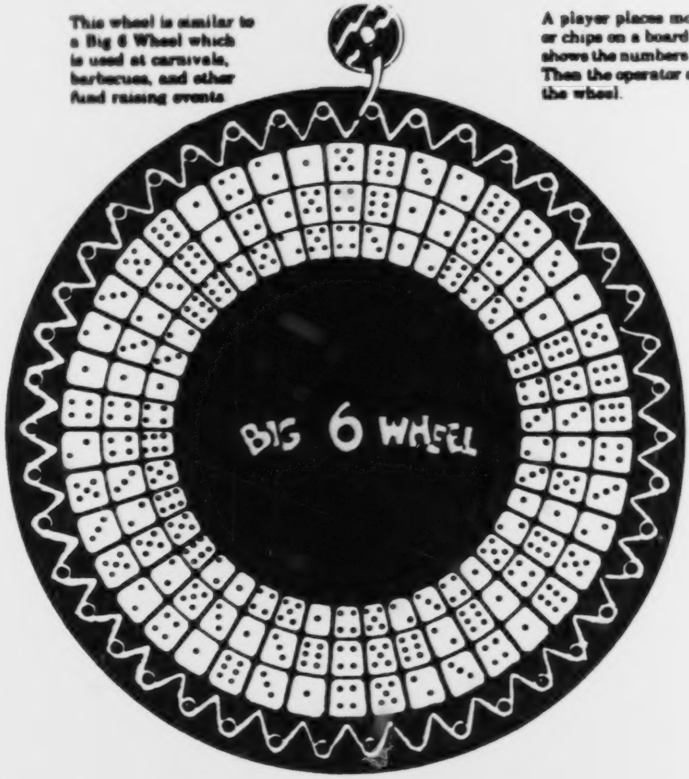
Making 26 such draws, the expected number of times for drawing a spade is $1/4 \times 26 = 6.5$; a ten is $1/13 \times 26 = 2$; an ace of diamonds $1/52 \times 26 = .5$.

Using the following table, have students record their data after making 26 draws. Then determine the experimental probability of drawing a spade, a ten, an ace of diamonds. Compare these results to the above.

	A	1	2	3	4	5	6	7	8	9	10	J	Q	K
♥ Heart														
♣ Club														
♦ Diamond														
♠ Spade														

Repeat the above experiment, increasing the number of draws.

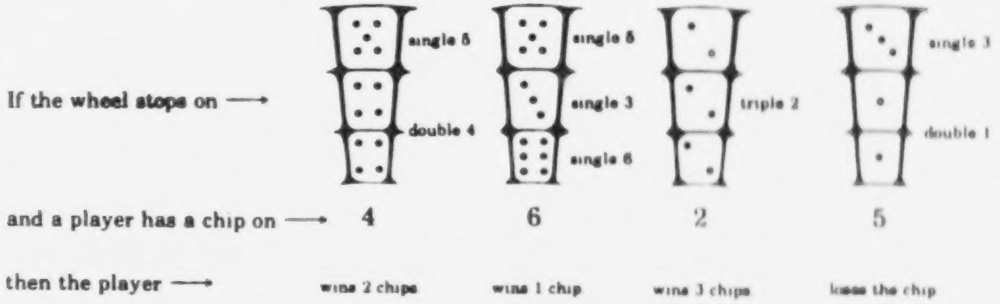
3. to determine probabilities in games of chance.



This wheel is similar to a Big 6 Wheel which is used at carnivals, barbecues, and other fund raising events.

A player places money or chips on a board that shows the numbers 1–6. Then the operator spins the wheel.

From: Bolster and Woodburn, Mathematics in Life (Toronto, ON: Scott, Foresman & Co., 1978).


If the wheel stops on →
single 5
double 4
single 5
single 3
single 6
triple 2
single 3
double 1
and a player has a chip on →
4
6
2
5
then the player →
wins 2 chips
wins 1 chip
wins 3 chips
loses the chip

Possible questions: Give the probability that the wheel will stop on a section that shows a 4 as $\frac{6}{42}$ or $\frac{1}{7}$ (there are 42 sections in all, and 6 sections contain a single 4); any 4 as $\frac{12}{42}$ or $\frac{2}{7}$; a double 4 as $\frac{3}{42}$ or $\frac{1}{14}$; a triple 4 as $\frac{3}{42}$ or $\frac{1}{14}$.

Students could be asked to determine the probability of winning at various games such as cards, dice, bingo, etc.

Resources:

1. Bolster, L. and Woodburn, H., Mathematics In Life (Toronto, ON: Gage Publishing Company, 1978), pp 170-185.
2. Ebos, F. and Zolis, P., Math Matters - Book 2 (Scarborough, ON: Nelson Canada Ltd., 1983), pp 338-346.
3. Itzkovitz, M. and Richards, T., Mastering Essential Mathematics Skills (River Falls, IL: Laidlaw Brothers, 1980), Unit 11.
4. Keedy, M. et al., Applying Mathematics (Don Mills, ON: Addison-Wesley Publishing Co., 1983), pp 416-419.
5. Manitoba Department of Education, Exercises to Accompany Mathematics 201 (Winnipeg, MB: The Author, 1982), Option IX. Available from the Manitoba Textbook Bureau.
6. Price, J. et al., Mathematics For The Real World, Canadian Edition (Weston, ON: Gage Publishing Co., 1979), Chapters 13, 14, 15.

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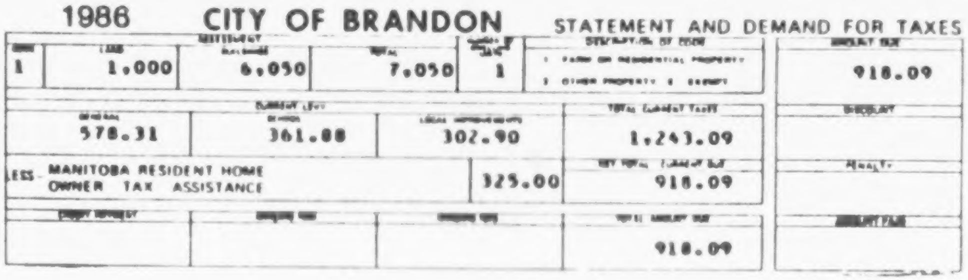
Op.IX. TAXATION (1 MODULE)

Property taxes and taxes in general are a fact of life. This module is designed to increase the student's awareness of taxation.

OBJECTIVES	NOTES TO TEACHERS
A. PROPERTY TAX	
The student should be able:	
1. to identify methods of collecting property tax;	For renters, property tax is usually included in rent. For homeowners with a mortgage, it is usually included in monthly payments (P.I.T.). For homeownerswith no mortgage, the homeowner must pay directly to city, town, or municipality.
2. to identify services covered by property tax;	Property tax covers fire and police services, welfare, local transit subsidy, recreation facilities, parks, streets, snow removal, etc. The education component of property tax covers some of the local education costs. Senior citizens get a reduction in this component of property tax. Teachers should get a copy of their local tax allocation.
3. to calculate homeowner's property tax.	Tax depends on 2 factors, the tax rate and the assessed value of property. The tax rate is expressed in mills (tax per $1000). Calculate total property tax on property assessed at $7050. The mill rate for "general" is 82.03 mills and the mill rate for "school" is 51.33 mills. $\frac{7050}{1000} \times 82.03 = \$578.31$ $\frac{7050}{1000} \times 51.33 = \$361.88$

Some homeowners pay a "local improvements" tax covering street paving, sidewalks, street lighting, etc. In the bill below, the local improvements are $302.90 so the total tax bill is $578.31 + $361.88 + $302.90 = $1243.09.



1986 CITY OF BRANDON STATEMENT AND DEMAND FOR TAXES

1	1,000	6,050	7,050	1	918.09
578.31	361.88	302.90	1,243.09		
LESS - MANITOBA RESIDENT HOME OWNER TAX ASSISTANCE		325.00	918.09		
			918.09		

For early payment of taxes, various discounts are allowed.

The assessed value of property increases if the house is enlarged, a recreation room is completed, a garage added, etc.

OBJECTIVES	NOTES TO TEACHERS
B. MANITOBA TAX CREDIT PLAN	Note: Personal Income Tax is an option topic in Math 204. Review it before completing this section.
The student should be able:	
1. to read and interpret Manitoba Tax credit plans;	Have students pay attention to clauses concerning who may claim.
2. to calculate cost of living credit;	Students will need information from T1 General or Special Income Tax forms.
3. to calculate Manitoba Property Tax credit both as renter and as homeowner.	
C. OTHER TYPES OF TAXATION	
The student should be able to identify other forms of taxation.	Other Forms: Current rates should be used for each of the following: 1. Federal Sales Tax - is included in the retail price. 2. Manitoba Payroll Tax - is a percentage of payroll and paid by employer. 3. Federal Excise Taxes - are levied on so-called luxury goods such as jewelry, watches, tobacco, liquor, gasoline and cosmetics. This tax is in addition to the Federal Sales Tax. For these items, taxes account for the major portion of the price. 4. Customs Duties - are charged by Canada Customs on goods entering the country. A personal exemption is allowed depending on the length of your stay in the foreign country. For details of this tax, a booklet "I Declare" is available from Canada Customs and Immigration.

5. Capital Gains Tax - is levied on any profits you make when you sell assets like stocks or real estate other than your principal residence. It does not apply to your personal assets.
6. Aircraft Boarding Tax - is included in the price of an airline ticket.
7. Cable or Pay TV Tax - is collected by the cable or pay TV company. It is payable to the Federal Government to fund more Canadian TV and film production.

Note: An accountant could expand on this list.

Resources:

1. Bolster, L.C. et al., *Consumer and Career Mathematics* (Toronto, ON: Gage Publishing Co., 1980), pp 204-211.
2. Manitoba Department of Education, *Exercises to Accompany Mathematics 201* (Winnipeg, MB: The Author, 1982), pp 204-211. Available from the Manitoba Textbook Bureau.
3. -------, *Exercises to Accompany Mathematics 301* (Winnipeg, MB: The Author, 1984), pp 5-26. Available from the Manitoba Textbook Bureau.

mathematics 301

THE 01 MATHEMATICS PROGRAM

01 Mathematics is an amalgamation of previous 01 (general), 02 (business) and 03 (vocational) mathematics programs in Manitoba. It is truly a general mathematics program, incorporating the best from several areas of mathematics and leading to a range of post-secondary education programs, vocational training and employment, as well as to more general life skills. This program is needed to familiarize students with a wider variety of mathematics topics and, thereby, to expand their future options.

Core topics of Mathematics 101 and 201 include arithmetic, consumer mathematics, geometry and algebra while core topics of Mathematics 301 include consumer mathematics, algebra, trigonometry and statistics. To produce a practical program, practical applications to current problems from business, consumer, trade and vocational areas are included. In this way, all 01 students, not only those registered in business and vocational courses, have an opportunity to receive instruction in business and/or vocational mathematics topics.

The amalgamated 01 program is particularly beneficial to smaller schools which, because they have been restricted in the past to offering two courses in mathematics at any one grade level, have been unable to offer business and vocational mathematics. It should be noted here, however, that larger schools may want to identify separate classes of 01 Mathematics with options dedicated to business, vocational or general mathematics.

GENERAL GOALS OF THE 01 PROGRAM

The student goals upon which this guide is based are included below. The program is designed:

1. to develop an awareness of the need/usefulness of mathematics;
2. to increase understanding of the concepts and to develop mastery of basic arithmetic skills using whole numbers, integers and decimal fractions through the solution of practical problems in the consumer, business, trade and vocational areas;
3. to increase proficiency in mental arithmetic and rapid calculation exercises;
4. to increase comprehension in the problem solving process;
5. to increase proficiency in making reasonable estimations in both geometric and numerical activities;
6. to develop further understanding and facility with metric measurement and language;
7. to increase consumer awareness with reference to both the market and work places;
8. to develop an understanding and a mastery of basic algebraic skills;

9. to increase inductive and deductive reasoning and other thinking skills;

10. to cultivate a positive attitude towards mathematics.

STRUCTURE OF THE MATHEMATICS 101 COURSE

Each module is designed for 14 periods (40 minutes each) of new work in a non-semestered system. This should allow approximately four periods per module for review, testing, diagnosis and enrichment. Ten modules comprise a full credit. Core modules are compulsory, and Mathematics 101 has the equivalent of five core modules. The remaining equivalent of five option modules are to be selected from the business, vocational and/or general mathematics topics.

The following chart gives a listing of the topics identified for the Mathematics 101 course and an indication of the allotted time in modules.

CORE TOPICS (Complete each of the following.)

C.I.	Arithmetic	(½ module)
C.II.	Consumer Mathematics	(1½ modules)
C.III.	Geometry	(1½ modules)
C.IV.	Algebra	(1½ modules)

OPTION TOPICS (Select the equivalent of 5 modules from the following.)

Calculator Mathematics		(½ module)
Op.I.	Operations of and Applications for Calculators	
Business Mathematics		
Op.II.	Basic Skills in Business Mathematics	(1 module)
Op.III.	Discount Calculations	(1 module)
Op.IV.	Merchandise and Service Business Calculations	(1 module)
Vocational Mathematics		
Op.V.	Measurement and Estimation	(1 module)
Op.VI.	Ratio and Proportion	(1 module)
Op.VII.	Percent Calculations	(1 module)

General Mathematics

Op.VIII.	Graphing	(1 module)
Op.IX.	Estimation	(½ module)
Op.X.	Mathematics Experiments	(½ module)
Op.XI.	Spatial Relations	(½ module)
Op.XII.	Statistics	(½ module)

STRUCTURE OF THE MATHEMATICS 201 COURSE

Each module is designed for 14 periods (40 minutes each) of new work in a non-semestered system. This should allow approximately four periods per module for review, testing, diagnosis and enrichment. Ten modules comprise a full credit. Core modules are compulsory, and Mathematics 201 has the equivalent of five core modules. The remaining equivalent of five option modules are to be selected from the business, vocational and/or general mathematics topics.

CORE TOPICS (Complete each of the following.)

C.I.	Consumer Mathematics	(1½ modules)
C.II.	Geometry	(1½ modules)
C.III.	Algebra	(2 modules)

OPTION TOPICS (Select the equivalent of 5 modules from the following.)

Calculators/Calculating Machines

Op.I.	Operations and Applications of the Ten-key Printing Electronic Calculator	(1 module)

Business Mathematics

Op.II.	Financial Operations	(1 module)
Op.III.	Income Statements	(1 module)
Op.IV.	Division of Business Profits/Losses	(1 module)

Vocational Mathematics

Op.V.	Calculations in Vocational Areas	(1 module)
Op.VI.	Mathematics in Specific Trades	(1 module)
Op.VII.	Electrical Trades	(1 module)

General Mathematics

Op.VIII.	Problem Solving Strategies	(1 module)
Op.IX.	Probability	(½ module)
Op.X.	Owning and Operating an Automobile	(½ module)
Op.XI.	Costs of Living On Your Own	(½ module)
Op.XII.	Leisure Mathematics	(½ module)

MATHEMATICS 301 PROGRAM

CORE TOPICS (75%)

(Do each of the following.)

			Page References
C.I.	Consumer Mathematics A. Budgeting B. Taxation C. Insurance D. Mortgages	(2½ modules)	195
C.II.	Algebra A. Algebraic Operations B. Equation Solving C. Graphing D. Variation	(3 modules)	245
C.III.	Trigonometry	(1 module)	283
C.IV.	Statistics	(1 module)	305

OPTION TOPICS (25%)

(Do the equivalent of 2½ modules from the following.)

			Page References
Op.I.	Annuities	(½ module)	321
Op.II.	Depreciation and Distribution of Overhead Costs	(1 module)	329
Op.III.	Digital Logic	(1 module)	339
Op.IV.	Investments (Stocks, Bonds, Real Estate)	(1 module)	355
Op.V.	Interpretation of Graphs	(½ module)	365
Op.VI.	Quadratic Equations	(½ module)	377
Op.VII.	Permutations and Combinations	(½ module)	385
Op.VIII.	Statistics (Standard Deviation and The Normal Curve)	(½ module)	393

C.I. CONSUMER MATHEMATICS (2½ MODULES)

A. BUDGETING

Families are important consumer units in the Canadian economic system because they purchase products and services which industry, business and government offer. Families may find money dwindling away because of impulse buying, failure to distinguish between real needs and psychological wants, the influence of advertising, buying without comparing prices of similar goods and services, or adding to the cost of purchased goods and services by buying on credit. A budget will aid a family in understanding what its income and expenses are and in planning to use its income wisely. Budgeting helps the family to plan its expenditures and to avoid the possibility of discovering that its money supply is depleted or that it is sinking deeper and deeper into debt. A knowledge of the principles of budgeting will enable students to develop an understanding of municipal, town/city, provincial and federal budgets.

This module covers mathematics related to the following concepts pertaining to budgeting: breakdown of average income dollar and preparing a monthly budget for a family.

OBJECTIVES	NOTES TO TEACHERS
The student should be able:	
1. to define and to explain the purpose of budgeting;	Budgeting could be defined simply as a plan for spending your money. Budgeting is important because: . It enables you to buy what you need and want within your income. . You will know where your money is being spent. . It decreases worry over money. . It enables you to control your spending. . You will be ready for expenses as they come up. . It lessens the number of family arguments over money. . It helps you to feel sure of yourself in handling money. Teachers might find it useful to review the section on personal budgets in C.II. Consumer Mathematics, Math 101.

2. to calculate the amounts a family would spend on an annual basis in various income categories, given the annual take-home pay and a recommended breakdown of the income dollar;

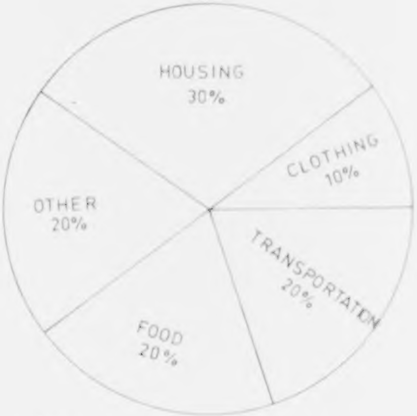
The following chart suggests what percent rate of take-home pay is recommended for various expenses.

Chart #1

Shelter	25%
Food	30%
Clothing	10%
Transportation	12%
Health & Personal Care	8%
Recreation	6%
Insurance	5%
Savings	4%
TOTAL	100%

This chart provides general guidelines only. A budget must meet the goals and needs of one's own particular situation. For example, some financial analysts would agree that the following breakdown of take-home pay would be more appropriate.

Chart #2



Note:

Housing - 30%

- A good guideline for total housing costs is 30% of your take-home pay.
- Total housing costs would include all costs related to running a home. For example: rent or mortgage payments, utility bills, maintenance and repair costs, taxes, insurance, etc.
- A budget that allocates more than 30% of income to housing may result in financial difficulties for the family.

Example:

Using the percent rates given in chart #1, calculate the annual dollar amounts which the families of the following would spend on the various income categories:

a) an unskilled labourer with an annual take-home pay of $10 500.
b) a technician with an annual take-home pay of $20 500.
c) a skilled labourer with an annual take-home pay of $18 600.
d) a mid-management person with an annual take-home pay of $25 600.
e) an executive/professional person with an annual take-home pay of $65 500.

Solutions:

Dollar amounts spent on various income categories:

	Unskilled Labourer	Technician	Skilled Labourer	Mid-Management	Executive/ Professional
Shelter:	$10 500 x 0.25 = $2 625.	$20 500 x 0.25 = $5 125.	$18 600 x 0.25 = $4 650.	$25 600 x 0.25 = $6 400.	$65 500 x 0.25 = $16 375.
Food:	$10 500 x 0.3 = $3 150.	$20 500 x 0.3 = $6 150.	$18 600 x 0.3 = $5 580.	$25 600 x 0.3 = $7 680.	$65 500 x 0.3 = $19 650.
Clothing:	$10 500 x 0.1 = $1 050.	$20 500 x 0.1 = $2 050.	$18 600 x 0.1 = $1 860.	$25 600 x 0.1 = $2 560.	$65 500 x 0.1 = $6 550.
Transportation:	$10 500 x 0.12 = $1 260.	$20 500 x 0.12 = $2 460.	$18 600 x 0.12 = $2 232.	$25 600 x 0.12 = $3 072.	$65 500 x 0.12 = $7 860.
Health and Personal Care:	$10 500 x 0.08 = $840.	$20 500 x 0.08 = $1 640.	$18 600 x 0.08 = $1 488.	$25 600 x 0.08 = $2 048.	$65 500 x 0.08 = $5 240.
Recreation:	$10 500 x 0.06 = $630.	$20 500 x 0.06 = $1 230.	$18 600 x 0.06 = $1 116.	$25 600 x 0.06 = $1 536.	$65 500 x 0.06 = $3 930.
Insurance:	$10 500 x 0.05 = $525.	$20 500 x 0.05 = $1 025.	$18 600 x 0.05 = $930.	$25 600 x 0.05 = $1 280.	$65 500 x 0.05 = $3 275.
Savings:	$10 500 x 0.04 = $420.	$20 500 x 0.04 = $820.	$18 600 x 0.04 = $744.	$25 600 x 0.04 = $1 024.	$65 500 x 0.04 = $2 620.

OBJECTIVES	NOTES TO TEACHERS
3. to calculate the amounts a family would spend on a monthly basis on the various income categories, given the annual income and the breakdown of the average income dollar;	Example: Using the information in chart #2 for the breakdown of the average income dollar and the annual salaries from the previous example, calculate the monthly dollar amounts the families of each of the persons would spend on the various income categories.
4. to explain the various sources of income;	Income may be categorized into three categories; 1) Regular Income: take-home pay. Budgets should be based on take-home pay (net salary), NOT income before deductions (gross salary). 2) Additional Income: any income you expect to receive regularly; for example: - family allowance - pensions - insurance benefits - child support - interest 3) Other Income: include such items as: - bonuses - overtime - tips Note: Unless this "other income" comes in regularly and/or is fairly certain it should not be relied upon.
5. to define and to explain fixed expenses;	Fixed expenses are definite amounts that must be paid at regular intervals. Examples of fixed monthly expenses include mortgage or rent payment, telephone, hydro, etc.

OBJECTIVES	NOTES TO TEACHERS
	Explain that when an item such as car insurance or life insurance is to be paid only once a year, it is recommended that a certain amount of money be set aside in each budget period. Therefore, when determining a monthly budget divide the annual amount by twelve to find the monthly expense.
6. to define and to explain variable expenses;	Variable monthly expenses are spending categories which usually occur every month, but in different amounts. When planning a budget, estimate these variable expenses.
7. to define and to explain a reserve fund;	Saving is one part of the budget that is often overlooked. Financial planners suggest that every individual have a cash reserve of 2 to 3 month's pay to meet unexpected expenses. This will prevent a person from encountering serious financial difficulty if something unexpected occurs.
8. to identify the steps required to organize a budget;	Budgets can be set up weekly, bi-weekly, monthly or yearly. It is recommended that a budget be set up to match a person's pay period. In this course, all budgets will be set up on a monthly basis. Steps to organize a budget include: - determine average monthly income - determine total monthly fixed expenses - determine total monthly variable expenses - determine reserve fund - prepare a monthly budget form

9. to prepare, on a budget form, a monthly budget for a family.

Example:

Allan Smythe, an administrative assistant, receives a salary of $450 a week. After his employer makes deductions from his gross weekly salary, his net weekly salary is $360 a week. His wife receives a monthly family allowance cheque of $52.25 for their two children. Their fixed expenses include:

a) mortgage payments on a home of $425 a month
b) payments on a car of $180 a month
c) average monthly telephone bill $9.60
d) average monthly hydro bill $70.00
e) yearly car insurance premium $258.00
f) yearly life insurance premium $240.00
g) yearly home insurance on a $40 000 home at a premium rate of $0.42 per $100 of insurance
h) property taxes on the home. The home is assesse[illegible] property tax purposes at $13 000 and the mill rate is 54 mills.

The family estimates its variable expenses to be:

a) groceries $400 per month
b) clothing $480 per year
c) car maintenance $300 per year
d) gasoline $70 per month
e) entertainment $450 per year
f) vacation $600 per year
g) newspapers and periodicals $90 per year

Have students prepare a monthly budget for the Smythe family for the month of June.

Explanation:

- Monthly take-home pay: $\frac{\$360 \times 52}{12} = \$1\ 560.$
- Monthly car insurance: $\frac{\$258}{12} = \$21.50$
- Monthly life insurance: $\frac{\$240}{12} = \$20.00$
- Home Insurance: $\frac{\$40\ 000}{100} \times 0.42 \div 12 = \$14.00$
- Property Taxes: $\$13\ 000 \times \frac{54}{1\ 000} \div 12 = \$58.50$
- Clothing: $\frac{\$480}{12} = \$40$
- Car Maintenance: $\frac{\$300}{12} = \$25$
- Entertainment: $\frac{\$450}{12} = \$37.50$
- Vacation: $\frac{\$600}{12} = \$50.00$
- Newspapers: $\frac{90}{12} = \$7.50$

Reserve Fund: $1612.25 x 2 ÷ 12 = $268.71

THE SMYTHE FAMILY

BUDGET FORM

for the month of June

INCOME:		
Regular Annual Income	$18 720.00	AVERAGE
Additional Annual Income	$ 627.00	MONTHLY
Other Income	$ -	INCOME
Total Annual Income	$19 347.00	1) $1 612.15
EXPENSES - MONTHLY FIXED EXPENSES		
Mortgage or Rent	$ 425.00	
Car Payment	$ 180.00	
Telephone	$ 9.60	
Hydro	$ 70.00	
Other Utilities	$ -	
Car Insurance	$ 21.50	
Life Insurance	$ 20.00	
Other: Home Insurance	$ 14.00	TOTAL
Property Taxes	$ 58.50	MONTHLY
Total Fixed Expenses	$ 798.60	2) $ 798.60
VARIABLE EXPENSES		
Groceries	$ 400.00	
Clothing	$ 40.00	
Car Maintenance	$ 25.00	
Gasoline	$ 70.00	
Credit Card Debt	$ -	
Entertainment	$ 37.50	
Vacations	$ 50.00	
Other: Newspaper & Periodicals	$ 7.50	
	$	
	$	TOTAL
	$	MONTHLY
Total Variable Expenses	$ 630.00	3) $ 630.00
RESERVE FUND		
Target Size (2 month's income)	$ 3 224.50	
Monthly Deposit required		4) $ 268.71

SUMMARY				
1) Net Income for Month			#1	$1 612.25
2) Fixed Expenses	#2	$ 798.60		
3) Variable Expenses	#3	$ 630.00		
4) Reserve Fund Payment	#4	$ 268.71		
Total 2 + 3 + 4				$1 697.31
5) Amount Available for Other Savings or Expenditures (or Deficit)			#5	$ (85.06)

Note: If the person is in a Deficit Position, the Reserve Fund Payment must be reduced by: RESERVE FUND PAYMENT - DEFICIT = NEW RESERVE PAYMENT
$268.71 - $85.06 = $183.65

Note:

1. The Smythe family is in a deficit position. Therefore, they must reduce their reserve fund payment from: $268.71 to $183.65, as illustrated on their budget form, in order to balance the budget.
2. In order to be consistent, whenever a person's take-home is weekly, this amount should be multiplied by 52 and then divided by 12 to get the average monthly income.
3. All yearly expenses should be divided by 12 to get the average monthly expense.

ENRICHMENT:

1. As part of a project, the students may prepare an Income and Expense Record as illustrated on p. 112 of the Math 101 course. In order to make a sound budget, estimates must be based on accurate figures. Looking at past figures and estimating future needs based on these amounts is a good starting point for a budget. Once these totals have been calculated the students could prepare a budget for themselves or their family.
2. Provide the students with or have them obtain copies of or information about budgets prepared by town/city, municipal governments, provincial and federal governments. Ask the students to determine and to discuss how these budgets influence family budgets.

BUDGET FORM

for the month of ______________

INCOME:			
Regular Annual Income	$		AVERAGE
Additional Annual Income	$		MONTHLY
Other Income	$		INCOME
Total Annual Income	$	1)	$
EXPENSES - MONTHLY FIXED EXPENSES			
Mortgage or Rent	$		
Car Payment	$		
Telephone	$		
Hydro	$		
Other Utilities	$		
Car Insurance	$		
Life Insurance	$		
Other: Home Insurance	$		TOTAL
Property Taxes	$		MONTHLY
Total Fixed Expenses	$	2)	$
VARIABLE EXPENSES			
Groceries	$		
Clothing	$		
Car Maintenance	$		
Gasoline	$		
Credit Card Debt	$		
Entertainment	$		
Vacations	$		
Other: Newspaper & Periodicals	$		
	$		
	$		TOTAL
	$		MONTHLY
Total Variable Expenses	$	3)	$
RESERVE FUND			
Target Size (2 month's income)	$		
Monthly Deposit required		4)	$

SUMMARY				
1) Net Income for Month			#1	$
2) Fixed Expenses	#2	$		
3) Variable Expenses	#3	$		
4) Reserve Fund Payment	#4	$		
Total 2 + 3 + 4				$
5) Amount Available for Other Savings or Expenditures (or Deficit)			#5	$

Note: If the person is in a Deficit Position, the Reserve Fund Payment must be reduced by: RESERVE FUND PAYMENT - DEFICIT = NEW RESERVE PAYMENT

B. TAXATION

There are three levels of government in Canada: municipal, provincial and federal. Each of these governments provides services for the people under its jurisdiction. To do this, governments must have money to pay their employees and to pay for the services provided. Governments prepare estimates and budgets for the cost of the services required by the people who elect them and for the money/income they collect from the people in the form of different types of taxes to provide these services. This portion of the consumer core module examines mathematics related to income tax, real property tax, federal sales tax, custom tariffs and excise taxes.

Income Tax

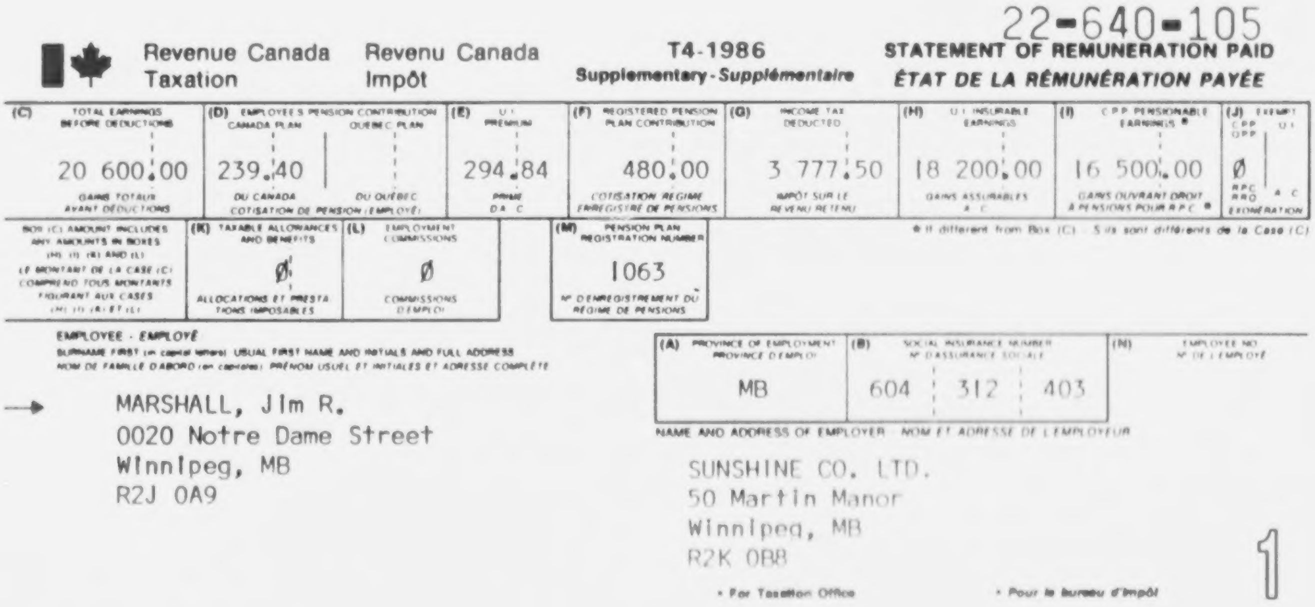
Note: Mathematics 201 introduced the student to the concept of an individual income tax return. The purpose of this unit is to complete an income tax return for a family. In the Mathematics 201 Exercises, there is a suggested approach to introducing the tax form. Teachers should review this material with the student as required. Both Mathematics 201 and Mathematics 301 suggest using the "Teaching Taxes" resources published by the federal government. A Teacher's Manual for Teaching Taxes and a Teaching Taxes Workbook are available from:

Teaching Taxes
Revenue Canada, Taxation
Distribution and Material Control
Ottawa, Ontario
K1A 1C2

These resources should be ordered in September. They will be sent to schools the following January.

OBJECTIVES	NOTES TO TEACHERS
The student should be able:	
1. to identify the four basic sections on an income tax return, given a blank copy of the return;	Main sections include: a) Identification b) Calculation of total income c) Calculation of taxable income d) Summary of tax and credits

OBJECTIVES	NOTES TO TEACHERS
2. to fill out the appropriate information in all four sections of a T1 General Form;	One approach to this section might be for the teacher to redo the SAMPLE A Question from the Mathematics 201 Exercises with the students. This will enable students to refresh themselves with the income tax return. Then, the teacher and students could work together through the following family example before the students attempt exercise 1.
3. to calculate the refund or the balance due for a family.	Example: The following T4 shows a record of Jim Marshall's earnings and deductions for the year. (Latest available forms and tables should be used.)



22-640-105

Revenue Canada Taxation | Revenu Canada Impôt | **T4-1986** **Supplementary - *Supplémentaire*** | **STATEMENT OF REMUNERATION PAID** ***ÉTAT DE LA RÉMUNÉRATION PAYÉE***

(C) TOTAL EARNINGS BEFORE DEDUCTIONS	(D) EMPLOYEE'S PENSION CONTRIBUTION CANADA PLAN	QUEBEC PLAN	(E) U.I. PREMIUM	(F) REGISTERED PENSION PLAN CONTRIBUTION	(G) INCOME TAX DEDUCTED	(H) U.I. INSURABLE EARNINGS	(I) C.P.P. PENSIONABLE EARNINGS *	(J) EXEMPT C.P.P. Q.P.P.	U.I.
20 600.00	239.40		294.84	480.00	3 777.50	18 200.00	16 500.00	0	
GAINS TOTAUX AVANT DÉDUCTIONS	DU CANADA COTISATION DE PENSION (EMPLOYÉ)	DU QUÉBEC	PRIME D'A.-C.	COTISATION RÉGIME ENREGISTRÉ DE PENSIONS	IMPÔT SUR LE REVENU RETENU	GAINS ASSURABLES A.-C.	GAINS OUVRANT DROIT À PENSIONS POUR R.P.C. *	R.P.C. R.R.Q.	A.-C. EXONÉRATION

BOX (C) AMOUNT INCLUDES ANY AMOUNTS IN BOXES (H), (I), (K) AND (L) LE MONTANT DE LA CASE (C) COMPREND TOUS MONTANTS FIGURANT AUX CASES (H), (I), (K) ET (L)	(K) TAXABLE ALLOWANCES AND BENEFITS	(L) EMPLOYMENT COMMISSIONS	(M) PENSION PLAN REGISTRATION NUMBER
	0	0	1063
	ALLOCATIONS ET PRESTATIONS IMPOSABLES	COMMISSIONS D'EMPLOI	Nº D'ENREGISTREMENT DU RÉGIME DE PENSIONS

* If different from Box (C) - S'ils sont différents de la Case (C)

EMPLOYEE - *EMPLOYÉ*
SURNAME FIRST (in capital letters) USUAL FIRST NAME AND INITIALS AND FULL ADDRESS
NOM DE FAMILLE D'ABORD (en capitales) PRÉNOM USUEL ET INITIALES ET ADRESSE COMPLÈTE

→ MARSHALL, Jim R.
0020 Notre Dame Street
Winnipeg, MB
R2J 0A9

(A) PROVINCE OF EMPLOYMENT PROVINCE D'EMPLOI	(B) SOCIAL INSURANCE NUMBER Nº D'ASSURANCE SOCIALE	(N) EMPLOYEE NO. Nº DE L'EMPLOYÉ
MB	604 312 403	

NAME AND ADDRESS OF EMPLOYER - NOM ET ADRESSE DE L'EMPLOYEUR

SUNSHINE CO. LTD.
50 Martin Manor
Winnipeg, MB
R2K 0B8

* For Taxation Office * Pour le bureau d'impôt

1

OBJECTIVES | NOTES TO TEACHERS

Jim's wife, Megan, earned $1400 working part-time during the year. They have one child, Sarah, who is two years old. Jim contributed $1200 into a Registered Retirement Savings Plan during the year. Also, Jim paid $126 for union dues during the year. The Marshalls earned $129 interest on a savings account. They live in a two-bedroom apartment for which they pay $375 a month rent. The family received taxable family allowance benefits of $323.16 during the year. Calculate the refund or balance due for the family. (Note: Mrs. Marshall will be required to complete a Schedule 10 - Child Tax Credit - in order to calculate the total family refund or balance due.)

Real Property Tax

The student should be able:

1. to calculate the rate of property tax in cents on a dollar, percent rate and mills, given the total budgetary requirement and the total assessment for a municipality;

Note: "Assessed Value" is NOT the same as "market value"; it is simply an artificial valuation, set by a provincial tax assessor for taxation purposes.

Example:

A municipality has a total property assessment of $375 000 000. The amount to be raised from real property taxes is $22 500 000. Find the tax rate expressed in cents per dollar, percent rate and mills, which the municiipality must set to collect the necessary amount.

$$\text{Rate:} = \frac{\text{Total Tax to be Raised}}{\text{Total Assessed Valuation}}$$

$$= \frac{22\ 500\ 000}{375\ 000\ 000}$$

$$= \underline{0.06}$$

OBJECTIVES | NOTES TO TEACHERS

This means a tax rate of six cents for every dollar of assessed valuation or 6% for every dollar of assessed value.

Students must know that there are 1000 mills in a dollar. Therefore, if the tax rate was \$0.06 per dollar, this would be equal to:

\$100 x 0.06 = 60 mills

MILLS are the standard method of expressing property taxes in Manitoba.

2. to calculate property tax on residential property, given the assessed value and the rate of taxation.

Example:

Joan Brown owns a home assessed at \$20 500. The current tax rate is 60 mills per dollar. What amount must she pay in property tax?

$$\text{Tax} = \text{Assessed Value} \times \text{Rate}$$
$$= \$20\,500 \times \frac{60}{1000} \text{ or } \frac{20\,500}{1000} \times 60$$
$$= \underline{\$1230}$$

Make students aware that items such as "local improvements" can increase the tax bill by a large amount above the basic rate established for the municipality.

Hidden Taxes

The term, "Hidden Taxes," refers to the fact that the government taxes many items of which the average customer is unaware. These hidden taxes include federal sales tax, custom duties and excise taxes.

The student should be able:

1. to calculate federal sales tax;

Note: A general federal sales tax of approximately 10% is imposed by the federal government on the manufactured sale price of all goods produced and manufactured in Canada. In addition, this tax applies to the duty paid value of all imported goods. The government, at its discretion, may establish exemptions in some categories depending on end use. Rates of federal sales tax vary. Each year's current rate should be checked.

Example:

Calculate the federal sales tax on an automobile that had a manufactured sale price of $6250.

Solution: The federal sales tax would be $6250 x 10% = $625.00.

This tax will be added on to the list price of the vehicle.

2. to calculate customs duties (tariffs) given a table of customs tariffs;

The table which follows identifies three categories of custom tariffs:

1) British Preferential Tariff: special rates for imports from Great Britain and members of the British Commonwealth of Nations (Australia, Indian, Nigeria, Bahamas, Bermuda, etc.).

2) Most Favoured Nation Tariff: rates set to encourage trade with certain nations with whom Canada has a trade agreement (i.e., United States, Switzerland, Japan, France, the Soviet Union, China, Italy, etc.).

3) General Tariff: rates set for imports from nations with whom Canada has no special agreement.

The tariff is collected in one of two ways:

1) Ad Valorem Duty: a percent rate of the price paid by the Canadian importer.
2) Specific Duty: the tax is set per unit of mass or measure: a specified number of cents per kilogram, per metre, etc.

CUSTOM TARIFFS

Goods	British Pref.	Most Favoured Nation	General	Goods	British Pref.	Most Favoured Nation	General
Automobiles	Free	15%	27.5%	Jewellery, gold	20%	25%	45%
Baskets, wire	10%	17.5%	36%	Jewellery, not gold	20%	25%	45%
Bottles, glass	15%	15%	32.5%	Pencils, lead	10%	15%	36%
Brushes, artist	Free	17.5%	30%	Perfume (no-alcohol)	15%	15%	40%
Cabinets, wood	15%	20%	45%	Phonographs and parts thereof	10%	15%	25%
Combs	10%	20%	30%	Radio and television apparatus	Free	15%	25%
Cellophane, not printed	10%	10%	20%	Tobacco, cut per pound	40¢	40¢	75¢
Drills, hand	10%	15%	36%	Watches	15%	20%	36%
Fabrics, wholly of cotton	22.5%	22.5%	36%				

The custom tariffs rates are determined by government policies and objectives and may change from time to time.

Example:

Calculate the ad valorem duty on 60 colour TV sets priced at $495 each which were shipped from Japan, a Most Favoured Nation.

Solution: $495 x 60 = $29 700

Duty or Tariff: $29 700 x 15% = $4455

3. to calculate excise taxes given a table of excise taxes.

Excise taxes are an additional tax levied on some luxury items whether they paid are produced in Canada or imported. The excise tax is based on the duty value of imported goods at the port of entry or on the manufacturer's sale price of Canadian products at the time of sale by the manufacturer.

EXCISE TAXES

Goods	Rate
Lighters	10¢ per lighter
Coin, disc or token operated games or amusement devices	10%
Smoker's accessories	10%
Cigars	20.5%
Matches	10%
Clocks and watches	10% • of the amount by which the sales price or duty-paid value of each watch exceeds $50.00.
Cigarettes (for each five cigarettes or fraction of five)	3¢
Tobacco, manufactured (per pound)	90¢
Playing cards (per pack)	20¢
Wine (per gallon) less than	25¢
Champagne and sparkling wines (per gallon)	$2.50

Example:

Calculate the excise tax on forty boxes of matches invoiced at $2.25 per box.

Solution: Total Cost: 40 x $2.25 = $90.00

Excise Tax: $90.00 x 10% = $9.00

C. INSURANCE

Students should be aware that people take risks every day of their lives - their houses may burn down, their cars may be stolen or damaged, they may suffer serious injury, they may injure someone else and have to compensate the injured party, etc. The major purpose of all insurance is to protect people against unexpected financial loss.

The student should be able to see how insurance companies calculate premiums for different types of insurance policies. At the end of this unit, students should be able to calculate the premiums for life, property and automobile insurance, using insurance rate tables.

Points to Emphasize:

Stress that insurance is a protection against financial loss rather than against death or theft. In all kinds of insurance, the basic cost is calculated on the risk factor.

Teaching Suggestion:

The tables in this section are provided by the Manitoba Public Insurance Corporation (MPIC). Teachers are encouraged to research policies by comparing premiums of competing insurance companies.

Kinds of Insurance

Three types of insurance or ways of providing against possible financial loss that will be examined are:

1. Life Insurance
2. Property Insurance
3. Automobile Insurance

OBJECTIVES	NOTES TO TEACHERS
Life Insurance	
The student should be able:	
1. to explain and use the insurance vocabulary;	Some basic insurance definitions include:

1) Beneficiary: a person named to the face of the policy in the event a person dies; for example, a wife receives the face of the policy when her insured husband dies.

2) Face of the policy or risk: the sum of money paid by the insurer to the insured in the event of a financial loss.

3) Insurer: the insurance company.

4) Policy: the insurance contract that contains the legally enforceable agreement between the insurer and insured.

5) Policy holder or insured: the person to whom the insurance company contracts to pay the face of the policy or the risk.

6) Premium: the amount of money the insured pays the insurance company for the policy. Payments are made in advance and may be made monthly, quarterly, semi-annually or annually.

2. to demonstrate an understanding of the four types of life insurance policies;

Note: Stress that the students must carefully consider the following when taking out life insurance.

1. a policy most suitable to the individual keeping in mind:
   a) financial situation
   b) age
   c) sex
   d) marital status
   e) purpose of insurance
   f) flexibility for future

2. the size of the policy that can be afforded

Four types of Life Insurance

1. Term Insurance
2. Straight Life Insurance (Whole Life or Ordinary Life Insurance)
3. Limited-Payment Life Insurance
4. Endowment Insurance

3. to define and to understand the nature of each type of life insurance policy, including:
   a) advantages,
   b) benefits received,
   c) characteristics;

1) Term Insurance

- insurance that provides only protection against loss
- it is purchased for a limited period of time
- if you die during the term your beneficiaries receive the amount; if you die after the term they receive nothing
- some policies are renewable but usually at a higher rate because, naturally, you will be older

Permanent Insurance

- permanent insurance differs from term insurance in three ways:
  1. provides lifetime protection
  2. premiums do not change regardless of how long a policy is in force
  3. permanent forms of life insurance are a combination of savings and insurance
- when you buy permanent insurance you pay more at the start than you would for protection alone
- the difference in premiums is placed in a fund that grows larger each year. The amount in the fund at any time is the cash value of the policy

2) Straight Life

- the insured pays premiums for as long as they want the protection; for example, up to age 65 (also called Whole Life)

3) Limited Payment Life

- premiums are paid for a stated number of years; for example, 20 years
- after the last premium is paid, the policy becomes paid up and remains in effect for the remainder of the insured person's life
- since fewer premiums are paid than straight-life, each premium will be higher than straight-life

4) Endowment Insurance

- actually an insured savings plan
- the policy holder agrees to pay premiums for a definite period of time. At the end of that time, the cash value of the policy is equal to its face value. The policy is then said to mature, and the insurance expires
- the policy holder may take the money in a lump sum or as a monthly income for a certain number of years or for life
- the most expensive type of insurance because of the rapidly rising cash values

4. to calculate premium payments on an annual basis;

Example: Have students use the table following to calculate the annual premium of a 15-year, $25 500 Term Insurance policy issued at age 25.

LIFE INSURANCE: ANNUAL PREMIUMS
(PER $1000 WORTH OF INSURANCE)

AGE AT ISSUE	TERM		WHOLE LIFE (TO AGE 85)	LIMITED-PAYMENT LIFE (TO AGE 65)	ENDOWMENT	
	10 YEARS	15 YEARS			TO AGE 60	TO AGE 65
15	$3.58	$4.08	$11.14	$11.82	$14.35	$14.98
16	3.64	4.10	11.48	12.21	14.85	15.86
17	3.70	4.13	11.82	12.60	15.34	16.74
18	3.76	4.15	12.16	12.99	15.84	17.62
19	3.83	4.17	12.50	13.38	16.34	18.50
20	3.90	4.20	12.84	13.77	16.84	19.37
21	3.96	4.22	13.25	14.26	17.48	20.17
22	4.02	4.24	13.66	14.75	18.13	20.97
23	4.09	4.26	14.07	15.24	18.78	21.77
24	4.16	4.28	14.48	15.73	19.45	22.57
25	4.23	4.31	14.87	16.23	20.07	23.38
26	4.29	4.42	15.39	16.90	20.92	24.48
27	4.36	4.53	15.91	17.58	21.77	25.58
28	4.43	4.64	16.44	18.26	22.62	26.68
29	4.50	4.75	16.97	18.94	23.48	27.79
30	4.57	4.86	17.48	19.61	24.34	28.90
31	4.74	5.10	18.15	20.54	25.52	30.49
32	4.91	5.34	18.83	21.47	26.70	32.08
33	5.09	5.58	19.51	22.40	27.88	33.67
34	5.27	5.82	20.20	23.32	29.05	35.26
35	5.46	6.05	20.87	24.24	30.22	36.85
36	5.78	6.44	21.71	25.51	31.88	39.21
37	6.10	6.83	22.56	26.78	33.55	41.57
38	6.42	7.22	23.41	28.05	35.23	43.93
39	6.74	7.61	24.26	29.32	36.90	46.29
40	7.10	8.00	25.10	30.61	38.55	48.65
41	7.58	8.60	26.20	32.50	41.00	52.49
42	8.06	9.20	27.30	34.39	43.45	56.33
43	8.54	9.80	28.40	36.28	45.90	60.17
44	9.02	10.40	29.50	38.17	48.34	64.00
45	9.53	10.99	30.60	40.05	50.82	67.83
46	10.28	12.00	32.02	43.07	54.74	75.25
47	11.03	13.01	33.44	46.09	58.66	82.67
48	11.78	14.02	34.86	49.11	62.58	90.09
49	12.53	15.03	36.28	52.13	66.50	97.50
50	13.32	16.04	37.72	55.15	70.41	104.91
51	14.75	17.70	39.58	60.86	77.97	
52	16.18	19.36	41.44	66.57	85.53	
53	17.61	21.02	43.30	72.28	93.09	
54	19.04	22.68	45.16	77.99	100.64	
55	20.47	24.34	47.02	83.68	108.19	
56	22.61	25.90	49.60			
57	25.15	27.46	52.18			
58	27.49	29.02	54.76			
59	29.83	30.59	57.35			
60	32.17	32.17	59.94			

PREMIUM RATE TABLE

Solution: From the table the cost per $1000 of Term Insurance at age 25 is $4.31. Therefore, the total annual premium will be

$\frac{\$25\ 500}{1\ 000} \times \$4.31 = \$109.905$ which is rounded off to $109.91 per annum.

5. to calculate semi-annual, quarterly and monthly payments using a Premium Rate Table and a Periodic Premium Table;

Note: Many people decide to pay their insurance premiums on a monthly, quarterly or semi-annually basis. Insurance companies will provide a Periodic Premium Table to calculate the necessary payment. Because of the extra paper-work for the insurance company, the rate is increased by a small percent rate; for example, the semi-annual rate should be 0.50 but in the Periodic Premium Table the rate is 0.51.

PERIODIC PREMIUM TABLE

If Period Is:	Multiply Annual Premium By:
Semi-annual	0.51
Quarterly	0.26
Monthly	0.087

Example: Sam Worthington decides to purchase a whole-life policy valued at $75 000 at age 32. What would be his total annual payment if he made:

a) annual payments
b) semi-annual payments
c) quarterly payments
d) monthly payments

Solution:

a) Annual: $\frac{\$75\ 000}{1\ 000} \times \$18.83 = \underline{\$1\ 412.25}$

b) Semi-annual: $1 412.25 x 0.51 = $720.25
total payment: $720.25 x 2 = $1 440.50

c) Quarterly: $1 412.25 x 0.26 = $367.19
total payment $367.19 x 4 = $1 468.76

d) Monthly: $1 412.25 x 0.087 = $122.87
total payment $122.87 x 12 = $1 474.44

Note: The total annual premium or payment increases if the periodic payment method is chosen.

Property Insurance

6. to explain and use the vocabulary concerning property insurance;

Property insurance is insurance against a loss to real property such as a house, garage, barn, cottage, and personal property such as furniture, books,clothing, jewelry.

Types of Property Insurance

1) Fire and Extended Coverage Insurance:

- house insurance that pays for loss to the home from:
  - a) fire
  - b) hailstorms
  - c) windstorms
  - d) aircraft
  - e) vehicles
  - f) explosions

OBJECTIVES | NOTES TO TEACHERS

- these policies do not cover damage from floods and war
- these policies do not cover personal belongings in the home (You may be required to buy a separate policy for personal property.)

2) Homeowner's Package Policy:

- most homeowners will purchase a homeowner's policy which includes fire and extended coverage insurance on both the home and its contents
- it also pays extra living expenses while living away from home while repairs are being made
- it also provides theft insurance and a limited amount of insurance on personal property not in the home when stolen or destroyed

Major Advantages:

- pays only one premium rather than several
- policy costs 20 to 30 percent less than if a separate policy were bought for each type of coverage

Note: The Homeowner features are listed as well as the difference between standard and select homeowner rate eligibility on the chart following.

THE HOMEOWNER FEATURES

1 Included in the basic premium **at no additional cost** are
- (a) Contents coverage — 50% of building limit for homeowners
  — 60% of building limit with Replacement Cost
- (b) Comprehensive Personal Liability Limit — $300,000
- (c) Credit Card and Depositors Forgery coverage — limit $1,500
- (d) Counterfeit Paper Currency — $250 per transaction/$500 aggregate
- (e) Fire Department Charges up to $500
- (f) Damage to own property by vehicle driven by insured — excepting lawns, trees, shrubs, fences, driveways and walks
- (g) Coverage on lawns is added to the Named Perils coverage provided on trees, shrubs, and plants — limit $250 per plant (applicable only to home owners)
- (h) Fourteen (14) days Automatic Removal Permit Personal Property insured while in transit and at the new premises (within Canada)
- (i) Liability arising from the use of Outboard Motors — included up to 25 H.P.
- (j) Inflation Protection Coverage at Principal Residences — automatically increases the amount of coverage on both **Dwelling** and **Personal Property** by:
  - 2% after 3 months
  - 4% after 6 months
  - 6% after 9 months
  - 7½% on renewal
- (k) Increased "Special Limits of Liability"
  - (i) Money — $250
  - (ii) Jewellery & Furs — $1,500
  - (iii) Watercraft — $1,000
  - (iv) Off premises theft cover — 10% of coverage C or $2,500 — (whichever is greater)
- (l) Theft coverage from locked vehicles does not require visible signs of forcible entry
- (m) Out-of-Canada excess medical and hospital expense coverages to $10,000 per policy term
- (n) Smoke damage from a fireplace

2 Select Homeowner Rate Eligibility — Standard and Broad Homeowners policies are eligible for select rates only if **all of the following requirements are met:**
- (a) Dwelling insured to at least 90% of current replacement value
- (b) Minimum sum insured on dwelling $40,000
- (c) Not more than one loss in last three years
- (d) Owner-occupied one or two family dwelling built within last 25 years. This age limitation may be increased to 40 years if it can be proven that within the last ten (10) years the following improvements have been made
  - (i) completely re-roofed,
  - (ii) entire electrical system replaced and fuses replaced by circuit breakers,
  - (iii) entire plumbing system (including all hidden pipes) replaced,
  - (iv) new heating system installed

3) Tenant's Package Policy:

- these policies are developed to meet the needs of tenants
- these are combination policies that cover a tenant's personal property against fire and theft, settle claims on a tenant by guests who suffer physical injury as a result of a tenant's negligence, and provide for living expenses in the event of damage to, or destruction of, the property

Setting Rates

One of the factors to consider when setting rates is the amount of insurance purchased. A second factor is the location of your residence.

The Manitoba Public Insurance Corporation (MPIC) uses 4 categories:

1) Metro Winnipeg - in the metro area and within 300 metres (1000 feet) of a fire hydrant.
2) Table 1 - all other locations within 300 metres (1000 feet) of a fire hydrant and in the area of a responding fire department.
3) Table 2 - all locations not within 300 metres (1000 feet) of a fire hydrant but within 8 road kilometres (5 miles) of a responding fire department.
4) Table 3 - all other locations.

OBJECTIVES

7. to calculate premiums of a Fire Insurance Policy using appropriate tables;

NOTES TO TEACHERS

Example: Find the annual premium for Dwelling Fire Insurance for $50 000 using the Select Table - Building (Fire and Extended Coverage) if the person has a $50.00 deductible and if the dwelling is not near a fire hydrant but is within 8 kilometres of a responding Fire Department.

Solution: Using the table below, the premium is $127.00

SELECT TABLE—BUILDING

(See Page PL23 for Eligibility)

FIRE AND EXTENDED COVERAGE

Net Premiums — not subject to Package Discount

Amount of Insurance	Metro Wpg		Table 1		Table 2		Table 3	
	$50 Ded	$100 Ded	$50 Ded	$100 Ded	$50 Ded	$100 Ded	$50 Ded	$100 Ded
20,000	36	32	35	31	46	41	70	63
22,500	40	36	38	34	49	44	76	68
25,000	44	40	42	38	52	47	81	73
27,500	47	42	45	40	57	51	89	80
30,000	52	47	49	44	64	58	99	89
32,500	58	52	56	50	72	65	107	96
35,000	65	58	63	57	77	69	119	107
37,500	72	65	71	64	86	77	129	116
40,000	78	70	78	70	95	85	139	125
42,500	85	76	86	77	103	93	149	134
45,000	94	85	93	84	111	100	160	144
47,500	100	90	100	90	119	107	170	153
50,000	105	94	105	94	127	114	180	162
55,000	114	103	114	103	140	126	201	181
60,000	123	111	123	111	153	138	222	200
65,000	134	121	132	119	166	149	243	219
70,000	143	129	141	127	179	161	264	238
75,000	152	137	150	135	192	173	285	256
Each additional 5,000	9	8	9	8	13	12	21	19

BASIC O.L.S. — REDUCE ABOVE PREMIUMS 20%

Note: If an individual wishes to purchase coverage for the contents of the building with fire and extended coverage, they would refer to the following table:

SELECT TABLES—CONTENTS

(See Page PL23 for Eligibility)

FIRE AND EXTENDED COVERAGE

Net Premiums — not subject to Package Discount

	Metro Winnipeg		Table 1		Table 2		Table 3	
Limit	$50 Ded	$100 Ded	$50 Ded	$100 Ded	$50 Ded	$100 Ded	$50 Ded	$100 Ded
8,000	18	16	15	13	18	16	27	24
9,000	20	18	16	14	20	18	29	26
10,000	21	19	17	15	21	19	31	28
11,000	22	20	18	16	24	22	36	32
12,000	23	21	20	18	29	26	39	35
13,000	25	22	23	21	32	29	41	37
14,000	28	25	27	24	33	30	45	40
15,000	30	27	29	26	35	31	49	44
16,000	33	30	32	29	38	34	53	48
17,000	36	32	35	31	40	36	58	52
18,000	39	35	38	34	43	39	62	56
19,000	42	38	41	37	47	42	66	59
20,000	45	40	43	39	51	46	70	63
Each Add $2,000	3	3	3	3	3	3	3	3

8. to calculate premiums of a Homeowner's Policy using appropriate tables;

Example: John Horton wishes to insure his home for $70 000 and decides to purchase a Homeowner's Package Policy. His home is rated in the Standard Rate Classification; Table 2. Find his annual premium if he wishes to have a $100 deductible.

Solution: There are two tables below. One is for "Standard Rates" and one is for "Select Rates". Students should choose the table appropriate to the question.

Therefore, using the "Standard Rate" table with a $70 000 limit in Table 2, with a $100 deductible, annual premium is $246.00.

HOMEOWNER "STANDARD RATES"
(STANDARD COVER WITH EXTENDED O.L.S.)

	Metro		Table 1		Table 2		Table 3	
Limit	$50 Ded	$100 Ded	$50 Ded	$100 Ded	$50 Ded	$100 Ded	$50 Ded	$100 Ded
$ 25,000	92	83	86	77	111	100	131	118
27,500	97	87	90	82	117	104	144	130
30,000	101	91	95	85	122	110	159	143
32,500	109	98	104	94	135	122	172	154
35,000	115	103	110	99	143	129	184	165
37,500	119	107	115	103	152	136	196	176
40,000	123	111	118	106	154	139	205	184
42,500	131	118	124	112	164	148	217	195
45,000	136	123	130	117	170	153	226	203
47,500	144	129	137	124	179	161	237	213
50,000	149	134	143	129	186	167	244	219
55,000	159	143	156	140	204	184	271	244
60,000	184	166	173	155	228	205	301	271
65,000	198	178	184	166	248	224	328	295
70,000	217	195	203	183	274	246	360	324
75,000	237	213	220	198	303	273	397	357
80,000	253	228	235	212	324	292	423	380
85,000	269	242	251	226	346	311	450	406
90,000	286	258	268	241	368	331	478	430
95,000	303	273	283	255	388	349	507	457
100,000	319	287	297	268	408	367	533	480
105,000	338	304	315	283	434	391	566	509
110,000	356	320	333	300	460	414	597	538
115,000	374	336	351	316	485	437	630	567
120,000	392	354	369	332	511	460	663	596
125,000	411	370	386	347	537	483	695	626
130,000	429	386	404	364	563	506	727	654
135,000	447	402	421	379	589	530	759	684
140,000	465	419	440	396	614	552	792	713
145,000	484	435	458	412	640	576	824	741
150,000	502	452	476	428	667	600	856	770
Ea Add $5,000 -	18	17	17	16	26	23	33	29
CTS/Ea $1,000 -	3	3	3	3	5	5	6	5

* BASIC O.L.S. — REDUCE ABOVE PREMIUMS BY 10%
* BROAD FORM — INCREASE ABOVE PREMIUMS BY 25%
* SINGLE FAMILY OCCUPANCY — Use above premiums
* TWO FAMILY DWELLINGS Semi-Detached or Row Housing — Increase above premiums by 20%

HOMEOWNER "SELECT RATES"
(STANDARD COVER WITH EXTENDED O.L.S.)

	Metro		Table 1		Table 2		Table 3	
Limit	$50 Ded	$100 Ded	$50 Ded	$100 Ded	$50 Ded	$100 Ded	$50 Ded	$100 Ded
40,000	108	97	101	91	126	113	167	150
42,500	116	104	107	96	132	119	176	158
45,000	120	108	111	100	138	124	183	165
47,500	127	114	117	105	144	130	191	172
50,000	132	119	122	110	151	136	199	179
55,000	149	134	134	121	167	150	221	199
60,000	162	146	149	134	186	167	244	220
65,000	174	157	159	143	202	182	267	240
70,000	192	173	173	156	222	200	292	263
75,000	211	190	190	171	246	221	322	290
80,000	223	201	203	183	262	236	343	309
85,000	239	215	216	194	281	253	367	330
90,000	257	231	231	208	299	269	389	350
95,000	269	242	241	217	312	281	412	371
100,000	283	255	257	231	331	298	434	391
105,000	300	270	273	246	352	317	461	415
110,000	316	284	287	258	373	336	487	438
115,000	331	298	302	272	394	355	513	462
120,000	348	313	318	286	416	374	539	485
125,000	363	327	332	299	437	393	566	509
130,000	380	342	349	314	458	412	591	532
135,000	396	356	364	328	478	430	618	556
140,000	412	371	379	341	499	449	643	579
145,000	428	385	394	355	520	468	670	603
150,000	444	400	410	369	541	487	696	626
Each Add $5,000 -	16	14	15	13	21	19	26	23
CTS/ Each $1,000 -	3	3	3	3	4	4	5	4

* BASIC O.L.S. — REDUCE ABOVE PREMIUMS BY 10%
* BROAD FORM — INCREASE ABOVE PREMIUMS BY 25%
* SINGLE FAMILY OCCUPANCY — Use above premiums
* TWO FAMILY DWELLINGS Semi-Detached or Row Housing — Increase above premiums by 20%

9. to calculate premiums using Tenants' and Condominium Owners' Contents Package Rates;

Example: Joe Renter lives in an apartment in Winnipeg. He feels that the contents of his apartment are valued at $15 000. If he purchases a tenants' package policy, what would be his annual premium if he chooses a $100 deductible?

Solution: Using the Tenants and Condominium Owners Contents Package Rates table listed below, the premium is $65.00.

TENANTS AND CONDOMINIUM OWNERS CONTENTS PACKAGE RATES

For tenants of private dwellings and apartments (other than rooming houses or risks situated in buildings containing a commercial occupancy)

	TENANTS				CONDOMINIUM OWNERS			
Limit	Metro Winnipeg		All Other Tables		Metro Winnipeg		All Other Tables	
	$50 Ded	$100 Ded	$50 Ded	$100 Ded	$50 Ded	$100 Ded	$50 Ded	$100 Ded
7,000	54	49	44	40	—	—	—	—
8,000	57	51	48	43	—	—	—	—
9,000	59	53	51	46	—	—	—	—
10,000	61	55	54	49	—	—	—	—
11,000	63	57	58	52	—	—	—	—
12,000	66	59	61	55	—	—	—	—
13,000	68	61	64	58	—	—	—	—
14,000	70	63	67	60	—	—	—	—
15,000	72	65	70	63	66	59	63	57
16,000	74	67	72	65	68	61	66	59
17,000	78	70	76	68	70	63	69	62
18,000	80	72	79	71	72	65	71	64
19,000	83	75	83	75	76	68	76	68
20,000	86	77	87	78	78	70	79	71
22,000	92	83	94	85	84	76	87	78
24,000	99	89	102	92	91	82	94	85
26,000	106	95	110	99	98	88	102	92
28,000	112	101	118	106	104	94	110	99
30,000	119	107	126	113	111	100	118	106
35,000	136	122	142	128	128	115	134	121
40,000	152	137	159	143	144	130	151	136
45,000	169	152	176	158	161	145	168	151
50,000	186	167	192	173	178	160	184	166
Each Add $1,000	3	3	3	3	3	3	3	3

OBJECTIVES	NOTES TO TEACHERS
10. to calculate the loss paid using the co-insurance clauses in insurance policies;	Many people do not insure their buildings for their full value. In order to encourage people to purchase proper insurance, many policies require that in order to receive full payment for any partial loss, the owner must insure his building for at least 80% of its replacement value. This is known as the 80% co-insurance clause. If the property is insured for less than 80% of its value, the owner must share part of the loss or damage caused by fire. This type of clause is commonly found in fire insurance policies on commercial buildings, but is rarely included in dwelling policies. We use the following formula to solve problems of this type: $\text{Amount paid by insurance company} = \frac{\text{Face of Policy}}{\text{80\% of the Property Value}} \times \text{loss}$ Example: A building valued at \$80 000 is damaged by fire to the extent of \$40 000. Calculate the loss paid by the insurance company if the building was insured under an 80% co-insurance clause for \$65 000. Solution: 80% of the value = \$80 000 x 80% = \$64 000. Therefore, since this person has more than the required minimum he/she would be paid the entire \$40 000 loss. Example: Given the above situation, what would be the loss paid by the insurance company if the building is insured for \$48 000? Solution: $\text{Amount Paid} = \frac{48\ 000}{64\ 000} \times 40\ 000 = \underline{\$30\ 000}$ Therefore, the owner would have to pay the difference himself.

Automobile Insurance

11. to explain the various factors affecting the cost of automobile insurance;

In Manitoba all vehicles are required to be insured by the Manitoba Public Insurance Corporation, or Autopac, as it is commonly referred to.

Factors Affecting the Cost of Insurance

1) Value of the Car: different makes, models and years of a vehicle determine the cost to repair and the Actual Cash Value (ACV) if the car is to be written off. More expensive cars, therefore, have a higher rate for their insurance.

2) Use of the Vehicle: insurance companies establish rates based on what the vehicle is used for.

   The most common uses of passenger vehicles are:

   - Preferred: a vehicle used for pleasure driving and can only be driven to and from work or school up to 1609 km (1000 miles) a year or four days a month.
   - All Purpose: a vehicle which is used to drive to and from work, or part way to and from work, but cannot be used for business purposes for more than 1609 km (1000 miles) per year.
   - School Car: used for pleasure driving and to and from or part way to and from school, but cannot be driven to and from or part way to and from work for more than 1609 km (1000 miles) a year, or more than four days a month.
   - Business (19): a vehicle:
     - registered in the name of a company
     - owned by the Government of Manitoba, an agency of the Government of Manitoba, or a municipality
     - otherwise registered and used for more than 1609 km (1000 m) during a registration year in connection with any business, trade or occupation (does not include charitable, non-profit and religious organizations)

- Farm Car: a passenger vehicle owned by a person who lives on a farm in Manitoba and is primarily occupied in farming or is retired from farming.

- Other uses include Farm Car--All Uses, U-Drive, Delivery Car, Public Bus, Taxi/Livery.

3. Territory or Place You Live:

Statistics indicate that certain areas of the province are likely to have more accidents and therefore would be required to pay a higher premium. For example:
  - Winnipeg - Territory 1
  - Southern Rural Manitoba - Territory 2

4. Deductibles: the amount for which you are responsible. Autopac has 3 possibilities:
  - $350
  - $250
  - $150

If you decide to reduce your deductible from $350, which is desirable for many reasons, you must pay for this option.

5. Third Party Liability: Coverage of a person against damages they cause to other people and/or to their property.
  - minimum amount $200 000
  - if you wish to purchase more you must pay an extra premium to get this coverage up to $300 000, $500 000, $1 000 000, or $2 000 000

6. Underinsured Motorist: a new concept which gives protection against loss suffered by a motorist with insufficient or non-existent insurance.

This policy insures you will be adequately compensated for your loss regardless of the financial circumstances of the other driver. It must be purchased in the same amount as your third party liability coverage.

Note: Every year, Autopac publishes a new booklet outlining the coverage provided by the policy and any recent changes in the regulations. These booklets are available from your local Autopac agent free of charge.

12. to calculate premiums using V.I.C. Tables and Premium Tables;

Example:

Find the annual Autopac premium for a 1984, V8, 4-door Chev Impala, with $150 deductible and a $1 000 000 third party liability, with the insurance use as "preferred," located in Metro Winnipeg (Territory 1).

Solution: Refer to 1988 V.I.C. Table and locate the described automobile. Under year, column 84, the basic licence plate fee is indicated as $32. Using the code A, refer to Territory 1 chart, rating group A, and preferred insurance use. Basic insurance = $415. For $150 deductible add $140 and the third party liability equals $10. Therefore, total premium = $597.00.

Territory 1 -	Licence Plate Fee =	$ 32.00
	Basic Insurance	415.00
	$50 deductible	140.00
	$1 000 000 3rd party	10.00
		$597.00

13. to compare rates paid for different territories and/or other variables such as use codes, third party liability, underinsured motorist coverage and deductibles.

Using appropriate tables, have students compare the costs of such factors as territorial rates, use codes, third party liability, underinsured motorist coverage, deductibles and the discounted rate for merit points. (Always use current tables.)

***IF BODY STYLE IS VAN OR STATION WAGON ADD 1 TO RATING GROUP —EXCEPT 0, 9, A, B, C or D**

VEHICLE DESCRIPTION

MAKE AND MODEL	NO OF CYL.	V.I.C.	89	88	87	86	85	84	83	82	81	80	79	78	77	76	75	74	73	72	71	70	69	68 Prior
CHEV G20 VAN 110WB				A	A	A	A	9	9	9	9	9	9	8	8	7	4	3	2	0	0			
**VAN*	6	ANE6A5		41	35	35	35	33	36	35	35	35	34	34	34	34	31	29	29	29	29			
**VAN*	8	ANE8A5		45	36	36	36	34	37	36	36	36	35	35	35	35	32	29	29	29	29			
CHEV G20 VAN 125 DIESEL					B	A	A	A																
**VAN*	6	B5G6A3					36	35																
**VAN*	8	B5G8A3			37	37	37	36																
CHEV G20 VAN 125WB				A	A	A	A	9	9	9	9	9	9	8	8	7	4	3	2	0	0	0		
**VAN*	6	ANF6A4		45	37	37	37	35	37	37	37	37	36	36	35	35	35	34	34	34	30	30		
**VAN*	8	ANF8A4		49	41	41	41	36	41	41	41	41	37	37	36	37	37	36	36	36	32	32		
-AMBULANCE	8	ANF884													52		52							
CHEV G30 SPORTSVAN 110																					0			
**VAN*	6	ANN6AT																			29			
**VAN*	8	ANN8AT																			29			
CHEV G30 SPORTSVAN 125				B	B	A	A	A	A	A	A	9	9	8	8	7	4	3	2	0	0			
**VAN*	6	ANP6AR											53	53	53	57	52	41	41	41	41			
**VAN*	8	ANP8AR		72	77	72	72	72	72	72	72	62	57	57	57	62	56	47	42	41	41			
CHEV G30 SPTVAN DIESEL					B	B	B	B																
**VAN*	8	B5K8AY			77	72	72	72																
CHEV G30 VAN 110WB													9	8	8	7	4	3	2	0	0			
**VAN*	6	ANG6A3											37	37	36	37	34	29	29	29	29			
**VAN*	8	ANG8A3											41	41	37	41	38	34	31	29	29			
CHEV G30 VAN 125 DIESEL					B	A	A	A																
**VAN*	8	B5H8A2			57	57	57	57																
CHEV G30 VAN 125WB				B	A	A	A	A	A	A	9	9	9	8	8	7	4	3	2	0	0			
**VAN*	6	ANH6A2		67	57	57	57	57					41	41	41	41	41	36	36	36	36			
**VAN*	8	ANH8A2		72	62	57	57	62	57	57	62	49	45	45	45	49	45	41	41	41	41			
CHEV G30 VAN 146WB													9											
**VAN*	8	AY38AG											41											
CHEV IMPALA							A	A	A	9	9	9	9	8	8	7	4	3	2	0	0	0	0	0
-CONVERT.	6	AAD616																						22
-CONVERT.	8	AAD816																		41	37	36	36	22
-2 DOOR	6	AAD626									31	31	33	33	33					37	35	34	34	22
-2 DOOR	8	AAD826									32	32	34	34	34	53	48	44	41	41	36	35	35	22
**ST. WAGON*	6	AAD636																						22
**ST. WAGON*	8	AAD836								36	37	37	39	41	39	66	60	54	43			37	37	22
-4 DOOR	6	AAD646					31	31	31	31	32	31	33	33	33	49				41	35	34	34	22
-4 DOOR	8	AAD846					32	32	32	32	33	32	34	34	34	53	48	44	41	41	36	36	35	22
CHEV IMPALA SS																		3	2	0	0	0	0	0
-CONVERT.	6	AAL61X																						22
-CONVERT.	8	AAL81X																						22
-2 DOOR	6	AAL62X																						22
-2 DOOR	8	AAL82X																44	41	41	36	35	36	22
-4 DOOR	6	AAL64X																						22
-4 DOOR	8	AAL84X																44	41	41	36	36	35	22
CHEV KINGSWOOD																				0	0	0	0	0
**ST. WAGON*	6	AAJ631																					37	22
**ST. WAGON*	8	AAJ831																		41	41	37	37	22
-AMBULANCE	8	AAJ881																					52	
			89	88	87	86	85	84	83	82	81	80	79	78	77	76	75	74	73	72	71	70	69	68 Prior

1988 AUTOPAC PREMIUMS TERRITORY 1

TIME PAYMENT ADMINISTRATION FEE $10.00

PASSENGER CARS (CLASS A-1)

INSURANCE USE	CODES	ALL PERILS DEDUCT.	RATING GROUPS (PER VIC TABLES)														THIRD PARTY LIABILITY AND UNDERINSURED MOTORIST			
																	300,000	500,000	1,000,000	2,000,000
			0	1	2	3	4	5	6	7	8	9	A	B	C	D	3/C	4/D	5/E	6/F
PREFERRED (01)	9	BASIC $350	190	220	240	255	265	282	301	321	346	381	415	461	506	550	12/4	32/6	62/10	87/15
	5	$250	26	29	35	42	46	53	59	65	72	72	72	72	72	72				
	4	$150	55	60	69	78	87	104	116	126	140	140	140	140	140	140				
ALL PURPOSE (02)	9	BASIC $350	245	294	312	327	344	365	389	412	447	516	540	632	690	756	12/4	32/6	62/10	87/15
	5	$250	26	29	35	42	46	53	59	65	72	72	72	72	72	72				
	4	$150	55	60	69	78	87	104	116	126	140	140	140	140	140	140				
FARMER (04)	9	BASIC $350	154	170	186	201	217	233	248	261	279	296	329	361	393	428	12/4	32/6	62/10	87/15
	5	$250	26	29	35	42	46	53	59	65	72	72	72	72	72	72				
	4	$150	55	60	69	78	87	104	116	126	140	140	140	140	140	140				
LEASE (03) OR BUSINESS (19)	9	BASIC $350	262	299	335	371	408	444	480	517	553	593	656	722	793	861	12/4	32/6	62/10	87/[illegible]
	5	$250	28	34	38	46	49	59	65	72	78	78	78	78	78	78				
	4	$150	60	64	77	86	95	113	126	140	155	155	155	155	155	155				

DELIVERY CARS (CLASS A-2)

DELIVERY CAR (37)	9	BASIC $350	262	299	335	371	408	444	480	517	553	593	656	722	793	861	12/4	32/6	62/10	87/15
	5	$250	28	34	38	46	49	59	65	72	78	78	78	78	78	78				
	4	$150	60	64	77	86	95	113	126	140	155	155	155	155	155	155				

FEE TO INCREASE MAXIMUM INSURED VALUE OVER $40,000—$15 PER $1,000 OF DECLARED EXCESS VALUE

FARM TRUCKS (CLASS A-6) AND FISHERMEN (CLASS A-5)

INSURANCE GROUPING	CODES	ALL PERILS DEDUCT.	1989 OR LATER	1982 TO 1988	1977 AND 1981	1970 TO 1976	1960 TO 1969	1959 AND PRIOR	300,000	500,000	1,000,000	2,000,000
UP TO 3630 kg. FARM TRUCK (14) FISHERMEN (24)	9	BASIC $350	330	256	199	154	143	143	12/4	32/6	62/10	87/15
	5	$250	28	28	28	23	23	23				
	4	$150	37	37	37	30	30	30				
3631 TO 9080 kg. FARM TRUCK (14) FISHERMEN (24)	9	BASIC $350	211	194	178	160	147	147	12/4	32/6	62/10	87/15
	5	$250	29	29	29	25	25	25				
	4	$150	41	41	41	34	34	34				
OVER 9080 kg. FARM TRUCK (14) FISHERMEN (24)	9	BASIC $350	271	251	233	209	191	191	12/4	32/6	62/10	87/15
	5	$250	34	34	34	29	29	29				
	4	$150	48	48	48	41	41	41				

FEE TO INCREASE MAXIMUM INSURED VALUE OVER $15,000—$10 PER $1,000 OF DECLARED EXCESS VALUE

D. MORTGAGES

Students should realize that some day they will be renting or purchasing a home. With the wide choice of housing facilities (semi-detached, mobile homes, etc.), students often have relatively little knowledge of the costs of buying and maintaining a home. An understanding of mortgage terms, sources of mortgage funds and a a checklist for things to look for when shopping for a home can be helpful.

Points to Emphasize

In the decreasing payments plan, the principal part of every payment remains unchanged and the interest part decreases. In the equal payments plan, the principal part of every payment increases and the interest part decreases.

Students should also be aware of the use of computers by financial institutes in calculating and amortizing mortgages. As an enrichment project, students can use computer programs.

Teaching Suggestions

Make use of local real estate agents, financial institutes and tax offices for proper documentation and current rates. Students could bring in advertisements from local newspapers of homes for sale or rent for discussion and preparation of various objectives.

OBJECTIVES	NOTES TO TEACHERS
The student should be able:	
1. to explain the meaning of terms associated with mortgages;	Integrate the terms as required. Terms are: Mortgage Mortgagor Mortgagee Offer to Purchase Agreement for Sale Title Real Estate Broker Closing date National Housing Act (NHA) Amortize Amortization Schedule Mills Second Mortgage Open Mortgage Closed Mortgage Equity Depreciation Statement of Adjustment Conventional Mortgage Loans

2. to demonstrate an understanding of financing the purchase of a home:
   - a) define the related terms,
   - b) understand sources of mortgage loans:
     - i) conventional first mortgage loans (75% of appraised value)
     - ii) National Housing Act (NHA) Loans (95% of property value),
   - c) explain methods of financing:
     - i) cash down payment and mortgage balance
     - ii) cash down payment and seller carries mortgage
     - iii) cash down payment and agreement of sale
     - iv) carry existing mortgage and second mortgage on balance;

Example:

Fred Small buys a house from his Uncle George. The purchase price is $70 000. Fred has saved $15 000 to be applied against the purchase. Uncle George will take back a mortgage for $45 000 at 10%, renewable in 5 years. Fred will have to take another mortgage for the balance of $10 000 at 16%.

a) What is a mortgage?
b) Who is the mortgagor? the first mortgagee? the second mortgagee?
c) What is the amount of the first mortgage?
d) What is the interest rate on the first mortgage? the second mortgage?
e) Why is the interest rate higher on one mortgage than on the other?

3. to calculate payments of mortgage loans using:

   a) the decreasing payments plan,

The student must understand that the mortgagor pays a fixed amount on the principal of the loan, plus interest on the unpaid balance of the loan, at regular intervals. (Review formula I = PRT.)

Note: Mortgage rates, releveant to the day, should be used.

Example:

Harold Brooks bought a home for $64 000. He paid $54 000 down and gave the seller a $10 000, 5-year mortgage at 18%. The mortgage was dated February 1st. Mr. Brooks agreed to make half-yearly payments of $1 000 plus interest on the unpaid balance, beginning July 31. Prepare a schedule of mortgage payments for 3 years.

Solution:

SCHEDULE OF MORTGAGE PAYMENTS

	Date	Regular Payment			Unpaid Balance	Owner's Equity
		Principal	Interest	Total		
1st year	February 1				$10 000.	$54 000.
			($10 000x.18x0.5)	($1000+900)	($10 000-1 000)	
	July 31	$1000.	$1900.	$1900.	9 000.	55 000.
2nd year	January 31	1000.	810.	1810.	8 000.	56 000.
	July 31	1000.	720.	1720.	7 000.	57 000.
3rd year	January 31	1000.	630.	1630.	6 000.	58 000.
	July 31	1000.	540.	1540.	5 000.	59 000.
4th year	January 31	1000.	450.	1450.	4 000.	60 000.

Explanation:

1. Payment on principal is the same each ½ year ($1 000).

2. Interest is calculated on previous unpaid balance ($10 000 x .18 x 0.5 = $900) at interest rate for ½ year.

3. Total payments are equal to principal payment plus interest ($1 000 + $900 = $1 900)

4. Unpaid balance is equal to payment on principal subtracted from previous unpaid balance ($10 000 - $1 000 = $9 000)

5. Owner's equity = previous owner's equity plus principal ($54 000 + 1 000 = 55 000).

Note:
(1) Principal payment remains the same
(2) Interest payment decreases
(3) Total payment decreases

b) Equal payments plan:

Under the equal payments plan, the monthly payment, including principal and interest, remains the same each month until the total amount of the mortgage is paid. Each regular payment is the same. The interest portion of the payment decreases and the principal portion of the payment increases.

i) calculations without the use of tables,

Example:

A house was purchased on August 4 for $75 000. The buyer made a down payment of $25 000 with the balance of the purchase price to be mortgaged over 25 years. Regular monthly payments of $759.00 are to be made beginning August 31. The interest rate is 18% per annum. Prepare an amortization schedule for 4 payments.

Solution:

AMORTIZATION SCHEDULE

Date	Regular Payment Principal	Interest	Total	Unpaid Balance	Owner's Equity
August 4				$50 000.00	$25 000.00
August 31	$9.00	$750.00	$759.00	49 991.00	25 009.00
Sept. 30	9.13	749.87	759.00	49 981.87	25 018.13
Oct. 31	9.27	749.73	759.00	49 972.60	25 027.40
Nov. 30	9.41	749.59	759.00	49 963.19	25 036.81
Dec. 31	9.55	749.45	759.00	49 953.64	25 066.36

Explanation:

1. Interest is calculated on previous unpaid balance.

   (Interest Aug. 31 = $\$50\ 000 \times .18 \times \frac{1}{12} = \$750$).

2. Principal is obtained by subtracting the interest from the total payment ($759 - 750 = $9.00).

3. Total payment remains the same each month.

4. Unpaid balance is equal to payment on the principal ($50 000 - 9.00 = $49 991) subtracted from the previous unpaid balance.

5. Owner's equity equals previous owner's equity plus payment on the principal ($25 000 + 9.00 = $25 009).

OBJECTIVES	NOTES TO TEACHERS
ii) calculations using tables;	Note: Amortization tables assist in determining the amount of an equal payment that will amortize a loan in a desired number of years. (Tables with current rates are available from local financial institutions.)

To find the monthly payment required to amortize a loan, multiply the appropriate payment for a $1 000 loan by the number of thousands in the loan.

Examples:

1) Find the monthly payment required to amortize a $40 000 mortgage at 12% for 25 years.

Solution: $\text{Payment} = \dfrac{40\ 000}{1\ 000} \times 10.32 = \$412.80$

2) Find the monthly payment required to amortize a $50 000 mortgage at 18.50% for 25 years.

Solution: $\text{Payment} = \dfrac{50\ 000}{1\ 000} \times 15.034 = \$759.22$

MONTHLY PAYMENT REQUIRED TO AMORTIZE A $1000 MORTGAGE

n	7%	7¼%	7½%	7¾%	8%	8¼%	8½%	8¾%	9%	9¼%	9½%	9¾%	10%	10¼%	10½%	10¾%	11%	11¼%	11½%	11¾%	12%	n
1	86.49	86.60	86.71	86.82	86.93	87.05	87.16	87.27	87.38	87.49	87.60	87.72	87.83	87.94	88.05	88.16	88.27	88.38	88.50	88.61	88.72	1
2	44.73	44.84	44.95	45.06	45.17	45.28	45.39	45.50	45.61	45.72	45.84	45.95	46.06	46.17	46.28	46.39	46.50	46.61	46.72	46.83	46.94	2
3	30.84	30.95	31.06	31.17	31.28	31.39	31.50	31.62	31.73	31.84	31.95	32.06	32.18	32.29	32.40	32.52	32.63	32.74	32.85	32.97	33.08	3
4	23.90	24.02	24.13	24.24	24.36	24.47	24.58	24.70	24.81	24.93	25.04	25.16	25.27	25.39	25.50	25.62	25.73	25.85	25.96	26.08	26.20	4
5	19.76	19.87	19.99	20.10	20.22	20.33	20.45	20.57	20.68	20.80	20.92	21.04	21.15	21.27	21.39	21.51	21.63	21.74	21.86	21.98	22.10	5
6	17.01	17.12	17.24	17.36	17.47	17.59	17.71	17.83	17.95	18.07	18.19	18.31	18.43	18.55	18.67	18.79	18.91	19.04	19.16	19.28	19.40	6
7	15.05	15.17	15.29	15.41	15.53	15.65	15.77	15.80	16.01	16.13	16.26	16.38	16.50	16.63	16.75	16.87	17.00	17.12	17.25	17.38	17.50	7
8	13.59	13.71	13.83	13.95	14.08	14.20	14.32	14.45	14.57	14.69	14.82	14.95	15.07	15.20	15.33	15.45	15.58	15.71	15.84	15.97	16.10	8
9	12.46	12.58	12.71	12.83	12.96	13.08	13.21	13.33	13.46	13.59	13.72	13.85	13.97	14.10	14.23	14.37	14.50	14.63	14.76	14.89	15.03	9
10	11.56	11.69	11.82	11.04	12.07	12.20	12.33	12.45	12.58	12.71	12.84	12.98	13.11	13.24	13.37	13.51	13.64	13.78	13.91	14.05	14.19	10
11	10.84	10.96	11.09	11.22	11.35	11.48	11.61	11.74	11.88	12.01	12.14	12.28	12.41	12.55	12.68	12.82	12.96	13.09	13.23	13.37	13.51	11
12	10.24	10.37	10.50	10.63	10.76	10.89	11.02	11.16	11.29	11.43	11.56	11.70	11.84	11.98	12.12	12.26	12.40	12.54	12.68	12.82	12.96	12
13	9.73	9.86	10.00	10.13	10.26	10.40	10.53	10.67	10.81	10.95	11.08	11.22	11.36	11.51	11.65	11.79	11.93	12.08	12.22	12.37	12.51	13
14	9.30	9.44	9.57	9.71	9.84	9.98	10.12	10.26	10.40	10.54	10.68	10.82	10.97	11.11	11.25	11.40	11.55	11.69	11.84	11.99	12.14	14
15	8.94	9.07	9.21	9.35	9.49	9.63	9.77	9.91	10.05	10.19	10.34	10.48	10.63	10.77	10.92	11.07	11.22	11.37	11.52	11.67	11.82	15
16	8.62	8.76	8.90	9.04	9.18	9.32	9.46	9.61	9.75	9.90	10.04	10.19	10.34	10.49	10.64	10.79	10.94	11.09	11.24	11.40	11.55	16
17	8.34	8.48	8.63	8.77	8.91	9.05	9.20	9.35	9.49	9.64	9.79	9.94	10.09	10.24	10.39	10.55	10.70	10.86	11.01	11.17	11.32	17
18	8.10	8.24	8.39	8.53	8.68	8.82	8.97	9.12	9.27	9.42	9.57	9.72	9.87	10.03	10.18	10.34	10.49	10.65	10.81	10.97	11.13	18
19	7.89	8.03	8.18	8.32	8.47	8.62	8.77	8.92	9.07	9.22	9.38	9.53	9.69	9.84	10.00	10.16	10.32	10.48	10.64	10.80	10.96	19
20	7.70	7.84	7.99	8.14	8.29	8.44	8.59	8.74	8.90	9.05	9.21	9.36	9.52	9.68	9.84	10.00	10.16	10.32	10.49	10.65	10.81	20
21	7.53	7.68	7.83	7.98	8.13	8.28	8.43	8.59	8.74	8.90	9.06	9.22	9.38	9.54	9.70	9.86	10.03	10.19	10.36	10.52	10.69	21
22	7.38	7.53	7.68	7.83	7.98	8.14	8.29	8.45	8.61	8.77	8.93	9.09	9.25	9.41	9.58	9.74	9.91	10.07	10.24	10.41	10.58	22
23	7.24	7.39	7.55	7.70	7.86	8.01	8.17	8.33	8.49	8.65	8.81	8.97	9.14	9.30	9.47	9.64	9.80	9.97	10.14	10.31	10.48	23
24	7.12	7.27	7.43	7.58	7.74	7.90	8.06	8.22	8.38	8.54	8.71	8.87	9.04	9.21	9.37	9.54	9.71	9.88	10.05	10.22	10.40	24
25	7.01	7.16	7.32	7.48	7.64	7.80	7.96	8.12	8.28	8.45	8.62	8.78	8.95	9.12	9.29	9.46	9.63	9.80	9.98	10.15	10.32	25
26	6.91	7.07	7.22	7.38	7.54	7.71	7.87	8.03	8.20	8.37	8.53	8.70	8.87	9.04	9.21	9.39	9.56	9.73	9.91	10.08	10.26	26
27	6.82	6.98	7.14	7.30	7.46	7.62	7.79	7.95	8.12	8.29	8.46	8.63	8.80	8.97	9.15	9.32	9.50	9.67	9.85	10.02	10.20	27
28	6.74	6.90	7.06	7.22	7.38	7.55	7.72	7.88	8.05	8.22	8.39	8.57	8.74	8.91	9.09	9.26	9.44	9.62	9.79	9.97	10.15	28
30	6.59	6.75	6.92	7.08	7.25	7.42	7.59	7.76	7.93	8.11	8.28	8.46	8.63	8.81	8.99	9.17	9.34	9.52	9.71	9.89	10.07	30
35	6.32	6.49	6.67	6.84	7.01	7.19	7.37	7.54	7.72	7.90	8.08	8.26	8.45	8.63	8.81	9.00	9.18	9.37	9.56	9.74	9.93	35
n	7%	7¼%	7½%	7¾%	8%	8¼%	8½%	8¾%	9%	9¼%	9½%	9¾%	10%	10¼%	10½%	10¾%	11%	11¼%	11½%	11¾%	12%	n

Note: Some trust companies give away booklets showing mortgage payments for interest rates varying from 8% to 20%.

OBJECTIVES

NOTES TO TEACHERS

4. to calculate and prepare a Statement of Adjustments.

Students must realize the adjustments necessary to the purchase price of a home, such as insurance, taxes, mortgage interest, water rates, which are owing or paid in advance at the closing date (the day the property is transferred to a new owner).

Example:

The purchase price for a home is $45 000. The terms of sale indicate that the buyer agrees to pay $15 000 down and to take over the existing mortgage of $30 000 at 11.75%, payable in equal monthly installments. The buyer makes a $2 000 deposit at the time the offer to purchase is signed. In preparation for making the final cash payment on the closing date, he is faced with a payment of $14 462.22, instead of the $13 000 he had expected. Prepare a Statement of Adjustments verifying the final cash payment due the seller.

Solution:

1. Prepare a statement using closing date March 15.
2. Purchase price of $45 000 is charged to buyer.
3. Credit the buyer with $2 000 deposit.
4. Seller pays interest on mortgage from last monthly payment to closing date (February 28 to March 15).

$$\$30\ 00 \times .1175 \times \frac{15}{365} = \$146.86$$

5. Charge the buyer with unused portion of policy. (9½ months = March 15 to December 31) Premium is $330 for 1 year.

$$\$330 \times \frac{9.5}{12} = \$261.25.$$

6. Taxes of $1 700 were paid for the year. The buyer pays for taxes from March 15 to December 31 (9½ months).

$\$1700 \times \frac{9.5}{12} = \$1\ 345.83$

Review calculations of Real Property Taxes.

Note: Some areas will include an adjustment of the water bill.

MR. MORTGAGE BUYER
STATEMENT OF ADJUSTMENTS
19__ 03 15

Particulars	Charged to Buyer	Credited To Buyer
Purchase Price	$ 45 000.00	
Deposit		$ 2 000.00
Mortgage		$30 000.00
Interest on Mortgage (February 28 - March 15)		144.86
Insurance	261.25	
Taxes (March 15 to December 31)	1 345.83	
Balance due Seller		$14 462.22
	$ 46 607.08	$46 607.08

ENRICHMENT PROJECTS:

1. Complete forms related to a mortgage.

   Related Forms:

   1. Offer to Purchase
   2. Applications for Mortgage
   3. Personal Statement of Affairs

   Note: It is intended that students completing these forms will have a better understanding of procedures involved. However, related forms may be pre-completed and distributed for student information and application. (Forms are available from local financial institutes, local real estate agents and lawyers' offices.)

2. Examine costs of owning a condominium.
3. Examine costs of owning a mobile home.
4. Examine the costs of owning a home as compared to renting.

   Example:

   Consider purchasing a house worth \$56 000 with a down payment of \$21 000. Expenses for the year are estimated as follows: mortgage interest \$6 314; taxes \$1 540; insurance \$280; repairs and maintenance \$600. The down payment is presently invested at 16.25% annually. Similar accommodations, if rented, would cost \$825 per month. Which will be cheaper, renting or buying?

Solution:

Cost of Ownership:

Interest on Mortgage	$ 6 314.00	
Taxes	1 540.00	
Insurance	280.54	
Repairs & Maintenance	600.00	
Interest on investment (21 000 x .1625)	3 412.50	(Cost if taken out of investment)
Cost per year	$12 147.00	
Cost per month	$12 147 ÷ 12 =	$1 012.25
Rental costs		-825.00
Renting would be less by		$ 187.25/month
		or $2 247.00/year.

Assumptions:

1. There is no increase in the value of the house.
2. No income tax is paid on interest earned on investment.

5. Write programs on amortization schedules for the use of a computer.

C.II. ALGEBRA (3 MODULES)

This is not intended to be an exhaustive study of algebra. While the algebra in the 01 courses may not have an immediate practical value, students should have some literacy in basic operations with exponents and polynomials.

OBJECTIVES	NOTES TO TEACHERS
A. ALGEBRAIC OPERATIONS (4 classes)	
The student should be able:	
1. to perform operations with bases having:	Students have worked with positive exponents in previous grades. Negative exponents may be new material to them.
a) positive exponents, b) negative exponents;	Students should know that 2^3 means 2 used as a factor three times in multiplication. Students should consider a negative exponent to mean reciprocal; e.g., reciprocal of $2^3 = 2^{-3}$ or $2^{-3} = \frac{1}{2^3}$ Laws of exponents should be demonstrated but not necessarily stated or proved. Limit questions to integral exponents. Examples: $2^3 = 8$ $3^5 \cdot 3^2 = 3^7$ $\frac{3^4}{3} = 3^3 = 27$ $5^{-2} \cdot 5^3 = 5^1 = 5$

OBJECTIVES

NOTES TO TEACHERS

$10^{-2}\ 10^{-3} = 10^{-5}$

$\frac{5^{-2}}{5^{-3}} = \frac{5^{3}}{5^{2}} = 5$

$\frac{10^{-5}}{10^{3}} = 10^{-8}$

$8^{3} \cdot 8^{-3} = 8^{0} = 1$

$a^{3} \cdot a^{4} = a^{7}$

Note: When using variables as bases, restrict examples to positive exponents only.

Negative exponents are introduced to help the students read numbers expressed in scientific notation.

2. to perform operations of addition, subtraction, multiplication and division with polynomials;

Much of this section is review but students need practice to refresh their skills. Limit operations to questions involving monomials and binomials.

Examples:

$3x + 5x$

$2x(3 + 5x)$

$(2x + 5) + (3x - 7)$

$3a - 9a$

$3a - (2a + 5)$

$(5a - 2) - (2a + 3)$

$3(2b)$

$-5(4x^2)$

$4(-7x^4)$

$(-3x)(-5x^2)$

$3(2a + 4)$

$-2(5a - 3)$

$(a + 7)(a + 4)$; $(2a - 5)(3a - 7)$

$2(a + 3) - 3(5a - 4)$

$\frac{15}{3}$; $\frac{x^5}{x^3}$; $\frac{25x^2}{5x}$; $\frac{15a^3 - 3a^3}{3a}$

Note: When using variables as bases, restrict examples to positive exponents only.

3. to write large numbers in scientific notation;

The intent is to increase a student's mathematical literacy. Some technical articles and science publications express numbers in scientific notation. Students using scientific calculators should be able to interpret numbers expressed in scientific notation on their calculator read-out.

Example: $45\ 000\ 000 = 4.5 \times 10^7$

OBJECTIVES	NOTES TO TEACHERS
4. to write numbers between zero and one in scientific notation;	Example: $0.000\ 000\ 036\ 5 = 3.65 \times 10^{-8}$
5. to convert numbers from scientific notation to expanded notation.	Example: $4.68 \times 10^{-2} = 0.046\ 8$ $2.3 \times 10^{4} = 23\ 000$ Make the Tower of Hanoi model (see Math 101). It takes $2^n - 1$ moves to move n discs. What length of time (in years) would be required to move 64 discs at one disc per second? $\frac{2^{64} - 1}{60 \times 60 \times 24 \times 365} = 5.8 \times 10^{11}$ The answer display to this question will be expressed in scientific notation on a scientific calculator.
B. EQUATION SOLVING (16 classes)	
The student should be able:	
1. to solve linear algebraic equations in one variable (2 classes);	See Math 101 Guide, p. 133. The equation solving techniques of grade ten should be reviewed as they were not included in Math 201. Equations should now include the following type:

$$\frac{3x + 4}{4} + 5 = \frac{x}{8}$$

Multiply by 8

$$8\left(\frac{3x + 4}{4} + 5\right) = \frac{x}{8}(8)$$

$$2(3x + 4) + 40 = x$$

$$6x + 8 + 40 = x$$

$$5x = -48$$

$$x = -\frac{48}{5}$$

2. to solve word problems by setting up a linear equation in one variable (2 classes);

Problems should be as realistic as possible (i.e., applied problems).

Example:

Ken's Quick Transfer needs to rent an extra truck for 1 month. The rental is going to cost $300/month plus 10¢/km. If $550 is budgeted for the truck rental, how many kilometres can be travelled to expend the budget?

Solution:

Let x = number of kilometres.

Therefore, $0.10x + 300 = 550$

$$0.10x = 250$$

$$x = 2500 \text{ km}$$

OBJECTIVES	NOTES TO TEACHERS
3\. a) to determine a solution for a linear equation of the form $ax + by = c$,	Students should be made aware that there are an infinite number of solutions to a single linear equation. Example: $2x + y = 4$ Some solutions are: (1,2), (0,4), (-1,6)
b) to determine whether an ordered pair is a solution for a system of linear equations (1 class),	An explanation of solution should be given. "Solution" is any ordered pair which satisfies both equations. Example: Is (4,1) a solution for the system $x + y = 5$, $3x - 2y = 10$? Solution: $(4) + (1) = 5$ $3(4) - 2(1) = 10$ $\therefore (4,1)$ is a solution

OBJECTIVES

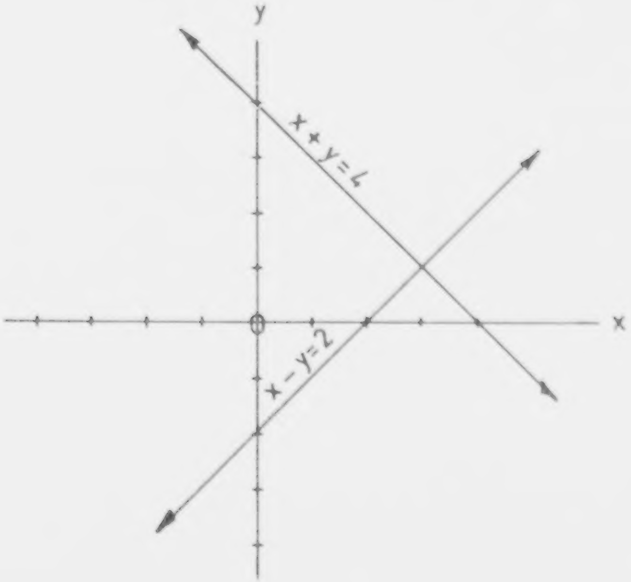
NOTES TO TEACHERS

c) to determine the solution of a system of two linear equations by graphing (3 classes),

Examples:

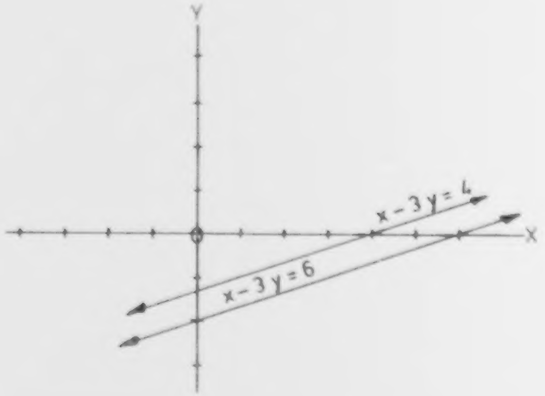
Graph and determine the solution of each of the following systems.

1) $x + y = 4$
$x - y = 2$



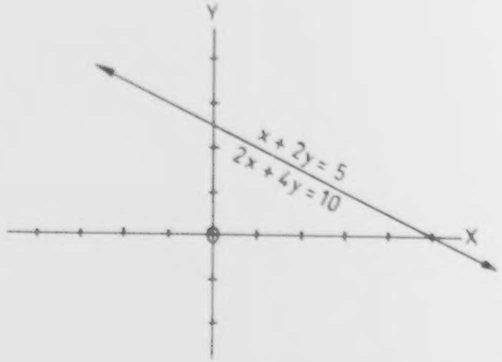
Both lines pass through (3,1).
(3, 1) satisfies both equations.
∴ (3,1) is a solution.

2) $x - 3y = 4$
$x - 3y = 6$



These lines are parallel with no intersection.
∴ solution is $\emptyset$ or there is no solution.

3) $x + 2y = 5$
$2x + 4y = 10$



These lines graph as the same line and all points of the first line are also points of the second line.
∴ solution is "all points on line(s)."

d) to solve algebraically a system of 2 linear equations (4 classes);

Questions should be limited to a system with 1 solution only. It is suggested that only the addition method or the substitution method be used.

Example 1:

Determine the solution of $2x + y = 6$ and $x - y = 6$

Solution: Add equations to get:

$$\begin{array}{rcl} 2x + y &=& 6 \\ \underline{x - y} &\underline{=}& \underline{6} \\ 3x &=& 12 \\ x &=& 4 \end{array}$$

Substitute $x = 4$ into equation (2) to get:

$$\begin{array}{rcl} (4) - y &=& 6 \\ -y &=& 2 \\ y &=& -2 \quad \therefore \text{ solution is } (4, -2) \end{array}$$

OR

$$\begin{array}{ll} 2x + y = 6 & \\ x - y = 6 & \quad x = y + 6 \end{array}$$

Substitute $y + 6$ for x in equation (1):

$$\begin{array}{rcl} 2(y + 6) + y &=& 6 \\ 3y + 12 &=& 6 \\ 3y &=& -6 \\ y &=& -2 \end{array}$$

Substitute $y = -2$ into equation (2) to get:

$$\begin{array}{rcl} x - (-2) &=& 6 \\ x &=& 4 \quad \therefore \text{ solution is } (4,-2) \end{array}$$

Example 2:

Determine the solution of $x - y = 4$ and $2x + 3y = -2$.

$x - y = 4$ ....(1)

$2x + 3y = -2$ ....(2)

Multiply equation (1) by 3

$3x - 3y = 12$

$2x + 3y = -2$

add

$5x = 10$

$x = 2$

Substitute in

$x - y = 4$

$2 - y = 4$

$-y = 2$

$y = -2$

Solution is (2,-2)

4. to solve word problems by setting up a system of 2 linear equations (4 classes).

Problems should be as realistic as possible.

Example:

A boy sells newspapers for 25¢ each and magazines for $1.25 each. If he knows he has made 17 sales for a total of $11.25, how many of each did he sell?

Solution:

Let x be the number of newspapers sold

Let y be the number of magazines sold

$\therefore x + y = 17$ ....(1)

$25x + 125y = 1125$ ....(2)

Multiply equations (1) by -25

$-25x - 25y = -425$ ....(3)

$25x + 125y = 1125$ ....(4)

Add

$100y = 700$

$y = 7$

$x + 7 = 17$

$x = 10$

∴ Number of newspapers sold = 10

∴ Number of magazines sold = 7

C. GRAPHING (12 periods)

In the traditional mathematics class, a graph is often thought of as a representation of an exact relationship which can be described by a mathematical equation. The equation is given and students are expected to draw the graph which represents the relationship described by the equation. Little attention is paid to what the variables represent, the treatment of errors, or why points might be plotted off the idealized graph.

In other classes, students are required to draw a graph of their results and are expected to pay particular attention to the quantities or variables. The assumption is made that the data embody some relationship which can be mathematically described and the goal is to decide which mathematical model best fits the experimental data.

One aim of this unit is to blend the traditional mathematics approach with other approaches so that students are not forced to learn one thing concerning graphing in mathematics and something which sounds totally different in other classes, but rather, have some understanding of the relation between different approaches.

Teachers may wish to integrate work from this section with that of Variation, found in Part D of this module.

OBJECTIVES

NOTES TO TEACHERS

Direct Linear Relations (4 periods)

The student should be able:

1. to obtain data experimentally for a direct linear relation;
2. to graph the data obtained in the previous objective;

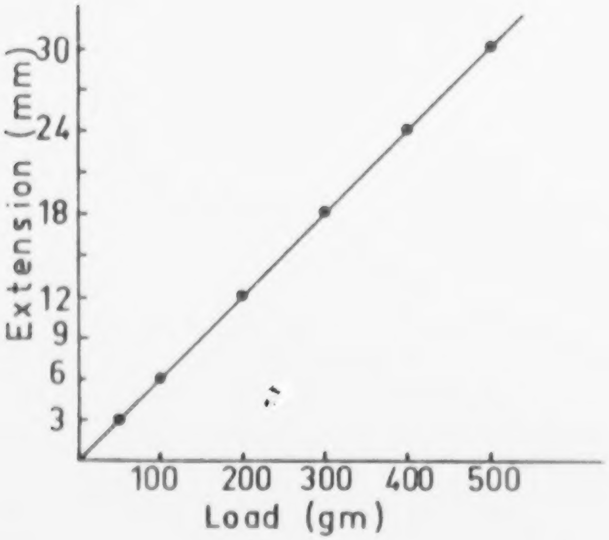
Review as necessary. Students should do one or two experiments followed by examples. See C.III on page 139 in the Math 201 outline for examples. Other examples could include the relation between elapsed time and distance travelled for an automobile travelling at constant speed, extension of a spring for a given load, etc.

Examples:

1. Extension of a Spring

For a given spring the stretch or extension under a specified load is as follows:

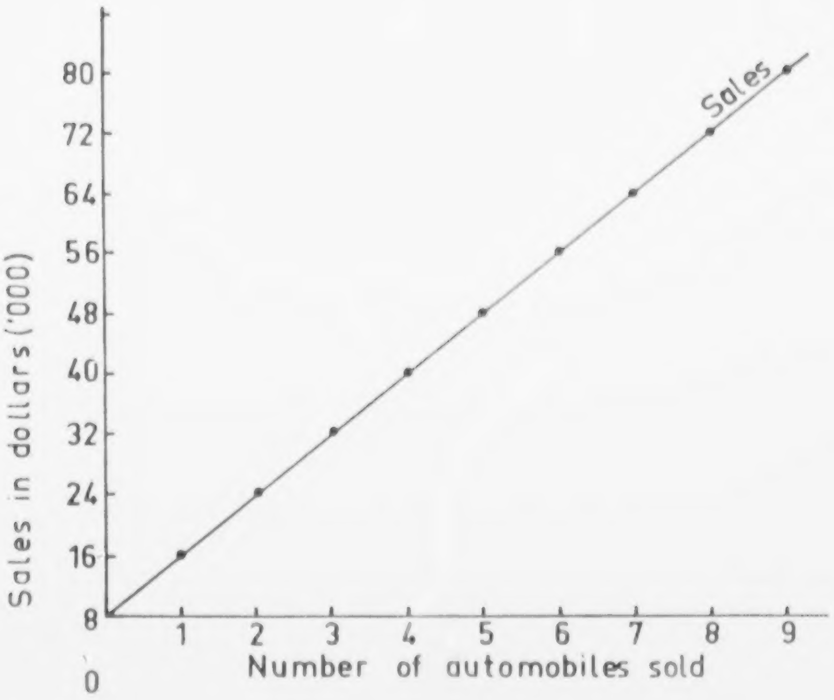
Extension in mm	0	3	6	12	18	24	30
Load in gm	0	50	100	200	300	400	500



Note: For purposes of this course, a direct linear relation is defined as a relationship between two variable quantities such that the graph is a straight line emanating from the origin.

2. Automobile Sales

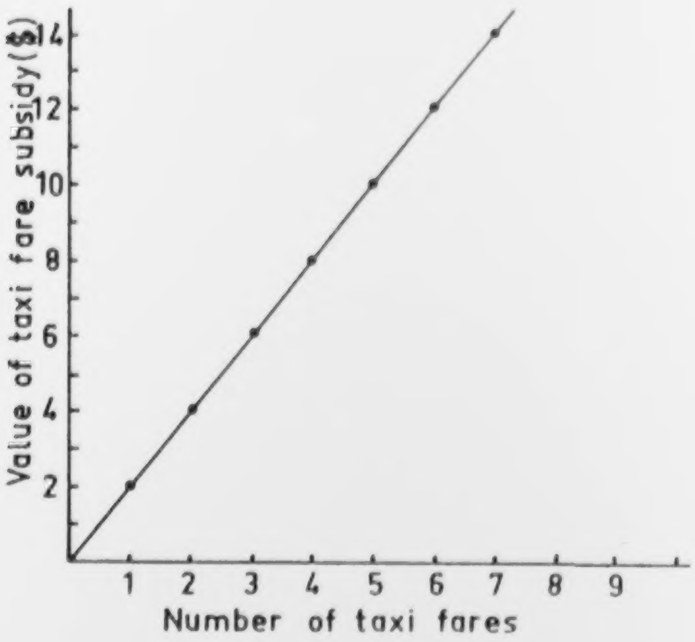
An automobile dealer finds that the average price of an automobile is $8000. This can be plotted as follows.



3. Taxi Subsidy

As an incentive to provide services to senior citizens, a local taxi service receives $2.00 from the city for each time a senior citizen uses a taxi cab.

This subsidy for senior citizens would appear graphically as follows:



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OBJECTIVES

NOTES TO TEACHERS

3. to identify reasons for deviations from the mathematical model;

Reasons might include error in judgment or measurement, temperature, reliability of the power source, tensile strength of spring, etc.

4. to interpolate and to extrapolate beyond the data obtained in Objective 1;

Interpolate - to read between given data.
Extrapolate - to read beyond the given data.

Example:

Using Example 1, Extension of a Spring, answer the following questions:

1. Interpolation - What extension on the spring is expected if the load is 75 gm?

   Answer: 4.5 mm

2. Extrapolation - What extension on the spring is expected if the load is 700 gm?

   Answer: 42 mm

5. to express an equation for the data obtained in Objective 1;

Example:

Using Example 2, Automobile Sales, determine the equation expressing the relation between the sales (S) and the number of automobiles (A).

A	S	
1	8 000	
		8 000 difference
2	16 000	
		8 000 difference
3	24 000	
		8 000 difference
4	32 000	

Since the A values differ by 1 and the S values differ by $8 000 (i.e., the average cost of an automobile), an equation for this direct relationship is $S = 8\ 000\ A$ where S is total sales and A is number of cars sold at $8 000 each.

OBJECTIVES

NOTES TO TEACHERS

6. to graph linear relations in the form y = mx where "m" is slope.

Note: When m = 0, the line is horizontal.

Partial Linear Relations (6 periods)

The student should be able:

1. to obtain data experimentally for a partial linear relation;

Students should perform one or two experiments followed by examples with data provided.

For purposes of this course, a "partial linear relation" is defined as the relationship which contains a constant and a variable portion. The graph of such a relation is a straight line not passing through the origin.

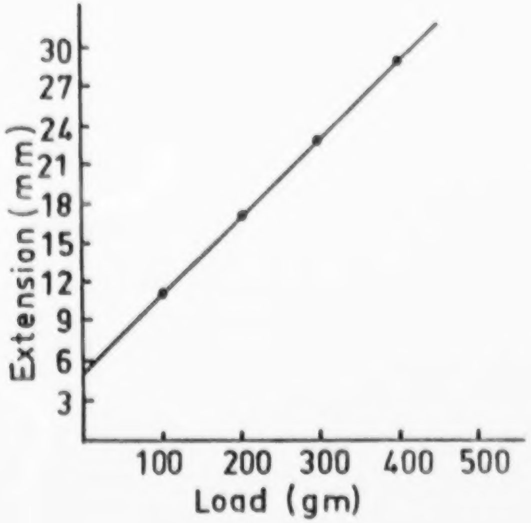
2. to graph the data obtained in Objective 1;

Example:

1. Extension of a Spring

Suppose a container is attached to a spring prior to adding the load. Since the container has some mass, the spring stretches 5 mm. Consider the following data table and use it to draw the graph.

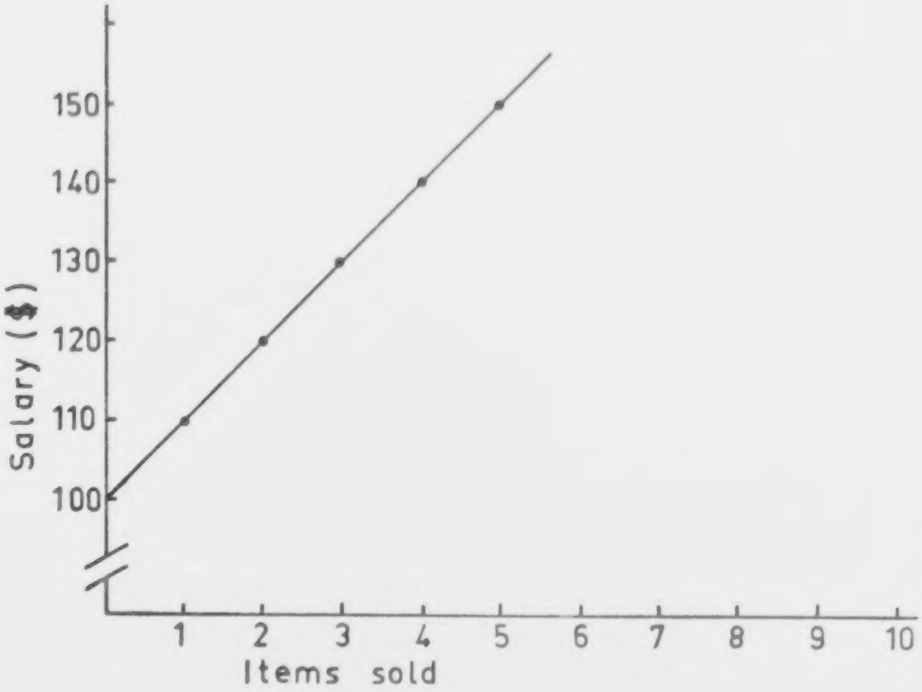
Extension in mm	5	11	17	23	29
Load in gm	0	100	200	300	400



In this example, the 5 mm extension for the container is constant throughout. This graph is the same as the one in A.2, Example 1, shifted up the vertical axis the equivalent of 5 mm.

2. Salary and Commission

An employee's salary is \$100 per week plus \$10 for each item sold. Draw a graph to represent the total salary as the number of items sold increases.



OBJECTIVES	NOTES TO TEACHERS
3. to identify reasons for deviations from the mathematical model;	There should be no deviation except due to calculation error in examples involving patterns/formula (i.e., salary and commission example). Deviations can occur in measurement situations because of inadequate measurement devices, worn or stretched equipment, etc.
4. to interpolate and to extrapolate beyond the data in Objective 1;	Example: Using Example 1, Extension of a Spring, Objective 2, answer the following questions: 1. Interpolation - What extension of the spring is expected if the load is 125 gm? Answer: 12.5 mm 2. Extrapolation - What extension of the spring is expected if the load is 700 gm? Answer: 47 mm
5. to express an equation for the data in Objective 1;	Example: Using Example 2, Salary and Commission, Objective 2, determine the equation expressing the relations between salary (S) and the number of items sold (n). Answer: $S = 100 + 10n$
6. to graph linear relations in the form $y = mx + b$ where "m" is slope and "b" is y- intercept;	Refer to C.III, Algebra in Mathematics 201. Note: If $m = 0$, the line is horizontal.

OBJECTIVES

NOTES TO TEACHERS

7. to graph a linear relation in the form Ax + By + C = 0 after converting to the form y = mx + b;

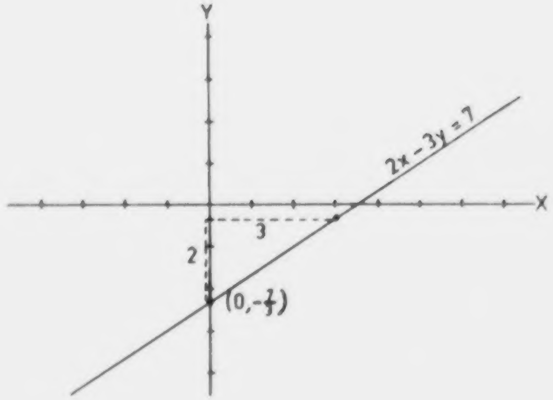
Example:

$2x - 3y = 7$
manipulate to

$y = \frac{2}{3}x - \frac{7}{3}$

compare with $y = mx + b$

y - intercept is $-\frac{7}{3}$, slope is $\frac{2}{3}$



8. to write the equation for a linear relation in the form y = mx + b where "m" is the slope and "b" is the y- intercept.

Limit this to examples where the values of the slope and intercept are given or where a graph containing the necessary information is provided. Most of the time should be spent on graphical examples.

Examples:

1. Write the equation in the form $y = mx + b$ for the straight line whose slope is 3 and whose y-intercept is -1. Answer: $y = 3x - 1$.

2. Write the equation in the form $y = mx + b$ for the straight line with a slope of 4 passing through the point (2,-1).

Solution:

$$\begin{aligned} y &= mx + b \\ -1 &= 4(2) + b \\ -1 &= 8 + b \\ b &= -9 \end{aligned}$$

Therefore, equation is $y = 4x - 9$

3. Write the equation for the straight line passing through A(0,-1) and B(-3,2).

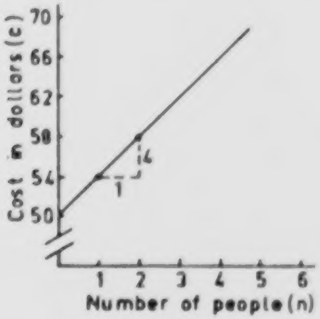
Solution: Since one point is A(0,-1) the value for the y-intercept is -1.

$y = mx + b$
$2 = m(-3) - 1$
$2 = -3m - 1$
$3 = -3m$
$m = -1$
Therefore, equation is $y = -1x - 1$ or $y = -x - 1$.

Note: When examples involving graphs are used, students may need to be shown how to determine slope from the graph. It is suggested that algebraic formulae be avoided.

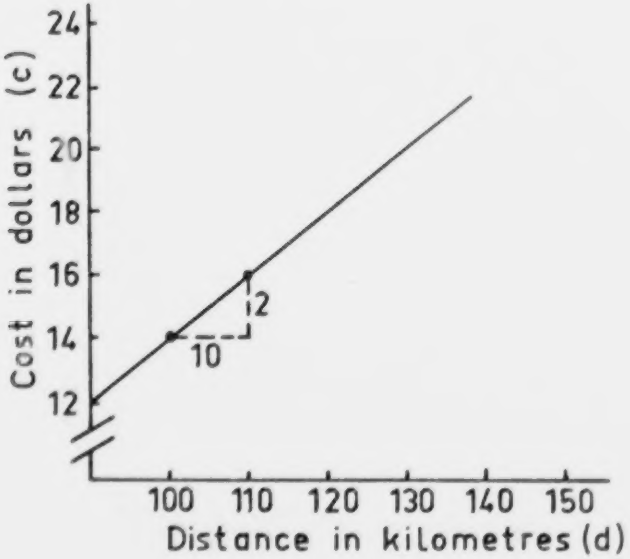
4. Determine the equation of the straight line shown on the accompanying graphs.

1) Cost of Banquet



Answer: $C = 4n + 50$

ii) Daily Cost of Renting a Car



Solution: slope (m) = $\dfrac{\text{change in cost}}{\text{change in distance}} = \dfrac{2}{10} = 0.2$

Equation is of the form $y = mx + b$

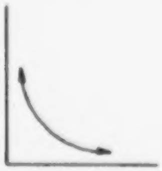
Therefore, answer is $\underline{C = 0.2d + 12}$

Non-Linear Relations
(2 periods)

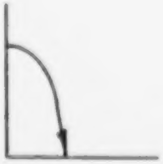
The student should be able to sketch simple non-linear relations, given appropriate data.

Students should realize that not all graphs are straight lines. Examples could involve the following:

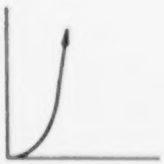
1. Inverse relations with graphs of the form

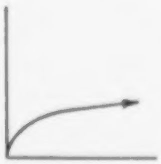


2. Parabolic relations whose graphs are of the form:



projectile motion





acceleration of auto

Students are expected to graph non-linear relations by plotting a series of points and sketching a curve through the points.

ENRICHMENT:

Interpolation and extrapolation involving non-linear relations.

D. VARIATION

This module on variation concentrates on three types of variation: direct, partial and inverse. Reserve joint variation and combined variation questions for enrichment purposes for those students who are able to handle them. The three main types of variation should be introduced in practical terms. For example: direct variation - the more you buy, the higher the cost; inverse variation - given an amount of money, the higher the cost, the fewer the number of items purchased; partial variation - the cost of posters printed depends upon a fixed amount (setting up the press) and a variable amount (the cost of paper, ink and labor to produce the posters).

OBJECTIVES	NOTES TO TEACHERS

Direct Variation

The student should be able:

1. to recognize direct variations expressed as:
   a) word statements,
   b) charts,
   c) graphs,
   d) equations;

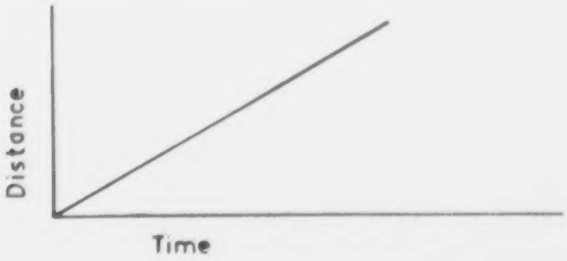
Examples:

a) The cost of running an automobile varies directly with the number of miles travelled.

b)

footballs purchased	1	2	3	4	5
total cost $	28	56	84	112	140

c)



d) $y = 4x$

OBJECTIVES

2. to determine the constant of variation;

NOTES TO TEACHERS

Examples:

1. Henry can cover a distance of 50 km in two hours riding his bicycle. What is the constant of variation (speed)?

   Solution: 50 km ÷ 2 h = 25 km/h

2. x varies directly with y. x is 4 when $y = 12$

   Solution:
   $x \propto y$ - variation statement
   $x = ky$ - equation using a constant of variation
   these statements are equivalent

   $4 = k \cdot 12$

   $k = \frac{1}{3}$

Note: Two types of problems should be given:

i) expressed in practical terms (as example 1 above)
ii) expressed in general terms such as x and y (as example 2 above)

OBJECTIVES	NOTES TO TEACHERS

3. to solve problems involving direct variation;

Examples:

1. The gas consumed varies directly as the distance driven. Find the amount of gas consumed over a distance of 150 km if 8 L of gas is used over a distance of 60 km.

Solution: $c \propto d$ where c is the number of litres consumed and d is the distance travelled.

$c = kd$

$8 = k \cdot 60$

$k = \frac{8}{60} = \frac{2}{15}$

substitute $k = \frac{2}{15}$ into the equation above

$c = \frac{2}{15} \cdot d$

$c = \frac{2}{15}(150)$

$c = \underline{20\ \text{L}}$

Alternate Method: Use a proportion without referring to a constant of variation.

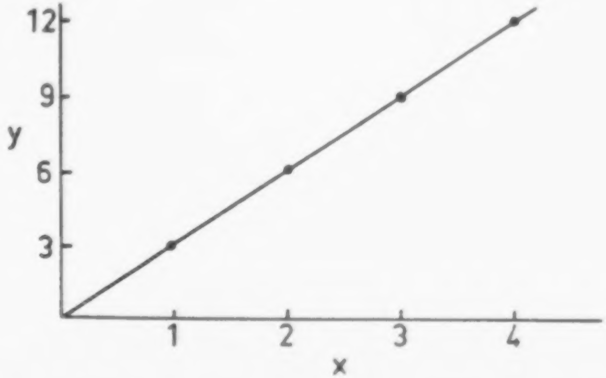
$$\frac{x_1}{x_2} = \frac{y_1}{y_2}$$

4. to change one form of direct variation to another.

Examples:

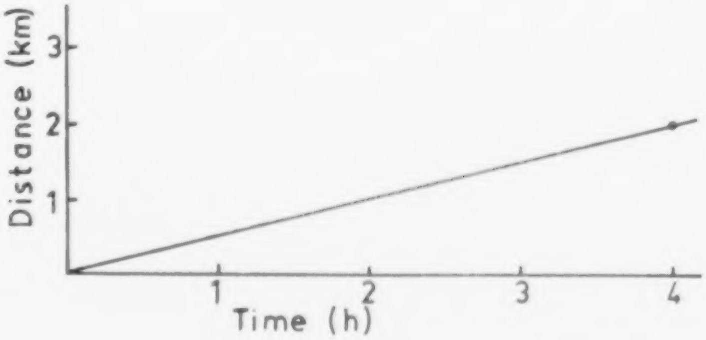
1. Express y = 2x in graphical and chart form.

Solution:



x	y
0	0
1	2
2	4
5	10
20	40

2. Express in equation and word statement forms.



Solution:

a) $d = \frac{1}{2} t$ or $t = 2d$

b) distance varies directly with time.

OR

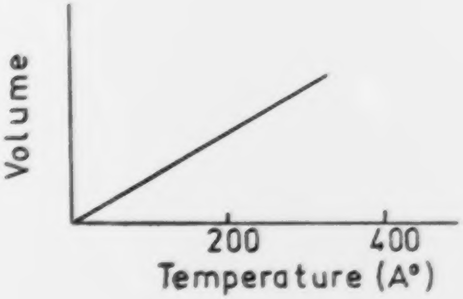
Peter travelled at the rate of ½ km per h.

3. Express "the volume of a quantity of gas varies directly as the absolute temperature," as an equation and as a graph.

Solution:

a) $V = kT$

b)



Partial Variation

The student should be able:

1. to recognize partial variations expressed as:
   a) word statements,
   b) charts,
   c) graphs,
   d) equations;

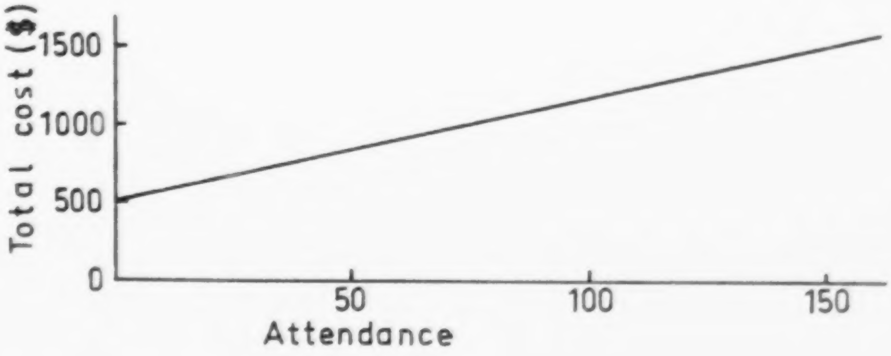
Examples:

a) The cost of putting on a dance includes some fixed costs such as hall rental and costs of a band, and some variable costs such as the number of meals to buy.

b) Costs of Holding a Dance

Number Attending	50	70	100	120	150	1	2
Total Cost ($)	750	850	1000	1100	1250	505	510

c)



d) $C = 500 + 5n$ where C is the total cost of holding a dance and n is the number of people attending.

2. to determine the constant of variation;

Examples:

1. The cost of sending a soccer team to a tournament is partly fixed and partly dependent upon the number of players going. If the cost of sending 12 players is \$160 and the cost of sending 15 players is \$184, find the additional cost per player.

   Solution: increase in players is 15 - 12 = 3
   increase in total cost is \$184 - \$160 = \$24
   additional cost per player is \$24 ÷ 3 = \$8 per player

2. In $T = P + kn$, if $T_1 = 500$ when $n_1 = 15$, and if $T_2 = 300$ when $n_2 = 5$, find k.

   Solution:

$$
\begin{aligned}
500 &= P + 15K \quad (1)\\
\text{and } 300 &= P + 5K \quad (2)\\
&\text{subtract (2) from (1)}\\
200 &= 10k\\
k &= \underline{20}
\end{aligned}
$$

3. to solve problems involving partial variation.

Examples:

1. The annual cost of operating an automobile is the sum of two quantities, one of which is constant and the other varies directly as the number of kilometres travelled. If the expenses of travelling 10 000 km total \$3 000, and travelling 15 000 km total \$3 600, what are the expenses for an annual total of 24 000 km?

Solution: $C = F + kn$ where C is the total cost,
F is the fixed cost per year,
n is the number of kilometres travelled,
and k is the cost per kilometre,

$$\begin{array}{l} 3\,000 = F + 10\,000\,k \quad (1) \\ \underline{3\,600 = F + 15\,000\,k} \quad (2) \end{array}$$

Multiply equation (1) by -1

$$\begin{array}{l} -\,3\,000 = -\,F - 10\,000\,k \\ +\,3\,600 = F + 15\,000\,k \\ \text{add} \end{array}$$

$$600 = 5\,000\,k$$

$$k = .12$$

substitute into the 1st equation:

$$\begin{aligned} 3\,000 &= F + 10\,000\,(.12) \\ 3\,000 &= F + 1\,200 \\ F &= \$1\,800 \end{aligned}$$

substitute F and k values into the original equation

$C = 1\,800 + .12n$

solve for $n = 24\,000$ km

$C = 1\,800 + .12(24\,000)$

$C = \underline{\$4\,680}$

2. In West Oil Land constituency, the approximate percent rate of votes for parties other than the governing party is partly fixed (due to a natural distrust of a party in power) and partly variable (depending on how many people are unemployed in the constituency). During the last election, the governing party did not receive 56% of the votes when 680 people were unemployed. During the previous election, it did not receive 32% of the votes when only 200 people were unemployed. What percent rate of votes would they not get in the next election if 1 280 people will be unemployed?

Solution:

$R = D + kn$ where R is the percent rate of votes lost in total
D is the percent rate of votes lost to the distrust of government
n is the number of unemployed
k is the percent rate of votes lost per person unemployed

$56 = D + k(680)$ (1) Multiply equation (1) by -1
$32 = D + k(200)$ (2) Add
$24 = 480k$
$k = 0.05\%$
then substitute this value into (1) or (2), say (2)
$32 = D + (0.05)(200)$
$32 = D + 10$
$D = 22\%$
now substitute values for D and k into the original equation
$R = 22 + 0.05n$
and solve for R if n = 1280 people.
$R = 22 + 0.05\ (1280)$
$= 22 + 64$
$R = \underline{86\%}$

Inverse Variation

The student should be able:

1. to recognize inverse variations expressed as:
   - a) word statements,
   - b) charts,
   - c) graphs,
   - d) equations;

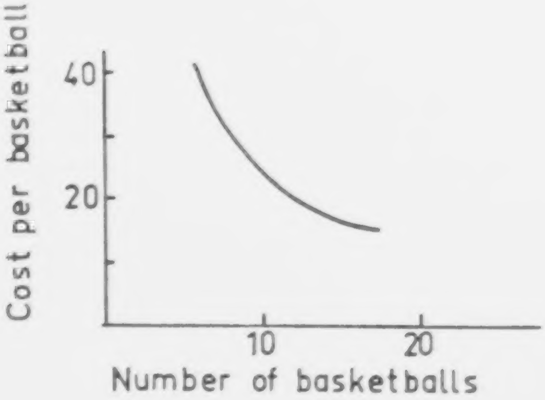
Examples:

a) A physical education program budgets $240 to purchase basketballs. The number of basketballs purchased is inversely proportional (varies inversely) to the cost per basketball.

b)

Number of Basketballs	Cost per Basketball
16	$15
12	20
10	24
8	30
6	40

c)



d) $y = \frac{\$240}{x}$ OR $xy = \$240$

2. to determine the constant of variation;

Examples:

1. It takes Jim 3 hours at 90 kilometers per hour to cover the distance from town A to town B. What is the distance covered?

   Solution: $t \propto \frac{1}{s}$ where t is the time travelled and s is the speed of travel

   $$t = k\left(\frac{1}{s}\right) \text{ or } t = \frac{k}{s}$$

   $$3 = \frac{k}{90}$$

   $$k = \underline{270} \text{ km}$$

2. If x varies inversely as y, find k when x = 42.5 and y = 12.

   Solution: $x \propto \frac{1}{y}$

   $$x = \frac{k}{y}$$

   $$42.5 = \frac{k}{12}$$

   $$k = \underline{510}$$

3. to solve problems involving inverse variation.

Examples:

1. The value of a car varies inversely as its age in years. If a car is worth \$3 000 when it is 4 years old, how much will it be worth when it is 7 years old?

Solution: $V = \frac{k}{A}$ where V is its value, and A is its age in years

$$\$3\ 000 = \frac{k}{4}$$

$$k = \$12\ 000$$

$$V = \frac{\$12\ 000}{7}$$

$$V = \underline{\$1714.29}$$

2. The time taken to complete a building project varies inversely as the number of people working. If 5 workers can complete the project in 10 days, how long will it take 8 workers?

Solution: $n \cdot t = k$ when n is the number of workers,
t is the time in days, and
k is a constant

$$(5)(10) = K$$
$$K = 50$$
$$n \cdot t = 50$$
$$8t = 50$$
$$t = \underline{6\tfrac{1}{4}} \text{ days}$$

OBJECTIVES

Types of Variation

The student should be able:

1. to identify the type of variation (direct, partial or inverse) from word problems and solve them;
2. to identify the type of variation from graphs.

NOTES TO TEACHERS

Alternate Method:

Use a proportion without referring to a constant of variation.

$$\frac{x_1}{x_2} = \frac{y_2}{y_1} \text{ OR } x_1y_1 = x_2y_2$$

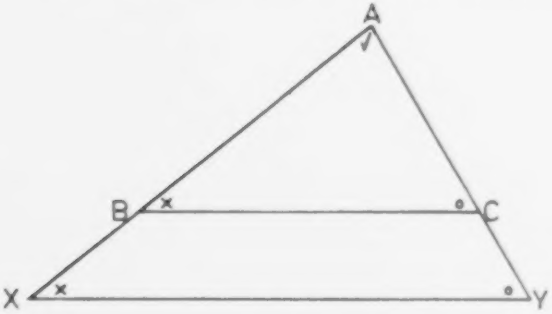
ENRICHMENT:

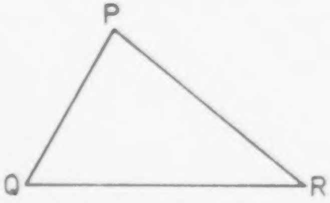
Do other types of variation.

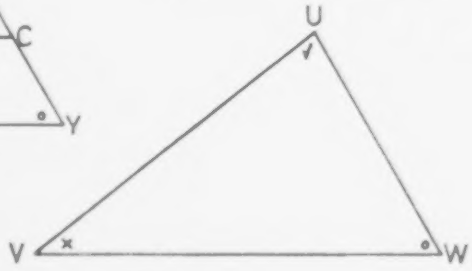
1. Joint Variation
2. Combined Variation

C.III. TRIGONOMETRY (1 MODULE)

OBJECTIVES	NOTES TO TEACHERS
The student should be able:	Consider using diagrams with most problems, especially with difficult ones.
1. to recognize similar triangles;	Two triangles are similar if their corresponding angles are the same.
2. to identify corresponding parts of similar triangles;	The teacher could construct a number of pairs of similar triangles, with corresponding parts marked.
3. to recognize that the ratio of two sides of a triangle is equal to the ratio of corresponding sides of a triangle similar to the first;	Students could measure the lengths of sides of the triangles, write internal ratios of triangles (ratios involving sides of one triangle only), and confirm that the ratios depend on the size of angles of the triangle, and not on the size of the triangle itself. Ratios could be used to predict the length of any side of a triangle, and the measurements used to confirm the length. All triangle types should be considered here: scalene, isosceles, equilateral, right, acute and obtuse.







$\triangle ABC$, $\triangle AXY$ and $\triangle UVW$ are similar since

$\angle ABC = \angle AXY = \angle UVW$

$\angle ACB = \angle AYX = \angle UWV$ given

$\angle BAC = \angle XAY = \angle VUW$

$\triangle PQR$ is not similar to $\triangle ABC$ or $\triangle AXY$ or $\triangle UVW$ since corresponding angles are not equal.

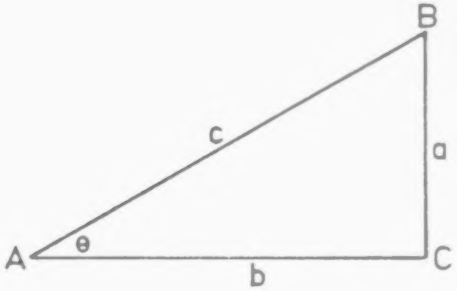
In $\triangle ABC$, $\triangle AXY$, $\triangle UVW$

$$\frac{AB}{BC} = \frac{AX}{XY} = \frac{UV}{VW}$$

$$\frac{AC}{AY} = \frac{AB}{AX} = \frac{BC}{XY}$$

$$\frac{BC}{AC} = \frac{XY}{AY} = \frac{VW}{UW}$$

OBJECTIVES	NOTES TO TEACHERS
4. to define the trigonometric ratios, sin Θ, cos Θ, and tan Θ, as ratios of the sides of a right triangle;	It is suggested that the terms "adjacent," "opposite" and "hypotenuse" be used in the definitions.



In a right triangle, the side opposite the right angle is the "hypotenuse", the side opposite Θ is the "opposite" and side next to Θ is the "adjacent" side. Note that the sides "opposite" and "adjacent" depend upon which vertex is labelled as "Θ". The sides other than the hypotenuse are called "arms."

$$\sin \Theta = \frac{\text{side opposite angle } \Theta}{\text{hypotenuse}}$$

$$\cos \Theta = \frac{\text{side adjacent to angle } \Theta}{\text{hypotenuse}}$$

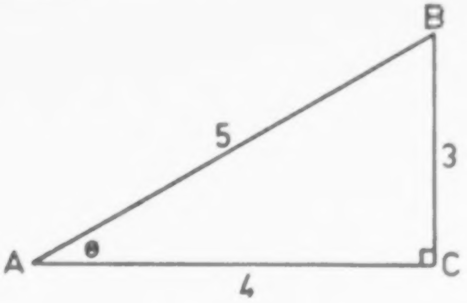
$$\tan \Theta = \frac{\text{side opposite}}{\text{side adjacent}}$$

OBJECTIVES

5. to determine the numerical value of sin θ, cos θ, and tan θ, given θ, $0 < \theta < 90$;

NOTES TO TEACHERS

It is suggested that calculators be used to determine the values. However, if the use of tables is required, it is suggested that the nearest degree be used.



Given:

$$\text{Sin } \theta = \frac{\text{opp}}{\text{hyp}} = \frac{3}{5} = 0.600\ 00$$

$$\text{Cos } \theta = \frac{\text{adj.}}{\text{hyp.}} = \frac{4}{5} = 0.800\ 00$$

$$\text{Tan } \theta = \frac{\text{obj.}}{\text{adj.}} = \frac{3}{4} = 0.750\ 00$$

Notes:

1. Calculators give trigonometric solutions to great accuracy. The number of figures in the solution depends on what is being measured.
2. The number of significant digits in the solution is no greater than the number in the least accurate piece of data given.

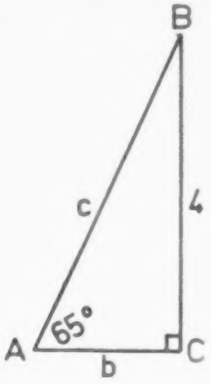
OBJECTIVES

NOTES TO TEACHERS

6. to find all parts of a right triangle, given one angle other than the right angle and one side;

Examples:

1.



Given: ∠A = 65° and side a = 4.00

Find: ∠B, b, c

∠B = 90° - 65°

$\boxed{\angle B = 25°}$

$\sin 65° = \frac{4.00}{c}$, $c = \frac{4}{\sin 65°} = 4.41$

or $\cos 25° = \frac{4.00}{c}$, $C = \frac{4}{\cos 25} = 4.41$

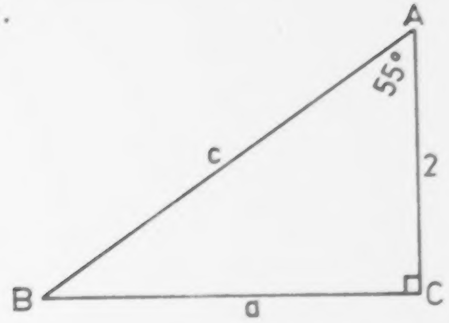
$\boxed{c = 4.41}$

$\tan 25 = \frac{b}{4.00}$, $b = 4 \tan 25 = 1.86$

$\boxed{b = 1.86}$

to 3 significant digits

2.



Given: ∠A = 55°
adjacent side b = 2

Determine: ∠B, a, and c

$\boxed{\angle B = 90° - 55° = 35°}$

$\tan A = \frac{a}{2}$, $a = 2 \tan 55° = 2.85$
or
$\tan B = \frac{2}{a}$, $a = \frac{2}{\tan B} = 2.85$

$\boxed{a = 2.85}$

$c^2 = 2.85^2 + 2^2$

$c^2 = 8.1225 + 4$

$c^2 = 12.1225$

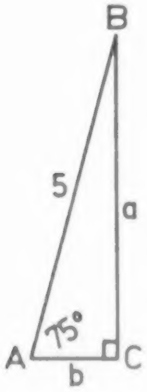
$\boxed{c = 3.48}$

3. Given: One angle and the length of the hypotenuse

$\angle A = 75°$, hypotenuse $(c) = 5$

Determine: $\angle B$, a, and b

$\boxed{\angle B = 90 - 75 = 15°}$



b	$\sin B = \frac{b}{5}$,	$b = 5 \sin 15$	$b = 1.29$ or 1.3 to 2 significant digits
	$\cos A = \frac{b}{5}$,	$b = 5 \cos 75$	
a	$\cos B = \frac{a}{5}$,	$a = 5 \cos 15$	$a = 4.82$ or 4.8 to 2 significant digits
	$\sin A = \frac{a}{5}$,	$a = 5 \sin 75$	

OBJECTIVES	NOTES TO TEACHERS

7. to determine the angles, $0 < \theta < 90$, given the numerical value of $\sin\theta$, $\cos\theta$ or $\tan\theta$, or the lengths of the sides of a right triangle;

Note: Scientific calculators may be used in place of trigonometric tables.

Examples:

1. Given: $\cos\theta = 0.70711$

Find: θ

NATURAL COSINES

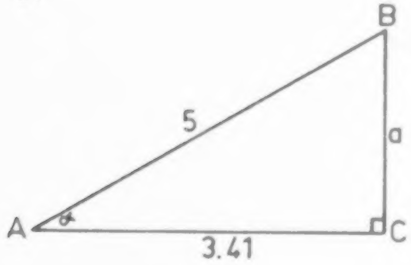
DEGREES	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	MEAN DIFFERENCES				
											1	2	3	4	5
45	·70711	70587	70463	70339	70215	70091	69966	69842	69717	69591	21	41	62	83	104
46	·69466	69340	69214	69088	68962	68835	68709	68582	68455	68327	21	42	63	84	106
47	·68200	68072	67944	67816	67688	67559	67430	67301	67172	67043	21	43	64	86	107
48	·66913	66783	66653	66523	66393	66262	66131	66000	65869	65738	22	44	65	87	109
49	·65606	65474	65342	65210	65077	64945	64812	64679	64546	64412	22	44	66	88	111

Using table $\theta = 45°$

Using calculation $\theta = 44.999$ or $45°$

Teachers may choose to do similar examples for sines and tangents.

2.



Given: $b = 3.41$, $c = 5$

Find: α to the nearest degree

$$\cos\alpha = \frac{3.41}{5} = 0.682$$

$\alpha = 47°$ using table above.

Using calculation 46.999 or 47°.

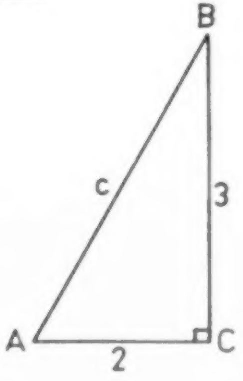
OBJECTIVES

NOTES TO TEACHERS

8. to find all parts of a right triangle, given any two sides;

Examples:

1.



Given: lengths of arms of triangle

a = 3 and b = 2

Find: a) ∠A and ∠B to the nearest degree

b) Find c

Solution:

Tan A = $\frac{3}{2}$, $\boxed{\angle A = 56.3°}$ to the nearest tenth of a degree

$\angle B = 90 - 56.31° =$

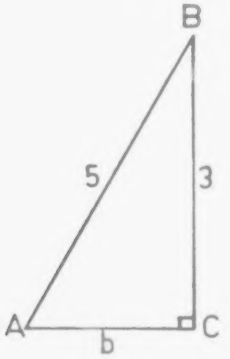
$\boxed{\angle B = 33.7°}$ to the nearest tenth of a degree

$c^2 = 2^2 + 3^2$

$c = \sqrt{13}$

$\boxed{c = 3.6}$ (to two significant digits)

2.



Given: $a = 3$, $c = 5$ (one arm and hypotenuse)

Find: a) b
b) $\angle A$, $\angle B$.

Solution:

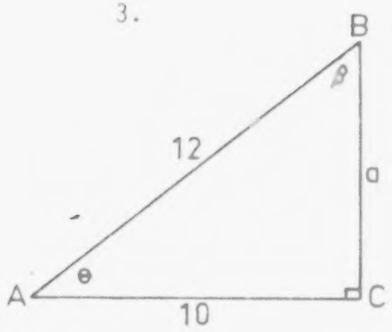
(a) $b + 3^2 = 5^2$

$\boxed{b = 4}$

(b) $\sin A = \frac{3}{5}, \angle A = 36.9°$

$\angle B = 90° - \angle A, \angle B = 53.1°$

3.



Given: $AB = 12$ $\angle ACB = 90°$
$AC = 10$

Find: a, θ and β

Solution:

Pythagoras Relation: $a^2 + 10^2 = 12^2$

$a^2 = 144 - 100$

$a^2 = 44$

$a = 6.63 = 6.5$

$$\cos \theta = \frac{10}{12} = 0.83300, \ \theta = {}^*33.6^\circ$$

$$\text{OR } \tan \theta = \frac{a}{10} = 0.66, \ \theta = {}^*33.4^\circ$$

$$\text{OR } \sin \theta = \frac{a}{12} = \frac{6.6}{12} = 0.55000, \ \theta = 33.4^\circ$$

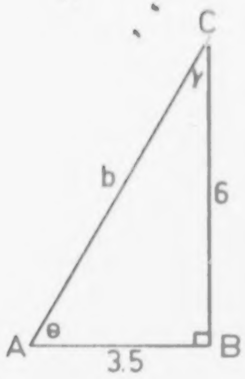
* difference due to round off error during calculation.

$$\beta = 180 - 90 - \theta = 56.5^\circ$$

$$\text{OR } \sin \beta = \frac{10}{12}$$

$$\beta = 56.4^\circ$$

4.



Given: $AB = 3.5$ $\angle ABC = 90^\circ$
$BC = 6.0$

Find: b, θ and γ

Solution:

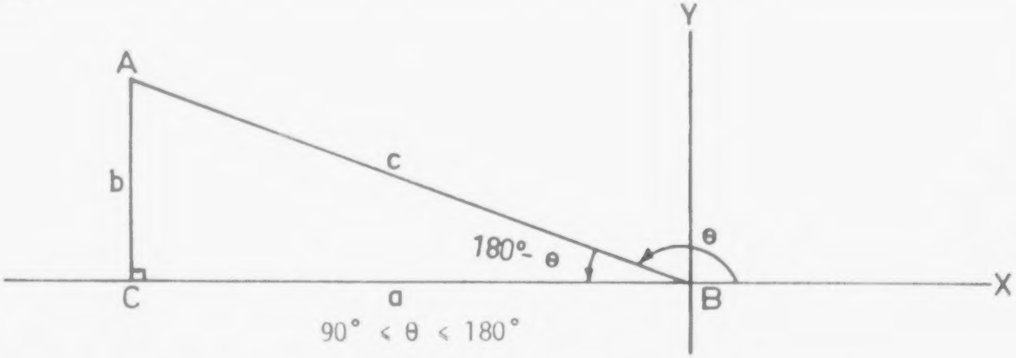
$$b^2 = 6^2 + (3.5)^2$$
$$= 48.25$$
$$= 6.946$$
$$\boxed{b = 6.9 \text{ (approx.)}}$$

$$\tan \theta = \frac{6}{3.5} = 1.171429 \qquad \boxed{\theta = 59.7^\circ}$$

$$\gamma = 180^\circ - \theta - 90^\circ \qquad \boxed{\gamma = 30.3^\circ}$$

9. to determine the numerical value of sin θ, cos θ and tan θ, given $90^\circ < \theta < 180^\circ$;

Value of trigonometric functions of angles from 90° to 180° are determined as follows:



θ is an exterior angle above and the ratios are defined as follows:

$$\sin\theta = \frac{b}{c} = \sin(180 - \theta)$$

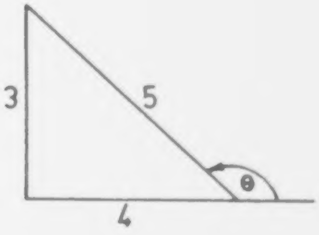
$$\cos\theta = \frac{a}{c} = -\cos(180 - \theta)$$

$$\tan\theta = \frac{b}{a} = -\tan(180 - \theta)$$

The trigonometric functions (cosine and tangent) are negative because "a" is measured in the opposite direction from the case where $0^\circ < \theta < 90^\circ$.

Examples:

1.

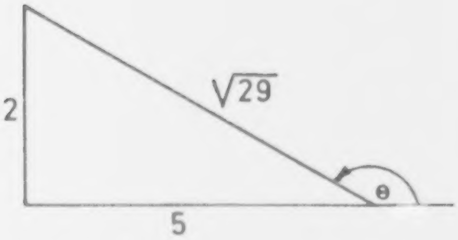


$\sin \theta = \frac{3}{5}$

$\cos \theta = -\frac{4}{5}$

$\tan \theta = -\frac{3}{4}$

2.



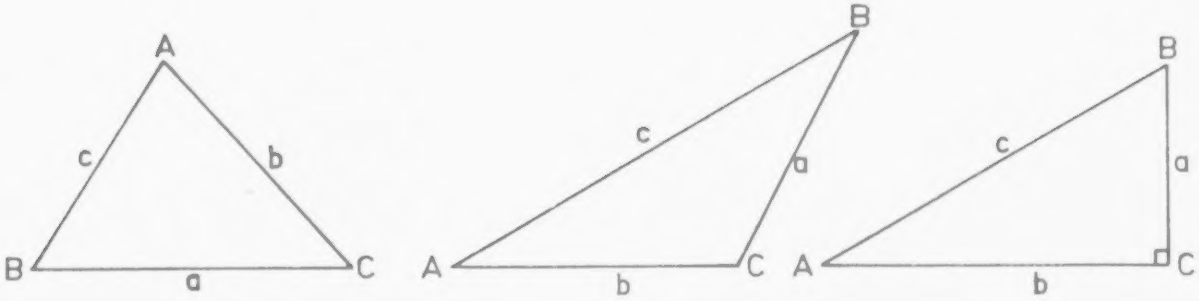
$\sin \theta = \frac{2}{\sqrt{29}}$

$\cos \theta = \frac{-5}{\sqrt{29}}$

$\tan \theta = -\frac{2}{5}$

Note: The largest angles found in triangles are less then 180°. Therefore, this is sufficient to do triangle trigonometry.

OBJECTIVES	NOTES TO TEACHERS
10. to apply the Law of Sines;	For any triangle, whether acute, obtuse, or right, the Law of Sines holds true.

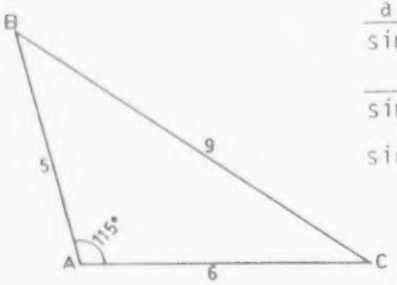


Law of Sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Example:

From the diagram of ΔABC, determine the size of ∠C to the nearest degree.



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{9}{\sin 115^\circ} = \frac{5}{\sin C}$$

$$\sin C = \frac{5 \sin 115^\circ}{9}$$

$$= 0.50350$$

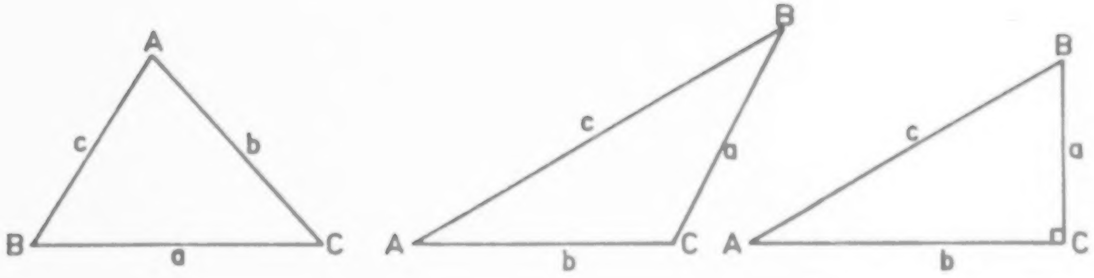
$$\angle C = 30^\circ$$

OBJECTIVES

11. to apply the Law of Cosines;

NOTES TO TEACHERS

For any triangle, whether acute, obtuse, or right the Law of Cosines holds true.

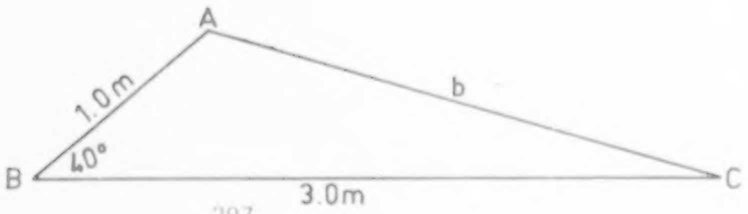

A
c
b
B
a
C
B
c
a
A
b
C
B
c
a
A
b
C

Law of Cosines:

1. $a^2 = b^2 + c^2 - 2bc \cos A$
2. $b^2 = c^2 + a^2 - 2ca \cos B$
3. $c^2 = b^2 + a^2 - 2ba \cos C$

Example:

From the diagram of $\triangle ABC$, find the distance from A to C.


A
1.0m
b
40°
B
3.0m
C

OBJECTIVES

NOTES TO TEACHERS

$$\begin{aligned} b^2 &= a^2 + c^2 - 2ac \cos B \\ &= 3^2 + 1^2 - 2(3)(1) \cos 40^\circ \\ &= 10 - 6\ (0.766\ 04) \\ &= 10 - 4.596\ 24 \\ b^2 &= 5.403\ 76 \end{aligned}$$

$$\boxed{b = 2.3 \text{ m}}$$

When determining the numerical value of sin θ, cos θ or tan θ, given $90^\circ < \theta < 180^\circ$:

Using calculators:

. the readout will specify whether the value of the function is positive or negative. The teacher may wish to explain why this occurs.

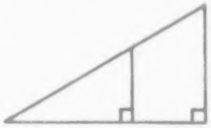
Using tables:

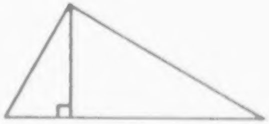
. explain standard position of angles, related angles, and which functions are positive or negative in the second quadrant.

12. to solve problems which require the use of either the Law of Sines or the Law of Cosines or right triangle relationships.

Use word problems, include those dealing with:

. angle of elevation
. angle of depression
. surveying
. overlapping triangles



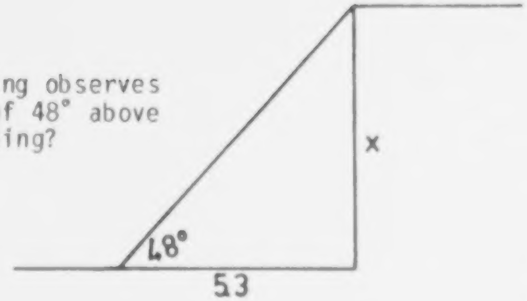


. adjacent triangles

Examples:

1. Definition of Ratios

A man standing 5.3 m from a building observes that the top appears at an angle of 48° above horizontal. How high is the building?



Solution: $\tan 48 = \dfrac{x}{5.3}$

$$x = (5.3)(\tan 48)$$

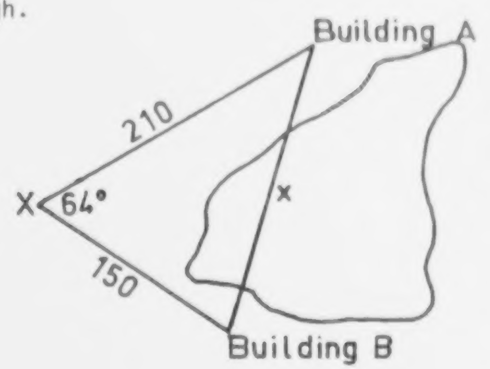
$$= (5.3)(1.1106)$$

$$\doteq 5.88$$

Therefore, building is 5.9 m high.

2. Law of Cosines

Two buildings are on opposite sides of a lake. From a point at one end of the lake the buildings are 64° apart. If the point is 150 m from one building and 210 m from the other, how far apart are the buildings?



Solution:
$$x^2 = (150)^2 + (210)^2 - 2(210)(150)\cos 64^\circ$$
$$= 22\,500 + 44\,100 - 63\,000\,(0.43837)$$
$$= 22\,500 + 44\,100 - 27\,617.38$$
$$= 38\,982.62$$
$$x = 197.44$$

Therefore, the buildings are approximately 197 m apart.

3. Law of Sines

Two artillery officers are 1.0 km apart on an east-west line. The western officer sees the target at 40° east of north, the eastern officer at 20° west of north. How far is each gunner from the target?

Solution: $\angle AXB = 180-50-70 = 60^\circ$

Law of Sines

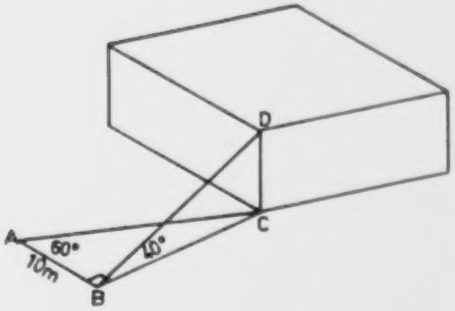
$$\frac{1}{\sin 60} = \frac{b}{\sin 70} = \frac{a}{\sin 50}$$

$$b = \frac{\sin 70}{\sin 60} = 1.085 \text{ km}$$

$$a = \frac{\sin 50}{\sin 60} = 0.884 \text{ km}$$

Western gunner is 1.1 km from target and eastern gunner is 0.9 km from target.

4. Two people are standing the same distance from a building, one of them straight out from a corner. The other person measures an angle of 60° between the corner and the first person and is 10 m from the other person. If the first person measures the angle of elevation (the angle above horizontal) of the top of the building as 40°, how tall is the building at the corner?



Solution: Let persons be A and B.

$AB = 10$ m

$\angle BAC = 60°$

$\angle CBD = 40°$

$$\frac{BC}{AB} = \tan 60° \therefore BC = 10 \tan 60 = 17.32$$

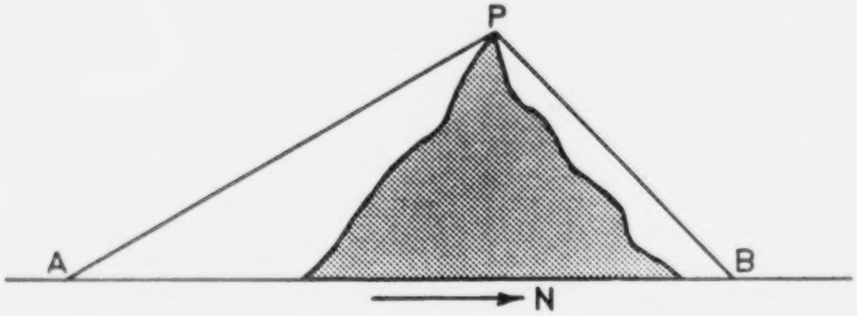
$$\frac{DC}{BC} = \tan 40° \therefore DC = 17.32 \tan 40 = 14.53 \text{ m}$$

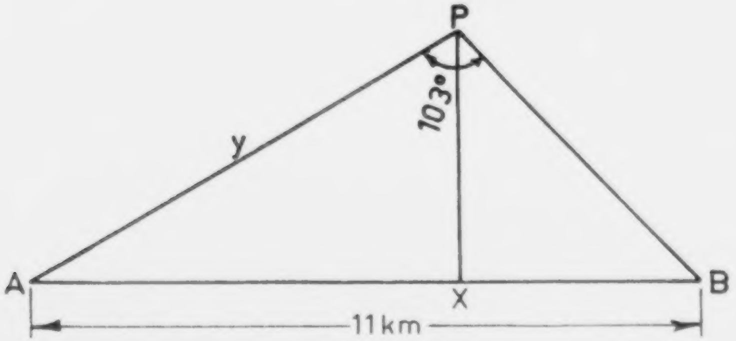
Therefore, the height of building is 14.5 m.

Note: If the angle ABC is not 90°, then Law of Sines must be used to find BC.

5. A person north of a mountain sees the peak of the mountain at 47° above horizontal. A person south of it sees it at 30° above horizontal (30° elevation). If the two people are on the same level and 11 km apart, how high is the peak above them?

Solution: Let the observers be located at B and A respectively.





First find length of AP = y using Law of Sines

$$\frac{11}{\sin 103} = \frac{y}{\sin 47}$$

$$y = \frac{11(\sin 47)}{\sin 103}$$

$$\doteq 8.2564$$

Now determine PX

$$\sin 30 = \frac{PX}{y}$$

$$\begin{aligned} PX &= y \sin 30^\circ \\ &= 8.2564 \sin 30^\circ \\ &= 4.1282 \end{aligned}$$

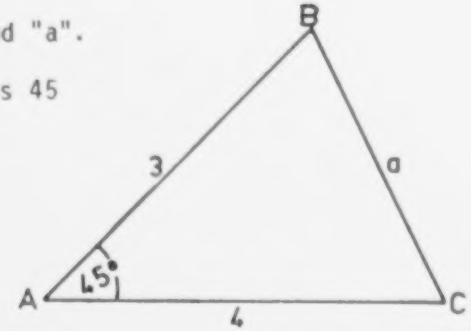
Therefore, the height of the mountain is 4.128 or 4.13 km.

6. Use Law of Cosines to find "a".

$$a^2 = 3^2 + 4^2 - 2(3)(4)\cos 45$$

$$= 8.029$$

$$a = 2.83$$



Use the Law of Sines to find ∠B and ∠C

$$\frac{b}{\sin B} = \frac{c}{\sin C} = \frac{a}{\sin A} = \frac{2.83}{\sin 45} = 4.00$$

$$\frac{b}{\sin B} = 4.00,\ \sin B = 1, \angle B = 90°$$

$$\frac{c}{\sin C} = 4.00,\ \frac{3}{\sin C} = 4.00,\ \sin C = \frac{3}{4}$$

$\boxed{\angle C = 49°}$ approximately

Note: Questions should include acute triangles, obtuse triangles, and right triangles.

C.IV. STATISTICS (1 MODULE)

Statistics is the study of the collecting and analyzing of data from which inferences and conclusions can be made. If students are required to collect data for analyzing, this collection should be done before beginning discussion of this topic.

If computers are available in the school, they may be incorporated into this module.

Some calculators are equipped with a statistics mode. Explain its use if students have access to such a calculator.

Sets of data at the end of this module may be used.

OBJECTIVES	NOTES TO TEACHERS
A. AREAS OF APPLICATION - USES OF STATISTICS	
The student should be able:	
1. to identify areas in which statistics are used.	Examples of some topics for discussion are: . mortality tables . insurance premiums . population movements . school construction and size . pension funds . game laws . evaluation of seat-belt laws
B. DATA GATHERING	
The student should be able:	
1. to distinguish between a census and a sample;	In a census, all elements of the population are polled. In a sample, only a representative percentage of the population is surveyed.

OBJECTIVES	NOTES TO TEACHERS
2. to list various methods of obtaining data;	Data may be obtained through interviews (telephone, mail-in, door-to-door survey), by measuring (velocity of cars from 0-10 seconds) or by counting (number of absences from school on Fridays).
3. to recommend methods of obtaining a representative sample.	The elements in a sample must have been chosen at random. Every element in the population must have an equal chance of being selected.
C. UNGROUPED OR RAW DATA	
The student should be able:	
1. to calculate from ungrouped data measures of central tendency: a) mode, b) mean, c) median;	The number which occurs most frequently in a set of data is called the mode. Data: 3, 2, 6, 3, 7, 12, 3, 14 Solution: The mode is 3. The arithmetic mean, $\bar{x}$ (bar x), is obtained by totalling the measures of a set and then dividing the sum by the number of measures in the set. Example: Find the mean of 78, 82, 83, 79 and 23. Solution: $\bar{x} = \frac{78 + 82 + 83 + 79 + 23}{5} = 69.$ If the data is arranged in order of magnitude, the middle measure is the median. If the number of the measures in a set is even, then the median is the mean of the two middle measures. Example: 23, 78, 79, 82, 83. The median is 79. Example: 2, 6, 7, 8, 15, 24. The median is $\frac{7 + 8}{2} = 7.5.$

OBJECTIVES	NOTES TO TEACHERS
2. to select the measure of central tendency most representative of the group of data;	Example: Find the mean, median, and mode of the following: 2, 8, 4, 9, 6, 5, 4, 3, 2, 8, 9, 3, 7, 9 Mean = 5.6 Median = 5.5 Mode = 9 Which is the best measure of central tendency? Each kind of measure is appropriate for certain types of data. One very high or very low number can distort the mean. The median is not affected by an extremely high or low number but may not be representative. For example, a group of 5 grandmothers and 4 infants might have a median age of 74, a mean of 42 and a mode of 80. In a normal distribution, the mean, mode and median would be identical. Most things, however, are not statistically normal. Examples: If a man lays with his head in an oven (40°C) and with his feet in an ice bucket (0°C), could we say that on the average (mean) he is quite comfortable? Did you know that the employees at Mr. Midas Skinflint's custom auto body earn an "average" of $20 000 a year? What you may not know is that each of his 3 employees has a salary of $7 000 while he pays himself $59 000 a year.
3. to calculate measures of dispersion of a set of ungrouped data: a) range,	At times, measures of central tendency have little meaning. The length of an airport runway cannot be built to accommodate the "average" aircraft; a crane cannot be built to lift the "average" load. In such cases it would be better to consider the extremes or the dispersion. The range is a measure of dispersion or variability. Variability is the difference between the largest and smallest elements of a set.

OBJECTIVES	NOTES TO TEACHERS

Example: Determine the range of the following data:

23, 46, 78, 12, 2, 9, 54, 97, 84, 13

Solution: Range 97 - 2 = 95

b) mean deviation.

The mean deviation is the mean of the absolute values of the differences from the mean. It is a measure of dispersion.

$$M.D. = \frac{\Sigma|\bar{x} - x|}{N}$$

Example:

Set A	6	8	10	12	14;	$\bar{x} = 10$
Set B	2	2	2	2	42;	$\bar{x} = 10$
Set C	8	8	10	12	12;	$\bar{x} = 10$

Set A	Difference from Mean	Set B	Difference from Mean	Set C	Difference from Mean
6	10 - 6 = 4	2	10 - 2 = 8	8	10 - 8 = 2
8	10 - 8 = 2	2	10 - 2 = 8	8	10 - 8 = 2
10	10 - 10 = 0	2	10 - 2 = 8	10	10 - 10 = 0
12	12 - 10 = 2	2	10 - 2 = 8	12	12 - 10 = 2
14	14 - 10 = 4	42	42 - 10 = 32	12	12 - 10 = 2
Sum of Differences	12	Sum of Differences	64	Sum of Differences	8
Mean Deviation	$\frac{12}{5} = 2.4$	Mean Deviation	$\frac{64}{5} = 12.8$	Mean Deviation	$\frac{8}{5} = 1.6$

Although all three sets have a mean of 10, their mean deviations are 2.4, 12.8 and 1.6, respectively, indicating that the measures in set B are most scattered and the measures in set C are least dispersed.

D. GROUPED DATA

The student should be able:

1. to organize ungrouped data in a frequency distribution table;

Example:

Twenty-five light bulbs were selected from a production lot of 1 000 bulbs and tested for hours of bulb life. Construct a frequency distribution table to illustrate the test results.

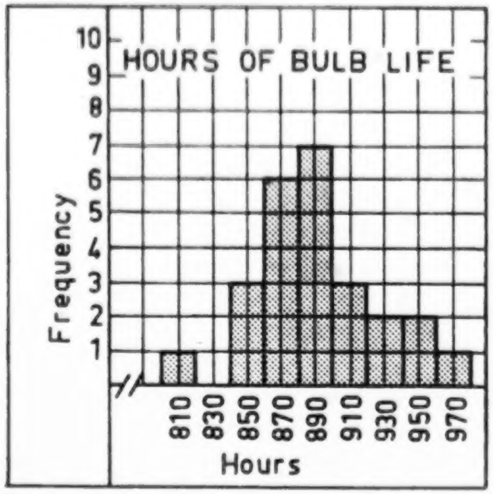
803	896	878	886	861
880	976	912	955	843
871	843	895	875	882
883	927	942	918	843
939	890	869	881	901

Solution: FREQUENCY DISTRIBUTION TABLE

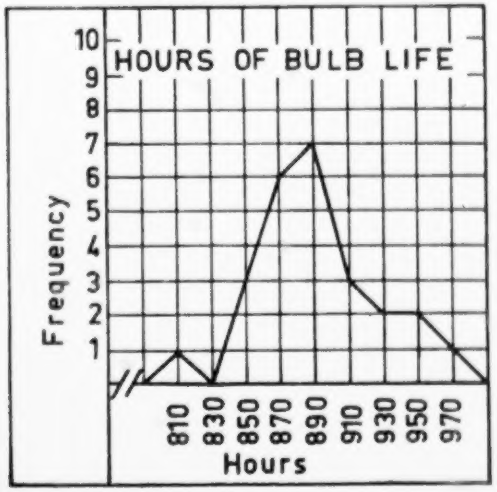
Class Boundaries	Class Mark	Tally	Frequency	Class Mark times Frequency	Cumulative Frequency
960 - 979	970	1	1	970	25
940 - 959	950	11	2	1 900	24
920 - 939	930	11	2	1 860	22
900 - 919	910	111	3	2 730	20
880 - 899	890	~~1111~~ 11	7	6 230	17
860 - 879	870	~~1111~~ 1	6	5 220	10
840 - 859	850	111	3	2 550	4
820 - 839	830		0	0	1
800 - 819	810	1	1	810	1
			25	22 270	

OBJECTIVES	NOTES TO TEACHERS
2. to construct a histogram to illustrate the information displayed in a frequency distribution table;	HISTOGRAM
3. to construct a frequency polygon to illustrate the information given in a frequency distribution table;	FREQUENCY POLYGON

HISTOGRAM



FREQUENCY POLYGON



4. to calculate from grouped data measures of central tendency including:

   a) mode,
   b) mean,
   c) median;

Calculate the mode, mean and median from the grouped data illustrated in the frequency distribution table.

FREQUENCY DISTRIBUTION TABLE

Class Boundaries	Class Mark x	Tally	Frequency f(x)	Class Mark times Frequency	Cumulative Frequency
960 - 979	970	1	1	970	25
940 - 959	950	11	2	1 900	24
920 - 939	930	11	2	1 860	22
900 - 919	910	111	3	2 730	20
880 - 899	890	~~1111~~ 11	7	6 230	17
860 - 879	870	~~1111~~ 1	6	5 220	10
840 - 859	850	111	3	2 550	4
820 - 839	830		0	0	1
800 - 819	810	1	1	810	1
			25	22 270	

The mode is the most frequently recurring measure. In grouped data, measures lose their individuality and take on the characteristics of the class. The individual measures also become distributed evenly within a class. From the table, the mode is 890. In the raw data, the mode for the same measures was 843.

The mean ($\overline{x}$) for grouped data is obtained by multiplying the frequency of the class marks by the class mark and dividing the sum by the number of measures.

$\overline{x} = \frac{\Sigma(x \cdot f)}{N} = \frac{22\ 270}{25} = \underline{\underline{891}}$ Where Σ represents "the sum of".

(The mean from the ungrouped data is 890.)

The median of a set of grouped data is the measure of the middle element. In the set of 25 illustrated in the previous Frequency Distribution Table, the median is the measure of the 13th element. The cumulative frequency column shows that the 13th element is in the group whose class boundaries are from 880-899 and would be the third of the seven elements or $\frac{3}{7}$ of the width of the class (20).

$$Md = \frac{3}{7} \times \underset{\substack{\uparrow \\ \text{class} \\ \text{width}}}{20} + \underset{\substack{\downarrow \\ \text{lower} \\ \text{limit}}}{880} = 8.57 + 880 = \underline{888.6}$$

(The median from the ungrouped data is 883.)

Example: Find the mean height and the median height of the 100 male students whose heights are summarized below.

Height	Class Mark	Frequency	Class Mark x Freq.	Cumulative Frequency
154-160	157	5	785	5
161-167	164	16	2 624	21
168-174	171	43	7 353	64
175-181	178	27	4 806	91
182-188	185	9	1 665	100
	TOTALS	100	17 233	

Solution: To find mean:

1. calculate class mark
2. multiply class mark by frequency
3. total this column
4. divide this total by the total of the frequency column, $\frac{17\,233}{100} = \underline{\underline{172.33}}$

To find median:

1. The median is between the 50th and 51st measures.
2. The sum of the first two class frequencies is 5 + 16 = 21, so to get the 50.5th measure, we need 29.5 more measures from the next group.
3. The measure we want must be $\frac{29.5}{43}$ of the way through the interval.
4. Because the third class interval actually corresponds to the heights 167.5 to 174.5, the median is $\frac{29.5}{43}$ of the way between 167.5 and 174.5.
5. The median is therefore

$$167.5 + \frac{29.5}{43}(174.5 - 167.5)$$

$$167.5 + 4.8$$

$$172.3.$$

5. to calculate the following measures of dispersion (or variability):

a) range,

The range of the bulb life is the difference between the highest measure (970) and the lowest measure (810) which is 970 - 810 = 160 hours.

b) mean deviation.

The mean deviation, or deviation from the mean, is a measure of variability. Previously, the mean bulb life was calculated at 891 hours.

Class Mark	Frequency	Deviation from the Mean	Deviation from Mean times Frequency
970	1	971 - 891 = 79	79
950	2	950 - 891 = 59	118
930	2	930 - 891 = 39	78
910	3	910 - 891 = 19	57
890	7	891 - 890 = 1	7
870	6	891 - 870 = 21	126
850	3	891 - 850 = 41	123
830	0	891 - 830 = 61	0
810	1	891 - 810 = 81	81
	25		669

Mean Deviation = 669 ÷ 25 = 26.8

$$M.D. = \frac{\Sigma f\,|\,\overline{x} - x\,|}{N}$$

Note: In calculating the mean deviation of grouped data some statisticians use N-1 instead of N (as a correction factor for samples) as a divisor, arguing that since one of the deviations from the mean is zero, the result is biased. In most sets of data, the arithmetic mean will not be equal to one of the elements. Therefore, in these exercises N is used consi[illegible]ntly.

E. PERCENTILES

The student should be able:

1. to calculate the percentiles for a given set.

A percentile is $\frac{1}{100}$ of a given set of elements arranged in order of magnitude.

The 50th percentile corresponds to the median.

Some test results (DAT) (SAT) (GRE) (LSAT) are communicated in percentiles only.

Example:

A student's score was ranked 75th on a test administered to 200 students. In which percentile would he/she be?

Solution:

If the student was ranked 75th then there would be 200 - 75 = 125 scores lower than his/hers. This would place him/her in the $\frac{125}{200} \times 100\% = 62.5$ percentile.

This means 62.5% of the students scored lower and 37.5% scored higher than he/she did.

Note: Sometimes, results are given in quartiles or quintiles.

First quartile: $\frac{1}{4}$ of the measures are less than or equal to it.

First quintile: $\frac{1}{5}$ of the measures are less than or equal to it.

Data Sets

The accompanying data sets have been constructed so there are sets with the same mean and different dispersions and sets with different means and the same dispersions. Possible interpretations are given as constructed experiments to create more realistic word problems. The same data set and problem(s) can be referred to for most of the objectives.

1. Weight of 100 professional football players measured in kg.
   Mean = 100.88
   Mean deviation = 6.30

110	103	88	102	85	116	98	92	105	109
97	105	111	84	93	107	101	92	109	88
91	103	104	99	105	96	96	106	111	104
97	108	101	95	112	96	104	107	87	101
98	110	98	107	101	101	113	108	101	91
116	96	105	105	99	95	99	90	110	94
93	94	100	107	114	112	95	91	100	101
97	101	99	98	104	96	98	81	105	103
104	90	110	106	96	80	103	106	97	93
105	102	111	108	93	102	96	116	103	104

2. Each of 100 workers are told to cut a piece of wood 100 mm long. The actual measurements are identified below.
   Mean = 100
   Mean Deviation = 0.63

101	100.3	98.8	100.2	98.5	101.6	99.8	99.2	100.5	100.9
99.7	100.5	101.1	98.4	99.3	100.7	100.1	99.2	100.9	98.8
99.1	100.3	100.4	99.9	100.5	99.6	99.6	100.6	101.1	100.4
99.7	100.8	100.1	99.5	101.2	99.6	100.4	100.7	98.7	100.1
99.8	101	99.8	100.7	100.1	100.1	101.3	100.8	100.1	99.1
101.6	99.6	100.5	100.5	99.9	99.5	99.9	99.	101	99.4
99.3	99.4	100	100.7	101.4	101.2	99.5	99.1	100	100.1
99.7	100.1	99.9	99.8	100.4	99.6	99.8	98.1	100.5	100.3
100.4	99	101	100.6	99.6	98	100.3	100.6	99.7	99.3
100.5	100.2	101.8	100.8	99.3	100.2	99.6	101.6	100.3	100.4

3. 100 people buying coffee put the following amounts of sugar (grams) in their coffee:
Mean = 10
Mean deviation = 0.63

11	10.3	8.8	10.2	8.5	11.6	9.8	9.2	10.5	10.9
9.7	10.5	11.1	8.4	9.3	10.7	10.1	9.2	10.9	8.8
9.1	10.3	10.4	9.9	10.5	9.6	9.6	10.6	11.1	10.4
9.7	10.8	10.1	9.5	11.2	9.6	10.4	10.7	8.7	10.1
9.8	11	9.8	10.7	10.1	10.1	11.3	10.8	10.1	9.1
11.6	9.6	10.5	10.5	9.9	9.5	9.9	9	11	9.4
9.3	9.4	10	10.7	11.4	11.2	9.5	9.1	10	10.1
9.7	10.1	9.9	9.8	10.4	9.6	9.8	8.1	10.5	10.3
10.4	9	11	10.6	9.6	8	10.3	10.6	9.7	9.3
10.5	10.2	11	10.8	9.3	10.2	9.6	11.6	10.3	10.4

4. Weights (kg) of a group of 13-year-olds in a judo competition:
Mean = 50
Mean deviation = 32.5

53.5	45	54	45.5	51.5	58	49	53	42	53.5
49.5	44.5	45.5	52.5	52	48.5	44	51	54.5	59.5
46	40	51.5	51.5	43.5	49.5	48	44.5	54.5	58.5
45.5	46.5	57.5	51.5	49.5	47	48.5	49	47.5	49
52.5	51.5	51	47	49.5	49	49.5	46	51	45
45.5	52	48	49.5	50.5	48	51	48.5	51.5	47
50.5	47	42.5	43	44	51.5	44.5	46.5	48	48.5
45.5	44.5	54.5	50	50.5	57	46.5	50	47.5	52.5
57.5	55	49.5	48.5	56.5	46	48	53	51	45
52	49	46.5	51.5	40	41.5	47	55.5	56	52

5. Length of time (days) that certain items sit on a store shelf. (Could be used to describe the rate of turnover for merchandise.)
Mean = 40
Mean deviation = 2.96

38	42.4	37.5	41.2	41.6	38.8	42	43.2	44.8	42.8
37.6	38.4	46.4	38	44	39.2	38.8	38	48	37.6
36.4	37.6	41.2	35.6	38.4	40.8	46	47.2	42	42.4
35.8	39.2	45.2	48	37.6	44	40.4	40.8	41.2	39.2
46.4	34	43.2	39.6	36.4	38	39.6	46.4	35.6	47.6
38.4	45.6	32	42.4	38.8	38	32	40.4	38.8	34.8
35.6	39.2	38	33.6	38.4	38.4	41.6	42	37.2	40.8
35.6	36	38.4	38.4	39.2	46.4	37.2	46	40.4	44
42.8	40.8	37.6	41.6	38.8	38.8	44	43.2	43.6	40.8
39.6	41.2	38	36	42	45.6	34.4	32	38	36

6. Cost in cents of 100 meals at a fast-food outlet.
Mean = 500
Mean deviation = 37

475	530	470	515	520	485	525	540	560	535
470	480	580	475	550	490	485	475	600	470
455	470	515	445	480	510	575	590	525	530
460	490	565	600	470	550	505	510	515	490
580	425	540	495	455	475	495	580	445	595
480	570	400	530	485	475	400	505	485	435
445	490	475	420	480	480	520	525	465	510
445	450	480	480	490	580	465	575	505	550
535	510	470	520	485	485	550	540	545	510
495	515	475	450	525	570	430	400	475	450

OBJECTIVES NOTES TO TEACHERS

General Questions by Objectives

Objective 2: Prepare histograms for each data set (students could use different interval sizes).

Objective 3: Observe similarities and differences between where the graph is located on the number line (where is its "centre") and whether it is more or less spread out. (Note that the mean and mean deviation are given for each data set. If a subset is used the mean and mean deviation may be slightly different.)

Objective 4: Calculate and compare mean, median, and mean deviation for each and compare to the histogram with respect to central tendency and degree of spread.

Objective 5: Calculate mean and median for whatever groupings were used in Objective #2.

Objective 6: Give each student a 10-element sample (a row or a column) and a 20-element sample, all from the same data set. Make a histogram (Objective #2) of the means of the 10-element samples and another of the 20-element samples. Which is spread out more? Where are their centres?

Objectives 7, 8: Calculate percentiles, quartiles, etc. for one of the data sets. Construct a cumulative frequency graph and read the quintiles from it.

Note: Standard Deviation and Normal Curve are dealt with in Op. V.

OBJECTIVES	NOTES TO TEACHERS
	Resources:

Resources:

1. Carli et al, Mathematics For A Modern World Book 4 (Agincourt, ON: Gage Publishing Company, 1975).
2. Dean & Moore, Foundations of Mathematics (Toronto, ON: Holt-Rinehart-Winston Publishers, 1968). Out of print.
3. Runynon, Richard, P, Winning With Statistics (Don Mills, ON: Addison-Wesley Publishing Company, 1977).

613 614

Op.I. ANNUITIES (½ MODULE)

An annuity is simply a series of periodic payments, usually made in equal amounts. Originally, the term referred only to annual payments but now, it implies almost any series of payments, regardless of the payment interval.

The payment interval is the time between payments. The term of the annuity refers to its full duration; i.e., the time between the beginning of the first payment interval and the end of the last payment interval.

Annuities are applied in various types of contracts and financial arrangements. Compound amount calculations are important in most of them. However, rental payments and Family Allowance payments do not normally imply a compund amount calculation. Mortgage payments and savings plan deposits certainly do involve compound amounts and balances. Interest payments on bonds are another example of an annuity, although these probably would not be equal amounts.

Annuities may be classified in at least three ways:

1. classification by the agreed-upon term
2. classification with respect to the dates of payments
3. classification by comparing the payment interval with the interest conversion period

Further details about these classifications are included in the teacher notes.

OBJECTIVES	NOTES TO TEACHERS
The student should be able:	
1. to calculate the compound amount of a given principal, invested for a specified time, given the annual interest rate and the interest conversion period;	Objectives 1 and 2 are included here to remind the student of the basic concept and types of computation involved in determining compound amounts and present values. Ordinary compound amount calculations involve a fixed principal which is incremented only by the accrued interest. The principal in an annuity grows because of the regular payments and the accrued interest.

Example:

Calculate the compound amount of $1 500.00 invested for 8 years at 12% per annum, compounded semi-annually.

Solution: Use $A = P(1+r)^n$ where
A = the compound amount
P = the principal
r = the interest rate per conversion period
n = the total number of conversion periods

Hence: $A = 1\,500(1 + 0.06)^{16}$ (the calculator steps follow)

Enter 1.06
y^x
16
=
x
1500
=
(answer) = $3 810.53

2. to calculate the present value (initial principal required) to be invested in order to develop a projected amount, given the term of investment, the annual interest rate and the interest conversion period;

Example:

Calculate the present value required to develop a compound amount of $10 000.00 over a 15-year period at 18% per annum, compounded quarterly.

Solution: Use $P = \frac{A}{(1+r)^n}$; hence $P = \frac{10\,000}{(1.045)^{60}} = \$712.89$

OBJECTIVES	NOTES TO TEACHERS
3. to list three types of annuities as they are classified by term;	Classification by the agreed-upon term: a) Annuity Certain: This type begins and ends on definite, pre-arranged dates. b) Perpetuity: These begin on a definite date but the length of the term is undefined. For example, the interest earned by investment is paid out in regular intervals. The principal is never reduced and the arrangement continues indefinitely. c) Contingent Annuity: The term of this type of annuity begins on a definite date but the ending date depends on some future expected event. For example, a student may be receiving aid in the form of an annuity while enrolled in the course but the benefits will be terminated upon completion of the course.
4. to list three types of annuities as they are classified by date of payments;	Annuities may also be classified with respect to the dates of the payments: a) Ordinary Annuity: Periodic payments are made at the end of each payment interval. b) Annuity Due: In this case, the payments are made at the beginning of the payment interval. c) Deferred Annuity: The payments will be made at the end of each payment interval, but the first payment interval does not begin until some later, specified time. For example, the student will repay the student loan by regular payments but the first payment interval does not begin until six months after graduation.
5. to explain the difference between a simple and a complex (general) annuity;	Annuities are also classified by comparing the payment interval with the interest conversion period:

a) Simple Annuity: As the term implies, there are no complications; i.e., the payment interval and the interest conversion period are exactly the same. They begin and end on the same dates.

b) Complex Annuity (General Annuity): The payment intervals and the interest conversion periods are not of the same duration. Either one may be the longer of the two. The following explains the computations of the mathematical equation.

If the payment interval (e.g., 6 months) is longer than the interest conversion period (e.g., quarterly), then a fictitious payment $E is calculated such that the payment interval and the interest conversion period can be the same. That is, two payments of $E, compounded according to the contract, would be exactly equal to the semi-annual payment as agreed upon in the contract. Then the annuity can be calculated as a simple annuity, using E, the new equivalent monthly payment.

This approach is also applied to the situation where the interest conversion period (e.g., 6 months) is longer than the payment interval (e.g., monthly). In this case, the six deposits, plus the accrued interest for a six-month period, become the new, equivalent payment, E, payable every six months. Again, the calculation is the same as a simple annuity.

The formula required for any calculation in this section will be given. The development, or proof, of the formula will not be shown but references have been provided. Computations will be done by applying the exponential function y^x on the hand-held calculator because charts and tables are usually incomplete and often out of date.

OBJECTIVES | NOTES TO TEACHERS

6. to compute the amount (Sn) of an ordinary simple annuity;

Example:

Compute the amount of an ordinary simple annuity payable at $25.00 monthly for 4 years bearing 15% per annum, compounded monthly.

Solution: Use $S_n = R\left[\frac{(1+i)^n - 1}{i}\right]$ where

S_n = the amount of an ordinary annuity
R = the regular payment
i = the interest rate per conversion period
n = the total number of intervals

For the development of the above formula, see Mullings, G.E. and Shao, S.P., *Mathematics for Management and Finance*, Canadian Edition (Agincourt, ON: Gage Publishing Ltd., 1979) pp. 354-56.

Calculation: $S_n = 25\left[\frac{(1+0.0125)^{48}-1}{0.0125}\right] = \underline{\$1\ 630.71}$

7. to compute the present value of an ordinary simple annuity, given the formula;

The present value of an annuity is the sum of all the present values of the many payments required.

Example:

Compute the present value of an ordinary simple annuity payable at $50.00 quarterly, for 6 years, bearing 12% per annum, compounded quarterly.

Solution:

Use $A_n = R\left[\dfrac{1-(1+i)^{-n}}{i}\right]$ where

A_n is the present value of an ordinary simple annuity and the other symbols have the meanings given under Objective 6.

The development of the above formula may be found in Mathematics For Management and Finance, pp. 360-61. (See Objective 6.)

Calculation: $A_n = 50\left[\dfrac{1-(1+0.03)^{-24}}{0.03}\right] = \underline{\$846.78}$

8. to compute the amount of an ordinary complex annuity, given the formula;

Example:

Compute the amount of an ordinary complex annuity payable at $40.00 per month for 3 years, bearing interest at 15% per annum, compounded semi-annually.

Solution: Use $S_n = R\left[\dfrac{(1+1)^{nc}-1}{(1+i)^{c}-1}\right]$ where

all the symbols mean the same as before and

$c = \dfrac{\text{payment interval}}{\text{conversion period}}$ (in this case $\frac{1}{6}$)

$$S_n = 40\left[\dfrac{(1+0.075)^{6}-1}{(1+0.075)^{\frac{1}{6}}-1}\right] = \underline{\$1792.13}$$

9. to compute the present value of an ordinary complex annuity, given the formula;

Example:

Compute the present value of the ordinary complex annuity described in Objective 8.

Solution: Use $A_n = R\left[\frac{1-(1+i)^{nc}}{(1+i)^c-1}\right]$ The answer is $1161.23.

Conclusion:

There is another type of annuity which presents another complication: a savings plan whereby you may deposit $30.00, for example, every month. The bank calculates the interest each month (based on the minimum monthly balance) but the interest is added to the principal only semi-annually. This becomes an ordinary complex annuity in which the interest rate (i) per conversion period is not clear. Another calculation is required, similar to the calculation of a fictitious payment, E, referred to earlier.

This type of savings scheme has been referred to previously in Mathematics 201, module C.I. Savings, Objective C-4. In that instance, the students made a chart in which all the deposits were entered manually and the ininterest was recorded according to the plan. This note has been added here simply to indicate that with some schemes, although they are easily described, it is almost impossible to capture the required calculation with one mathematical formula.

ENRICHMENT:

Determine the outstanding principal of an amortized loan after n payments.

The outstanding principal after a number of payments is the present value of an annuity formed by the remaining unpaid payments. The formula shown below is somewhat complex, but the exponential function, y^x, and the mem-

OBJECTIVES	NOTES TO TEACHERS

memories in the hand-held calculator, enable us to perform the calculation without great difficulty. Business math calculators are available on the market. Many of the required functions and sequences are built into these calculators.

Enrichment Exercise

Determine the outstanding principal after 7 years of monthly payments on a Canadian mortgage if the original contract was signed for \$32 000.00 at 15% (annual rate). The payments are \$410.00 per month for 20 years.

The formula is: $B_n = P(1+r_m) - A\left[\frac{(1+r_m)^n - 1}{r_m}\right]$ where

B_n = the balance after n payments
P = the original loan principal
r_m = the monthly interest rate (to coincide with monthly payments)
A = the regular payment
n = the number of payments completed

Note: r is determined in various ways:

i) $r_m = (1+r_a)^{\frac{1}{12}} - 1$ This seems to be typical in the U.S.A.
ii) $r_m = (1+\frac{r_a}{12}) - 1$ Typical of Canadian banks.
iii) $r_m = \frac{r_a}{12}$ Credit unions.

(r_a is the annual interest rate in all cases.)

With the Canadian version of r_m, the answer should be: \$28 827.08.

Op.II. DEPRECIATION AND DISTRIBUTION OF OVERHEAD COSTS (1 MODULE)

A. DEPRECIATION

Fixed assets are assets that will be used for a number of years in the operation of a business. Usually, an asset is considered as fixed when it will be in the business for a period exceeding one year.

Some examples of fixed assets include: building, vehicles, equipment, furniture. Other assets, such as cash, accounts receivable and inventory, are known as current assets or liquid assets and do not depreciate.

Depreciation is a decrease in value of a fixed asset due to wearing out or obsolescence. The amount that a fixed asset depreciates every year is an expense to the business.

This module will demonstrate some of the methods by which depreciation expenses are calculated.

OBJECTIVES	NOTES TO TEACHERS
The student should be able:	
1. to define the terms:	
a) original cost,	"Original cost" includes all expenditures needed to purchase a fixed asset. This includes discounts, freight, sales taxes, etc.
b) disposal, scrap or salvage value,	"Disposal," "scrap" or "salvage value" refers to the value of the asset when it is replaced. This amount must be estimated because the actual value will not be known until the fixed asset is actually disposed.
c) estimated useful life;	"Estimated useful life" is the period of time the fixed asset is expected to be useful to the business. This estimated useful life is an estimate because the actual useful life is not known until the item is actually disposed.

OBJECTIVES	NOTES TO TEACHERS
2. to calculate the total amount of estimated depreciation;	Example: Ace Manufacturing purchased a new machine for $16 000. The firm estimated the machine would have a scrap value of $1 000 at the end of 10 years. What total amount of depreciation must the firm spread out over the 10 years? Solution: Cost - Scrap Value = Total Depreciation $16 000 - 1 000 = $15 000, which must be depreciated over the 10-year period.
3. to calculate annual or monthly depreciation expense, using the straight-line method;	The "straight-line method" is the simplest method of calculating depreciation. The amount of depreciation is divided equally into each year or month of expected useful life $\text{Annual Depreciation} = \frac{\text{Total Depreciation}}{\text{Estimated Useful Life In Years}}$ Example: A construction company purchased a new front-end loader for $150 000. The estimated useful life is 8 years, at which time the disposal value is estimated to be $15 000. Calculate the amount of annual depreciation expense, using the straight-line method. What would be the monthly depreciation expense? Solution: Total depreciation: $135 000.00 Annual depreciation: $ 16 875.00 Monthly depreciation: $ 1 406.25

4. to define the term "book value";

"Book value" is the original cost minus the total accumulated depreciation of a fixed asset.

5. to calculate the book value;

Example:

What would be the book value at the end of the first, second and third year of useful life for a truck costing $40 000 and depreciation at an annual amount of $8 000?

Solution:

First year:	Cost	$ 40 000
	Less: Depreciation	8 000
	Book Value Year 1	$ 32 000
Second year:	Book Value Year 1	$ 32 000
	Less: Depreciation	8 000
	Book Value Year 2	$ 24 000
Third year:	Book Value Year 2	$ 24 000
	Less: Depreciation	8 000
	Book Value Year 3	$ 16 000

To calculate the book value at the end of a particular year, students could use the formula: cost less (annual depreciation x number of years) = book value.

6. to calculate the percent rate of depreciation;

Businesses often refer to the depreciation on a fixed asset as a percent rate rather than an amount. The percent rate is based on the original cost, not the original cost less scrap value. This may present a problem for some because the annual depreciation must include this scrap value as shown in Objective 2.

Example:

An electronic typewriter that cost $2500 when new, has an estimated disposal value of $300 after 8 years. What is the annual rate of depreciation?

Solution:

Total depreciation $2 500 - 300 = $2 200
Annual depreciation $2 200 ÷ 8 = $275
Annual percent rate of depreciation = $\frac{275}{2\ 500} \times 100 = 11\%$

7. to calculate the annual depreciation expense, using the declining-balance method.;

In the previous sections, we assumed an equal amount of depreciation each year. Many businesses argue against this method and state that an asset depreciates at a larger amount in its earlier years.

The "declining-balance" or capital-cost-allowance method recognizes this point and allocates a larger rate/amount of depreciation in the beginning years of useful life than at the end.

Example:

An asset was purchased at a cost of $10 000. The disposal value is estimated to be $1 000 at the end of 9 years. If the accountant decides to use the declining balance method of depreciation, what would be the annual depreciation charge at the end of the first, second, third and fourth years?

Solution:

To solve this problem one must first find the straight-line percent rate of depreciation.

$10 000 - $1 000 = $ 9 000 total depreciation
9 000 ÷ 9 = $ 1 000 annual depreciation
$\frac{1\ 000}{10\ 000} \times 100 = 10\%$ annual rate of depreciation

The declining-balance percent rate in practice is twice the straight-line rate; i.e., 10% x 2 = 20%.

The book value at the end of each year multiplied by 20% equals:

Year	Book Value	Percent Rate of Depreciation	Amount of Depreciation
1	$10 000	20	$2 000
2	$ 8 000	20	$1 600
3	$ 6 400	20	$1 280
4	$ 5 120	20	$1 024

The different methods of calculating depreciation will not usually result in the same final book value.

The book value in year 1 is the cost of the asset. The declining balance method does not account for salvage value.

The book value for each of the years is found by subtracting the depreciation for the year from the book value at the start of that year.

8. to calculate the annual depreciation expense, using the sum-of-the-years digits method;

This method of calculating depreciation is also based on the assumption that a fixed asset depreciates more in the initial years of useful life.

Example:

Wilson Plumbing purchased a new back-hoe for $45 000. The estimated useful life of the machine is 5 years, at the end of which the scrap value is estimated to be $15 000. Calculate the annual depreciation, using the sum-of-the-years digits method.

Solution:

This method requires the calculation of the total depreciation in the same manner as the straight-line method.

Total depreciation = cost - scrap value
= \$45 000 - \$15 000
= \$30 000

The next step is to add the digits of the years involved; i.e., 1 + 2 + 3 + 4 + 5 = 15

Each year's depreciation will be a fraction of this sum-of-the-years digits.

In our example, this fraction will always have the denominator 15. The first year's depreciation will be $\frac{5}{15}$ of the total depreciation; the second year's $\frac{4}{15}$; the third $\frac{3}{15}$; the fourth $\frac{2}{15}$ and the fifth $\frac{1}{15}$.

This fraction is multiplied by the total depreciation.

Solution:

1st year's depreciation	$\frac{5}{15} \times 30\ 000 =$	\$10 000
2nd year's depreciation	$\frac{4}{15} \times 30\ 000 =$	\$ 8 000
3rd year's depreciation	$\frac{3}{15} \times 30\ 000 =$	\$ 6 000
4th year's depreciation	$\frac{2}{15} \times 30\ 000 =$	\$ 4 000
5th year's depreciation	$\frac{1}{15} \times 30\ 000 =$	\$ 2 000
Total depreciation		\$30 000

9. to compare the annual depreciation charge, using the above three different methods.

Example:

A front-end loader costs $250 000. Its estimated useful life is 8 years and the estimated scrap-value is $20 000. Compare the amount of depreciation in the first, second and third years using the straight-line method, the declining-balance method or the sum-of-the-years digits method.

B. DISTRIBUTING OVERHEAD COSTS

Many firms maintain separate financial records for each department in the enterprise. If so, expenses such as heat, light, or rent must be allocated among the various departments.

This module examines a number of different ways this allocation can take place.

The student should be able:

1. to allocate overhead costs according to floor space occupied;

Example:

The factory of Wilson Industries pays rent amounting to $10 000. Find the amount that should be allocated to each of its four departments if Department A occupies 500 m^2, Department B occupies 1 000 m^2, Department C occupies 2 000 m^2 and Department D occupies 1 500 m^2.

Solution:

Total floor space: 500 + 1 000 + 2 000 + 1 500 = 5 000 m^2

Department A $\frac{500}{5\ 000} \times \$10\ 000 = \$\ 1\ 000$

Department B $\frac{1\,000}{5\,000} \times \$10\,000 = \$\ 2\,000$

Department C $\frac{2\,000}{5\,000} \times \$10\,000 = \$\ 4\,000$

Department D $\frac{1\,500}{5\,000} \times \$10\,000 = \underline{\$\ 3\,000}$

Total allocated $= \underline{\underline{\$10\,000}}$

2. to allocate overhead costs according to the number of employees;

Note: Firms would use this method to allocate the cost of staff support services provided to employees.

Example:

Potvin and Son has four departments with the following number of employees in each department: Department A - 25, Department B - 15, Department C - 30 and Department D - 30. Total expenses related to personnel services for the month are $5 000. Calculate each department's share of expenses.

Solution:

Total employees 25 + 15 + 30 + 30 = 100

Department A $\frac{25}{100} \times \$5\,000 = \$1\,250$

Department B $\frac{15}{100} \times \$5\,000 = \$\ \ 750$

Department C $\frac{30}{100} \times \$5\,000 = \$1\,500$

Department D $\frac{30}{100} \times \$5\,000 = \underline{\$1\,500}$

Total Expense $= \underline{\$5\,000}$

3. to allocate overhead expenses based on sales volume.

Many retailers allocate expenses based on sales volume of each department.

Example:

A store has four departments. Total sales for the store were $10 000 during the month of May. Overhead expenses during the month totalled $2 000. Sales volumes for each department during May were as follows:

Department A	$2 500
Department B	$1 500
Department C	$4 000
Department D	$2 000

Calculate the amount of overhead expenses allocated to each department, based on the sales volume.

Solution:

Step 1: Calculate the percent rate that overhead expenses represent of total sales.

$\$2\ 000 \div \$10\ 000 \times 100 = 20\%$

Step 2: Multiply each department's sales by 20% to allocate the amount of overhead expense to be allocated to that department.

Department A $2 500 x 20% = $ 500
Department B $1 500 x 20% = $ 300
Department C $4 000 x 20% = $ 800
Department D $2 000 x 20% = $ 400

Total expenses = $2 000

Op.III. DIGITAL LOGIC (1 MODULE)

The Industrial Revolution produced many new machines and a great variety of automatic equipment. As a result, mechanical and electrical devices are part of our society. Modern equipment of all types features control circuits with sensors, switches and relays, always increasing in sophistication. The development in the twentieth century has resulted in a special level of sophistication in control circuitry; i.e., digital devices. Pin-ball machines, digital clocks, meters, calculators, computers and video games are all examples of applications of digital techniques.

The two main categories of basic components in digital devices are special switches called GATES and storage cells called LATCHES or FLIP-FLOPS. The gates are special switches in that they have more than one input, but only one output. The usual toggle switch, as we know it, has one input and one output. The latches and flip-flops are very important since they store bits of information. They serve as memory cells.

Digital logic is a study of multiple inputs into various devices and the resulting output, which depends on the nature of the device. All signals and pulses are either high or low; one or zero. Therefore, it is a two-valued system. The binary number system, with digits 1 and 0, is the most compatible mathematical form.

Boolean algebra is a unique form of algebra which has been developed to express digital circuits in a more abstract form. There are fourteen basic axioms and these are effective in simplifying logic circuits. The reason for simplifying and reducing the number of devices required is, of course, economic. Boolean algebra and its application will not be included in this module.

The main objective of this vocational module is to introduce four of the six basic gates. Their symbols and functions will be studied and the inputs and outputs will be listed in a truth table. A truth table is a simple chart listing all the arrangements of 0 and 1 on the inputs and showing the corresponding output.

The internal circuit of a simple latch will be studied to demonstrate how several gates may be connected to form a memory cell. Finally, an application of a digital logic circuit will be discussed.

The binary number system should be reviewed to the extent that the student is able to recognize the first 16 binary numbers; i.e., from 0 to 15. This will be useful when attempting to list all the possible arrangements of 0 and 1 on two, three, four or more inputs of a gate. See Table 1.

When there are two inputs to a gate, the truth table will involve $2^2 = 4$ rows, with three inputs there will be $2^3 = 8$ rows and with four inputs there will be $2^4 = 16$ rows. These subsets of inputs are indicated in Table 1.

In order to remember how to fill in the zeros and ones, notice the pattern in each column. Column A is 8 zeros and 8 ones, column B is 4 zeroes and 4 ones and column C is 2 zeros and 2 ones.

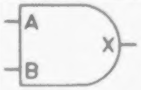
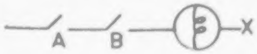
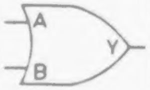
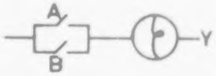
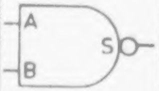
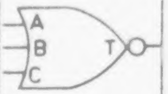
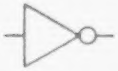
TABLE I

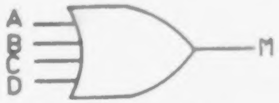
TRUTH TABLE INPUTS

Inputs:

A	B	C	D	Decimal Equivalent
0	0	0	0	0
0	0	0	1	1
0	0	1	0	2
0	0	1	1	3
0	1	0	0	4
0	1	0	1	5
0	1	1	0	6
0	1	1	1	7
1	0	0	0	8
0	0	0	1	9
1	0	1	0	10
1	0	1	1	11
1	1	0	0	12
1	1	0	1	13
1	1	1	0	14
1	1	1	1	15

TABLE II

FOUR LOGIC GATES and the inverter				
Name	Symbol	Function	Controlling Input	Truth Table Inputs and Outputs
AND		 Similar to switches connected in series.	Low or zero. Any low input produces a low output.	A B X 0 0 0 0 1 0 1 0 0 1 1 1
OR		 Similar to switches connected in parallel.	High or one. Any high input produces a high output.	A B Y 0 0 0 1 1 1 1 0 1 1 1 1
NAND (not AND)		The function is the same as for AND but the output is inverted.	Low or zero. Any low input produces a high output.	A B S 0 0 1 0 1 1 1 0 1 1 1 0
NOR (not OR)		The function is the same as for OR but the output is inverted. This gate has three inputs. Any of the above gates may have more than two inputs.	High or one. Any high input produces a low output.	A B C T 0 0 0 1 0 0 1 0 0 1 0 0 0 1 1 0 1 0 0 0 1 0 1 0 1 1 0 0 1 1 1 0
Inverter		The bubble on any input or output indicates an inversion of the logic level.	If the input is high, the output is low and vice versa.	

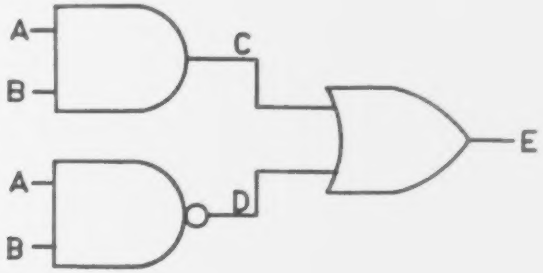
OBJECTIVES	NOTES TO TEACHERS
The student should be able:	
1. to draw the symbols of AND, OR, NAND and NOR gates;	See Table II.
2. to indicate all possible arrangements of 1 and 0 at the input of 2-, 3- and 4-input gates;	See Table I.
3. to explain the function of AND, OR, NAND and NOR gates;	See Table II.
4. to state the controlling input of AND, OR, NAND and NOR gates;	See Table II.
5. to prepare a truth table for a 2-input gate;	See Table II.
6. to indicate the symbol and the function of an inverter;	See Table II.
7. to recognize the decimal numbers 0 to 15 as coded in the binary system;	See Table I.
8. to prepare a truth table for a 4-input gate;	Prepare a truth table for a 4-input OR gate. 

A	B	C	D	M
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	1
0	1	0	0	1
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

The input set is the same as the body of Table I.
The OR gate puts out a high pulse if any input is high.

9. to prepare a truth table showing the inputs and outputs at all stages in a combinational circuit;

Prepare a truth table listing all the inputs and outputs in the following combination of gates:

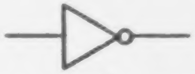


A	B	C	D	E
0	0	0	1	1
0	1	0	1	1
1	0	0	1	1
1	1	1	0	1

10. to explain the function of an inverter;

Inverters are indicated by:

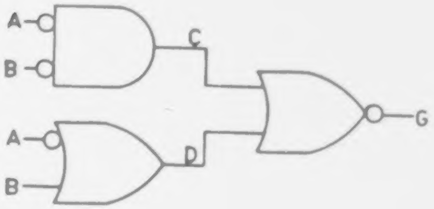
a) a bubble on a gate symbol

b) as  in a line.

The inverter simply complements the signal. That is, the signal is switched or inverted to the other logic level.

11. to prepare a truth table to show the output of a logic circuit which includes gates with inverted inputs;

Prepare a truth table to show the output of the following circuit:



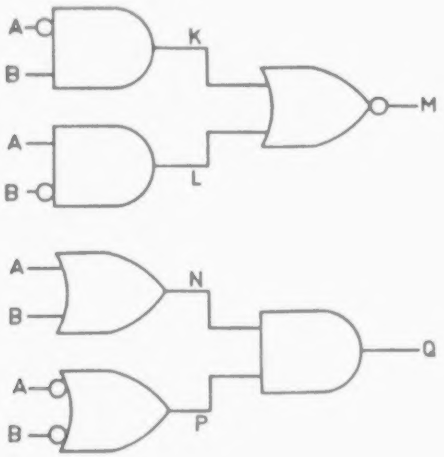
A	B	C	D	G
0	0	1	1	0
0	1	0	1	0
1	0	0	0	1
1	1	0	1	0

OBJECTIVES

NOTES TO TEACHERS

12. to prepare a truth table to test and compare the final output of two different logic circuits;

Prepare a truth table to compare the outputs of these two circuits; i.e., compare M and Q.



				**			**
A	B	K	L	M	N	P	Q
0	0	0	0	1	0	1	0
0	1	1	0	0	1	1	1
1	0	0	1	0	1	1	1
1	1	0	0	1	1	0	0

** M and Q are complements of each other. This difference in output is caused by an inverter. Which inverter should be eliminated?

13. to differentiate between combinational and sequential logic circuits;

Combinational vs. Sequential Logic

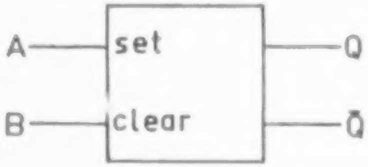
All the circuits we have considered are combinational (or combinatorial). The main implication is that any and every change in input effects an immediate change in the output.

In a sequential circuit, the output is not changed with every change in input.

The device which changes the nature of the circuit is a latch (or flip-flop). The latch output may be changed by a high input or by a low input. It does not change with every change of input. The output depends on a previous input. Therefore, a bit of information has been retained. This introduces the idea of storage or memory. In order to determine the output, it is important to know the sequence of the inputs. The output of a sequential circuit is shown as a wave which features time delays and gaps between pulses. Recall that a truth table was sufficient to show the output for a combinational circuit.

14. to recognize the schematic symbol of a latch and explain its main function;

The Latch (Flip-Flop):



The schematic represents a simple set/reset (or clear) latch with the following definitions:

SET condition: $Q = 1$, $\bar{Q} = 0$

CLEAR condition: $Q = 0$, $\bar{Q} = 1$

The inputs A and B come from spring-loaded switches and the activating pulse is discontinued as soon as the switch is released. When the switch is depressed, the input is one; as soon as it is released, the input is zero. Zero is a non-activating pulse for this latch.

The main idea is that the outputs Q and $\bar{Q}$ do not change when the switch is released. The activating pulse is high.

Consider the following chart, which lists the inputs and outputs for a simple latch. (The input array in this chart will not be a simple subset of Table 1.)

TABLE III

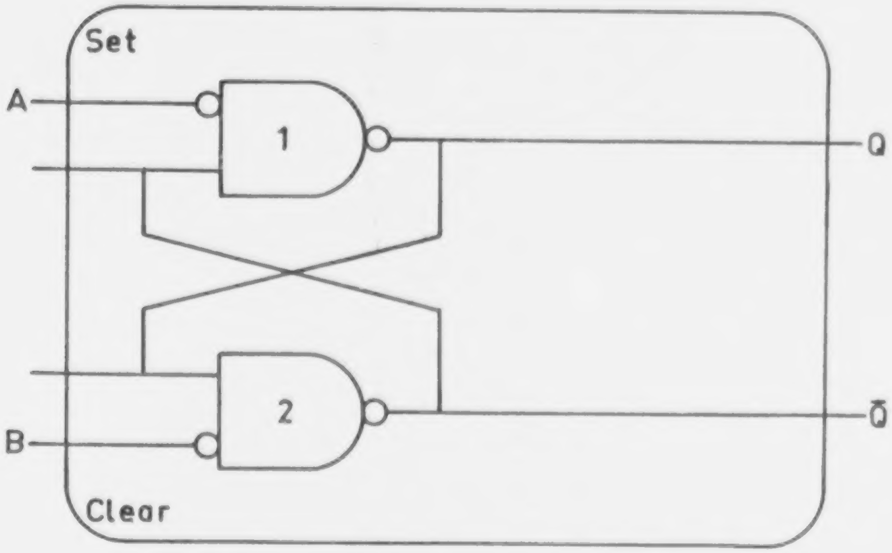
A	B	Q	$\overline{Q}$	Comments
0	0	?	?	Undefined state
1	0	1	0	SET - stable
0	0	1	0	Remains set
0	1	0	1	CLEAR - stable
0	0	0	1	Remains clear
1	1	?	?	Avoid **

Note: When both switches are depressed, the output is ambiguous. The wiring in this simple latch is not sophisticated enough to be protected against this avoid situation. With a few gates this can be overcome so that the avoid state always produces a toggle (inversion) and the output becomes predictable and useful.

15. to sketch and explain the function of the internal gate combination of a simple latch;

Consider the internal logic circuit of a latch. See the following schematic. Consider the inputs listed in Table III, as they would be applied to A and B. A low input on either A or B does not produce a change in Q or $\overline{Q}$.

Latch Circuit - initialize in SET condition
- both A and B at 0 level



SET: $Q = 1$; $\overline{Q} = 0$

CLEAR: $Q = 0$; $\overline{Q} = 1$

A and B are spring-loaded switches. When pressed, the input is high and they activate A or B, and cause a change at the outputs Q and $\overline{Q}$.

When released, the input changes to low but the outputs do not change. The effect of the last high pulse is retained. Hence, the latch retains one binary bit and functions as a storage or memory cell.

OBJECTIVES NOTES TO TEACHERS

Operation of the Latch Circuit

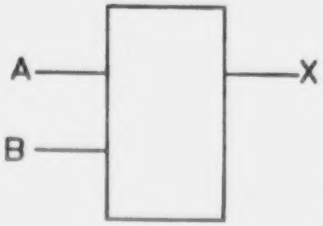
Notice that the latch circuit involves 2 NAND gates, each having one inverted input. Since one cannot know the state of Q and $\overline{Q}$ at the outset, we assume one or the other. Assume the initial condition is SET and A and B are both 0 (switches released).

1. The high on Q is connected to gate 2 and the low on $\overline{Q}$ is connected to gate 1.
2. When A is depressed, input A is high but this is inverted to low before it enters gate 1.
3. This will not affect the output of gate 1 because the other input is already low and it takes only one low signal to shut the gate off.
4. The low output of gate 1 is inverted to high, because it is a NAND gate and Q remains high. Since the latch was SET to begin w[illegible] the high signal on A produces no change.
5. When A is released, the input signal goes low and is inver[illegible] to high when entering gate 1. But the low $\overline{Q}$ is still applied to the gate at the other input so the output cannot change. The above step demonstrates the importance of the controlling signal of any gate. The low $\overline{Q}$ signal at the input of gate 1 effectively inhibits the gate so that the other input has no effect. The latch was SET and remains SET regardless of input changes at A.
6. When B is depressed, however, there is an immediate change in Q and $\overline{Q}$. A high signal at B is inverted to low as it enters gate 2 and this signal inhibits gate 2, just the same as gate 1 was inhibited (shut off) in step 5. As a result of NAND, gate 2 puts out a high signal and $\overline{Q}$ = 1. This signal is cross-connected with gate 1 and Q becomes 0. Now the latch is RESET or clear.
7. Follow the above steps carefully until you see how the logic circuit functions to retain the effect of the last high pulse on A or B. Consider several applications of high pulses on inputs A and B. Follow the circuit action, in each case, through to the effect on Q and $\overline{Q}$.

8. The overall effect of this internal circuit is shown by the following diagram. The output wave X is the result of alternate A and B signals (high). The output wave diagram shows time delays and the importance of sequencing.

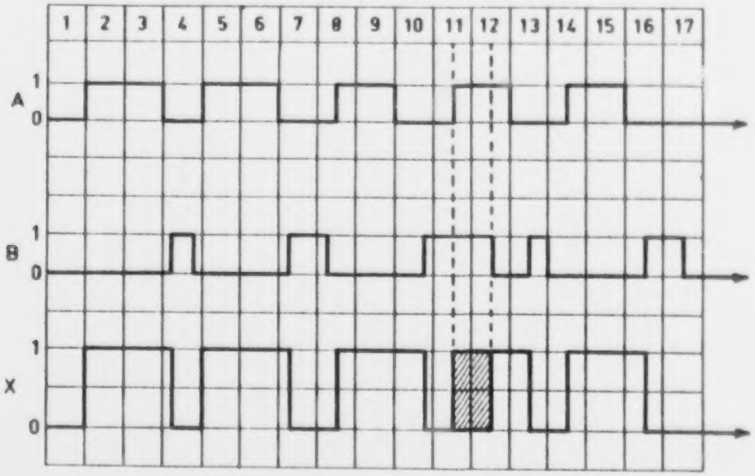
16. to indicate the output of a simple latch, on a wave diagram, given the wave diagrams of the inputs;

Simple SET/RESET Latch with Output X



The initial condition is CLEAR.
Therefore, X is low.
The other output is now shown. It is simply the complement of X.
The X output is referred to as the "normal output."

Notice the avoid state in the output (shaded in) when both inputs are high.



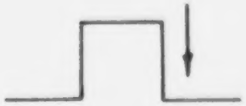
17. to prepare a schematic of a four-bit binary counter.

The Binary Counter

A latch can be designed to respond to a low signal instead of responding to a high. In this case, the output will toggle every time the input goes from high to low. Latches are called "flip-flops" because of this toggling action.

Flip-flops are commonly designed to be controlled by either a regular clock pulse or a counting pulse, which may be intermittant or irregular.

When the clock pulse goes low, or the counter switch is released, the flip-flop changes state. The flip-flop is said to respond to the negative edge of the clock pulse.



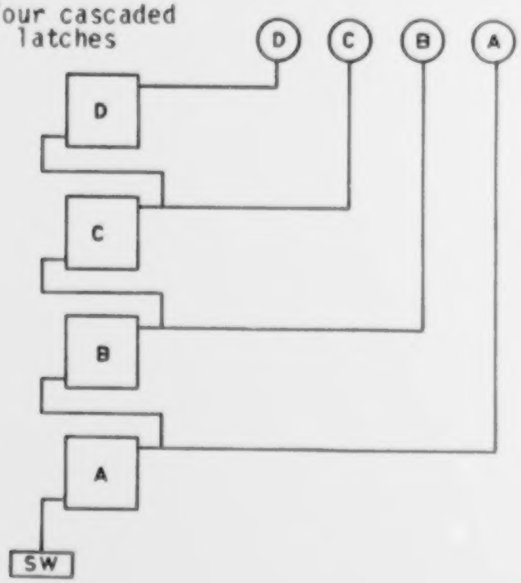
This symbol indicates that a device is negative-edge triggered. Some devices are positive-edge triggered. Simple latches are also level triggered. This means the inputs are enabled for the full duration of a high clock pulse.

This flip-flop is the basic component used to build a binary counter. The following diagram illustrates how four flip-flops are cascaded to produce a four-bit counter which counts from 0 to 15 in binary numbers. Notice that $15_{10} = 1111_2$. At this point all the LED's will be lit. (LED means light emitting diode.)

Upon the next counter pulse, the four LED's will be off, indicating zero. This is because five bits are required to express 16_{10} in the base 2; $16_{10} = 10\ 000_2$. Hence, the device counts from 0 to 15_{10} and starts over again.

Binary Counter Four LED's

Four cascaded latches



	LSB			MSB		MSB			LSB
P	A	B	C	D		D	C	B	A
0	0	0	0	0		0	0	0	0
1	1	0	0	0		0	0	0	1
2	0	1	0	0		0	0	1	0
3	1	1	0	0		0	0	1	1
4	0	0	1	0		0	1	0	0
5	1	0	1	0		0	1	0	1
6	0	1	1	0		0	1	1	0
7	1	1	1	0		0	1	1	1
8	0	0	0	1		1	0	0	0
9	1	0	0	1		1	0	0	1
10	0	1	0	1		1	0	1	0
11	1	1	0	1		1	0	1	1
12	0	0	1	1		1	1	0	0
13	1	0	1	1		1	1	0	1
14	0	1	1	1		1	1	1	0
15	1	1	1	1		1	1	1	1
16	0	0	0	0		0	0	0	0

The right hand side of the chart shows the binary numbers in the normal order; i.e., MSB to LSB from left to right as they would appear in the LED's.

OBJECTIVES	NOTES TO TEACHERS

Notes:

1. The wiring of the input circuit to the latches has not been shown. The usual SET and RESET inputs have been simplified to one activating pulse.
2. The released switch represents a high signal. The depressed switch enters a low signal. The low signal activates all the changes.
3. All the latches start in the clear condition: all the lights are off.
4. All latches change state when the input goes low.
5. After the first pulse, latch A will be high.
6. Upon the second pulse, latch A changes state to low. This output is connected to latch B. As A output goes low, B changes state to high.
7. Trace through the steps and complete the chart.
8. This procedure will demonstrate the process of counting from 0 to 15 in binary numbers. The next pulse reverts the counter back to zero.

Op.IV. INVESTMENTS (STOCKS, BONDS, REAL ESTATE) (1 MODULE)

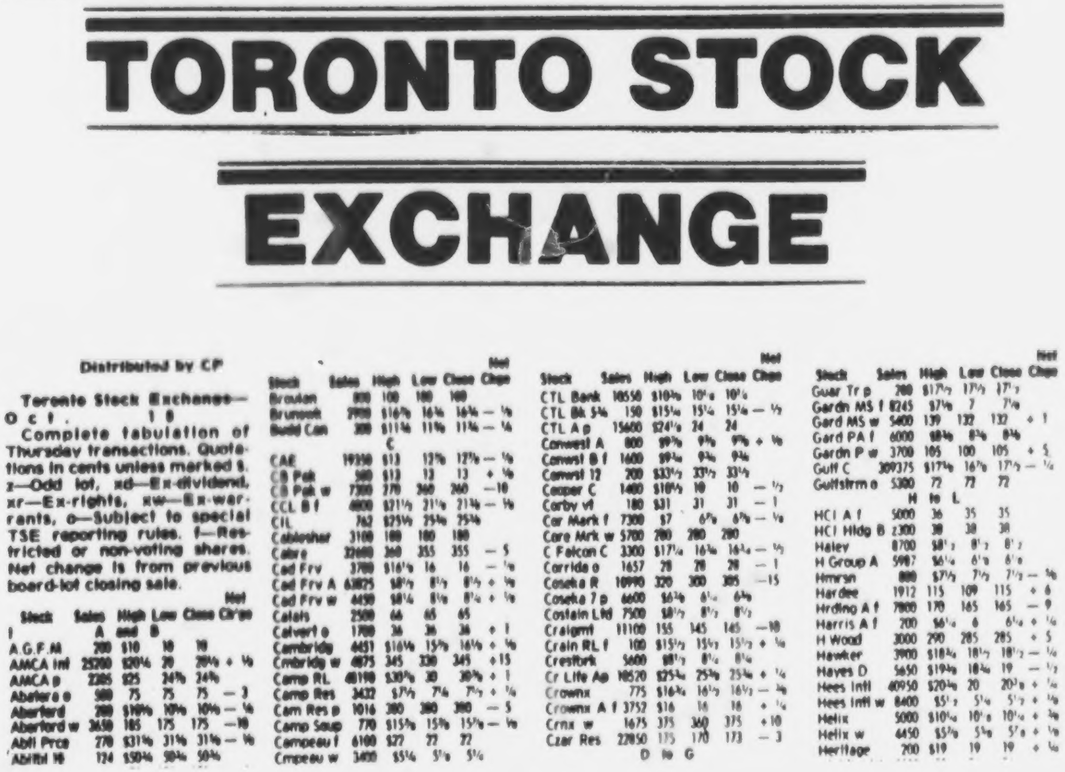
OBJECTIVES	NOTES TO TEACHERS

A. STOCKS

The student should be able:

1. to demonstrate an understanding of stock quotations as listed in a newspaper;

Use a copy of the stock market pages from a newspaper to explain this concept. The following is a copy of a report of the Toronto Stock Exchange from The Winnipeg Free Press.



TORONTO STOCK EXCHANGE

Distributed by CP

Toronto Stock Exchange—Oct. 19

Complete tabulation of Thursday transactions. Quotations in cents unless marked $. z—Odd lot, xd—Ex-dividend, xr—Ex-rights, xw—Ex-warrants, o—Subject to special TSE reporting rules. f—Restricted or non-voting shares. Net change is from previous board-lot closing sale.

Stock	Sales	High	Low	Close	Net Chge
I	A	and	B		
A.G.F.M	200	$10	10	10	
AMCA Intl	25200	$20¼	20	20¼	+ ⅛
AMCA p	2305	$25	24⅞	24⅞	
Abatera o	500	75	75	75	— 3
Aberford	200	$10⅝	10½	10½	— ⅛
Aberford w	3650	185	175	175	—10
Abti Prce	270	$31⅝	31⅜	31⅜	— ⅛
Abitibi	124	$50¾	50¾	50¾	
Brascan	800	100	100	100	
Brunswk	2900	$16⅞	16¾	16¾	— ⅛
Budd Can	300	$11¼	11⅛	11⅛	— ¼
		C			
CAE	19350	$13	12⅞	12⅞	— ⅛
CB Pak	500	$13	13	13	+ ⅛
CB Pak w	7300	270	260	260	—10
CCL B f	4800	$21½	21⅛	21⅛	— ⅛
CIL	762	$25⅛	25⅛	25⅛	
Cableshar	3100	180	180	180	
Cabre	32600	360	355	355	— 5
Cad Fry	3700	$16⅛	16	16	— ⅛
Cad Fry A	63875	$8½	8½	8½	+ ⅛
Cad Fry w	4450	$8¼	8⅛	8¼	+ ⅛
Calais	2500	66	65	65	
Calvert o	1700	36	36	36	+ 1
Cambridg	4451	$16⅛	15⅞	16⅛	+ ⅛
Cmbridg w	4875	345	330	345	+15
Camp RL	48190	$30⅝	30	30⅝	+ 1
Camp Res	3432	$7½	7¼	7½	+ ¼
Cam Res p	1016	380	380	380	— 5
Camp Soup	770	$15⅞	15⅞	15⅞	— ⅛
Campeau f	6100	$27	27	27	
Cmpeau w	3400	$5¼	5⅛	5⅛	

Stock	Sales	High	Low	Close	Net Chge
CTL Bank	10550	$10⅜	10¼	10¼	
CTL Bk 5⅝	150	$15¼	15¼	15¼	— ½
CTL A p	15600	$24⅛	24	24	
Conwest A	800	$9⅞	9⅞	9⅞	+ ⅛
Conwst B f	1600	$9¾	9¾	9¾	
Conwst 12	200	$33½	33½	33½	
Cooper C	1400	$10½	10	10	— ½
Corby vf	180	$31	31	31	— 1
Cor Mark f	7300	$7	6⅞	6⅞	— ⅛
Core Mrk w	5700	280	280	280	
C Falcon C	3300	$17¼	16¾	16¾	— ½
Corrida o	1657	28	28	28	— 1
Coseka R	10990	320	300	305	—15
Coseka 7 p	6600	$6¾	6¼	6¾	
Costain Ltd	7500	$8½	8½	8½	
Craigmt	11100	155	145	145	—10
Crain RL f	100	$15½	15½	15½	+ ¼
Crestbrk	5600	$8½	8¼	8¼	
Cr Life Ap	10520	$25¾	25⅝	25¾	+ ¼
Crownx	775	$16¾	16½	16½	— ⅜
Crownx A f	3752	$16	16	16	+ ¼
Crnx w	1675	375	360	375	+10
Czar Res	22050	175	170	173	— 3
	D	to	G		
Guar Tr p	200	$17½	17½	17½	
Gardn MS f	8245	$7⅛	7	7⅛	
Gard MS w	5400	139	132	132	+ 1
Gard PA f	6000	$8¾	8¾	8¾	
Gardn P w	3700	105	100	105	+ 5
Gulf C	309375	$17¾	16⅞	17½	— ¼
Gulfstrm o	5300	72	72	72	
	H	to	L		
HCI A f	5000	36	35	35	
HCI Hldg B	2300	38	38	38	
Haley	8700	$8½	8½	8½	
H Group A	5987	$6¼	6⅛	6⅛	
Hmrsn	800	$7½	7½	7½	— ⅛
Hardee	1912	115	109	115	+ 6
Hrdlng A f	7800	170	165	165	— 9
Harris A f	200	$6¼	6	6¼	+ ¼
H Wood	3000	290	285	285	+ 5
Hawker	3900	$18¾	18½	18½	— ¼
Hayes D	5650	$19⅜	18¾	19	— ½
Hees Intl	40950	$20⅜	20	20⅜	+ ¼
Hees Intl w	8400	$5½	5¼	5½	+ ⅛
Helix	5000	$10¼	10⅛	10¼	+ ⅜
Helix w	4450	$5⅞	5⅝	5⅞	+ ⅛
Heritage	200	$19	19	19	+ ¼

Notes:

1. The first publication of the Winnipeg Free Press each year contains a statement of the maximum and minimum values of each stock for the previous year.
2. Sometimes stock names will be abbreviated.
3. "Sales" means the number of shares of stock sold during the day.
4. "High" means the highest price paid per share during the day by a buyer.
5. "Low" means the lowest price paid per share during the day by a buyer.
6. "Close" means the price paid per share by the last purchaser of the day.
7. "Net ch'ge" means "net change" and indicates the increase or decrease in closing prices from the previous day.
8. Remember that quotations are in cents unless there is a $ sign before the price given in the High Column.

Sample Questions:

1. What were the sales of CAE for the day?
2. What was the highest price that Budd Can sold for during the day?
3. What was the lowest price that CAE sold for during the day?
4. What was the closing price of Campbell Soup for the day?
5. Campeau f closed at $22 for the day. What was the closing price for the previous day?

2. to calculate the cost of buying stock, using stock quotations;

Example:

What would 200 shares of Haley L cost a purchaser if it was purchased at the closing price?

Solution:

Closing price is $8½ = $8.50
therefore cost = $8.50 x 200 = $1 700.00

3. to calculate the commission that must be paid in stock transactions;

Note: Commission rates and tables will vary. The schedule provided is only one possible example. It may be useful to contact a stockbroker and obtain a copy of that firm's schedule.

Base Rates of Commission for Orders Up to and Including $20 000

a) For shares selling under $14.00, the commission is 2.5% of the value of the order.
b) For shares selling at $14.00, and up to and including $30.00, the commission is 22.575 cents per share plus 0.887 5% of the value of the order.
c) For shares selling over $30.00, the commission is 1.64% of the value of the order.

Minimum Commission

Orders under $10.00	Discretion of the broker
Orders of $10-$50.00	$2.00
Orders over $50.00	$5.00

Examples:

1. Find the commission on the purchase of 2 000 shares of Calais Stock selling at $0.65.

Solution:

Cost of stock: 2 000 x 0.65 = $1 300.00
Commission would be 2.5% of the value of the order because the stock costs under $14.00.
Therefore: commission = $1 300.00 x 2.5% = $32.50

OBJECTIVES	NOTES TO TEACHERS

2. Find the commission on the purchase of 200 shares of GAZ 15 p selling at $26.00.

Solution:

Cost of Stock 200 x $26.00 = $5 200.00
Commission would be 22.575 cents per share plus 0.887 5% of the value of the order because the stock costs over $14.00 but under $30.00.
Therefore, commission based on the shares sold:
200 x $.227 57 = $45.15
commission based on total value:
5 200 x 0.887 5% = $46.15
total commission paid $45.15 + $46.15 = $91.30

4. to calculate the total cost of purchasing stock;

Example:

Find the cost of purchasing 150 shares of Con Gas B stock at a price of $38.00.

Solution:

Purchase price: 150 x $38.00 = $ 5 700.00
Commission: $5 700 x 1.64% = 93.48
Total cost: $5 700 x $93.48 = $ 5 793.48

5. to calculate the net proceeds from selling stock;

Note: Emphasize that when a person sells stock, the commission paid on the transaction will reduce the proceeds received from the sale.

Example:

Find the proceeds from the sale of 75 shares of Camp R-L at a price of $12 3/4.

Solution:

Sale price: 75 x $12.75 = $956.25
Commission: $956.25 x 2.5% = 23.91
Net Proceeds: $956.25 - $23.91 = $932.34

6. to calculate the capital gain or loss on the sale of shares;

Note: A capital gain occurs when the net proceeds of a sale of stock exceed the total purchase price of the stock. A capital loss occurs when the net proceeds of a sale of stock is less than the total purchase price of the stock.

Example:

Henry Turner bought 50 shares of running stock at $29.00, and sold them later for $34 5/8. Find the capital gain or loss.

Solution:

Purchase price:
Cost: 50 x $29 = $1 450
Commission: $1 450 x 0.887 5% = $12.87
50 x 0.257 5 = $11.29
Total cost: $1 450 x $12.87 + $11.29 = $1 474.16

Selling price:
Sale: 50 x $34.625 = $1 731.25
Commission: $1 731.25 x 1.64% = $28.39
Net Proceeds: $1 731.25 - $28.39 = $1 702.86

Capital gain is: $1 702.86 - $1 474.16 = $228.70

OBJECTIVES

NOTES TO TEACHERS

7. to calculate the yearly dividend from stocks if the dividend was paid quarterly, semi-annually or annually;

Example:

What yearly income will a person receive from common stock that pays a dividend of $0.95 per share quarterly if he/she owns 150 shares?

Solution:

Dividend for 1 quarter:	150 x .95 = $142.50
Yearly dividend	$142.50 x 4 = $570.00

8. to calculate the annual rate of return on stock.

Note: The Math 01 program has stressed the following formula involving percent questions:

$A = R \times B$

where
A = amount
R = rate
B = base

When dealing with rate of return questions:
A = amount of net income, profit or yield
R = percent rate of return
B = amount originally invested

Example:

Find the rate of return on a common stock that pays a dividend for each quarter of $.72 if the total cost of the share was $50.

Solution:

Yearly dividend: $.72 x 4 = $2.88

$$A = R \times B$$
$$2.88 = R \times 50$$
$$\frac{2.88}{50} = R$$
$$5.76\% = R$$

Notes:

1. If a student had $10 000.00 invested in certain stocks, trace what happens to the value of these stocks over an extended period of time. Have a student report to your class re the value every second Friday for 3 months. Also, indicate the total gain or loss in each report.
2. Be aware that financial mathematics textbooks seem to disagree on the rate of return. Some suggest the rate of annual dividend should be based on the current market price of the share, rather than the amount originally invested. In this course, the amount originally invested is used as the base.

B. BONDS

The student should be able:

1. to calculate the market price of a bond;

Note: The market price of a bond is quoted as a certain percent rate of the par value. For example, a price quotation of "95" means 95% of the par value of the bond while a quotation of "120" means 120% of the par value of the bond.

The "par value" is the amount that appears on the bond. It is the amount that will be repaid to the bondholder at the time the bond matures.

Example:

What would you pay for ten bonds if you purchased the bonds at 92? The bonds had a par value of $1 000.

Solution:

Market price of 1 bond: $ 1 000 x 92% = $920
Purchase price of 10 bonds: $ 920 x 10 = $9 200

OBJECTIVES

NOTES TO TEACHERS

2. to calculate the cost of a bond including commission;

Example:

J. R. Smith purchases six \$1 000 Bonds at 96 3/4 plus a commission of 1/2% on the price of the bond. What would be the total amount of the investment?

Solution:

Cost of 1 bond: \$1 000 x 96.75% = \$967.50
Cost of 6 bonds: \$967.50 x 6 = \$5 805.00
Commission: \$5 805 x .5% = \$29.03
Total Investment: \$5 805 + \$29.03 = \$5 834.03

Note: Brokers charge a commission of about 1/2% for buying and selling bonds. This may vary.

3. to calculate bond income;

Bond income is always calculated on the par value and never the market value.

Example:

What will be the annual income from twelve bonds having a par value of \$1 000 each and paying 16% interest?

Solution:

Interest on 1 bond: \$1 000 x 16% = \$160
Total income: \$160 x 12 = \$1 920

4. to calculate the rate of return on bonds.

Example:

A \$1 000 bond, bearing interest at 17%, was purchased at 98 plus a commission of 1/2% on the price of a bond. What is the rate of income on the investment?

Solution:

Cost of bond: \$1 000 x 98 = \$980
Commission: \$980 x .5% = \$4.90
Total investment: \$980 + \$4.90 = \$984.90
Interest or income: \$1 000 x 17% = \$170
Rate of return: $A = R \times B$

$$170 = R \times 984.90$$

$$\frac{170}{984.90} = R$$

$$0.17266 = R$$

Therefore, rate of return is equal to 17.3%

Note: It is not intended in this course to examine bonds sold between interest dates where interest has been accrued. If time permits, this topic might be an enrichment topic.

C. REAL ESTATE

The student should be able:

1. to calculate the annual net income from an investment in real estate;

When discussing income, emphasize the difference between gross rentals (income) and net income (profit).

When talking about investment, emphasize that investment refers to the amount of cash invested in the property, not the value of the property. A mortgage carried is not part of the investment but rather one of the expenses of the investment.

Example:

McGregor bought a home for \$35 000, paying \$15 000 in cash and giving a 15% mortgage for the remainder. During the first year, \$400 a month was received for rent. Expenses for the year were:

Interest at 15% on the amount of the mortgage
2% depreciation on the house
Taxes, repairs, insurance, and other expenses $850

What was the net income for the year?

Solution:

Gross income:	$400 x 12 = $4 800
Expenses:	
a) Interest:	$20 000 x 15% = $3 000
b) Depreciation:	$435 000 x 2% = $ 700
c) Taxes, etc.:	$ 850
Total Expenses:	$4 550
Net Income:	$4 800 - $4 550 = $250

2. to calculate the rate of return on real estate.

Example:

Using the figures from example 1 calculate the rate of net income/return earned from investment.

Solution:

Remember that the investment is the cash investment, not the total purchase price.

$A = R \times B$
$250 = R \times 15\ 000$

$$\frac{250}{15\ 000} = R$$

$0.0166 = R$
Therefore, rate of return is 1.7% on the investment.

OP. V. INTERPRETATION OF GRAPHS (½ MODULE)

OBJECTIVES	NOTES TO TEACHERS

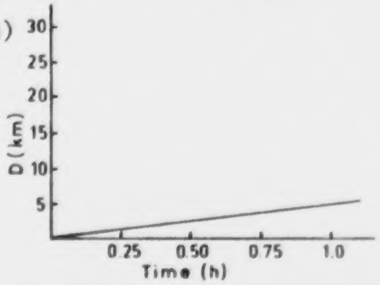
1. The student should be able to discuss and to analyze a given graph.

Several examples are included here to indicate the type of discussion and analysis that is expected of the student.

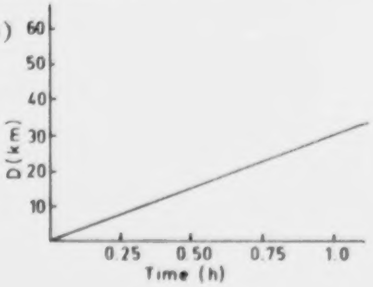
Example 1:

SPEED OF CARS

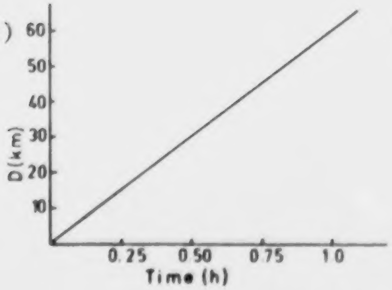
(a)



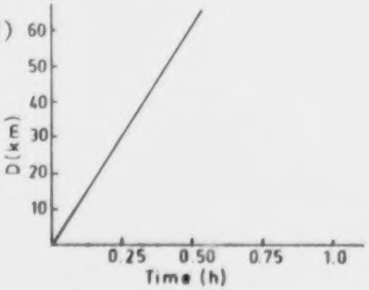
(b)



(c)



(d)

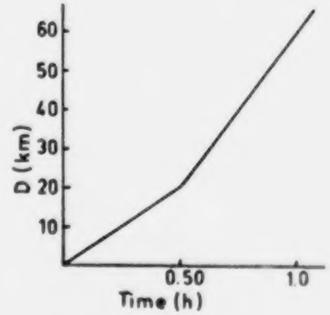


1. What is the speed of the car in (a)?
2. What is the speed of the car in (b)?
3. What is the speed of the car in (c)?
4. What is the speed of the car in (d)?

The concept of slope could be discussed here.

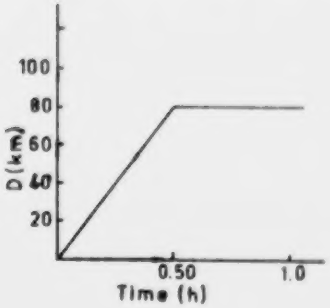
Answers:

1. 5 km/h 2. 30 km/h 3. 60 km/h 4. 160 km/h



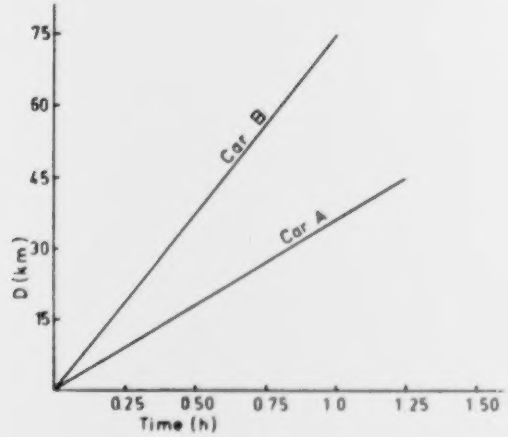
1. What was the average speed of the car during the first half hour?
2. What was the average speed of the car during the second half hour?
3. What was the average speed during the one-hour period?

Answers:
1. 40 km/h
2. 80 km/h
3. 60 km/h



1. What was the average speed of the car during the first half hour?
2. What was the average speed of the car during the second half hour?
3. What was the average speed of the car for the one-hour period?

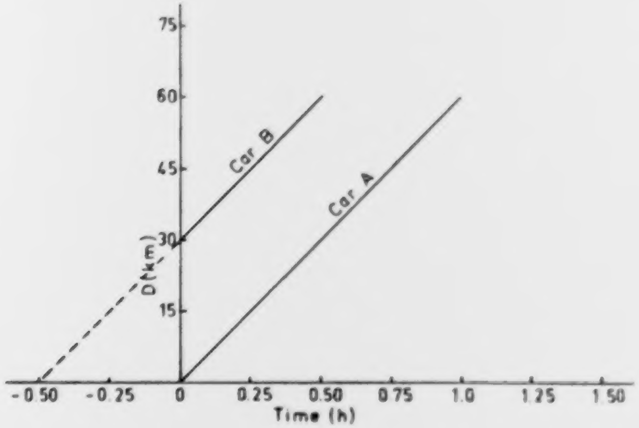
Answers:
1. 160 km/h
2. 0 km/h
3. 80 km/h



1. Which car is going faster?
2. How much faster is it going?

Answers:

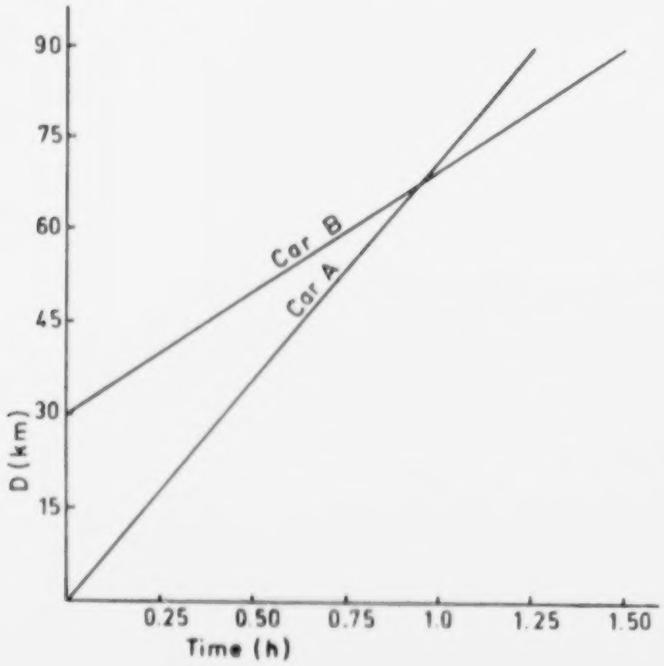
1. Car B
2. .39 km/h



1. How fast is car A going?
2. How fast is car B going?
3. What do parallel lines indicate about speed?
4. What is the time-intercept for car B? Explain.

Answers:

1. 60 km/h
2. 60 km/h
3. Cars are travelling at the same speed.
4. -0.50. Either the Car B began travelling one-half hour before Car A, or it began 30 km ahead of Car A.

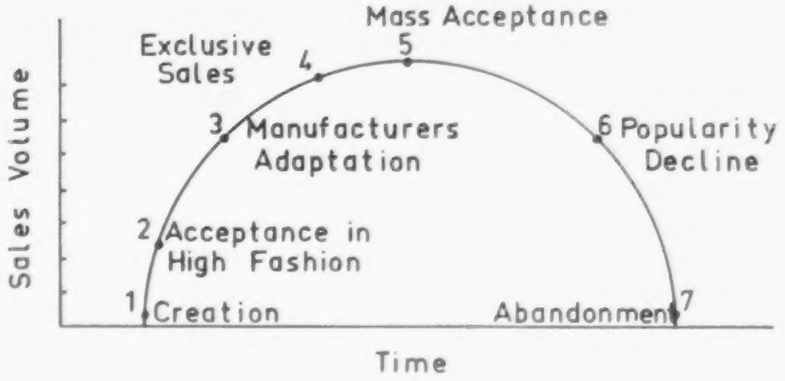


1. Which car is going faster?
2. What is the physical significance of the point of intersection of the two graphs?

Answers:

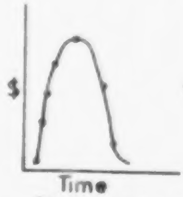
1. Car A
2. Car A overtakes Car B 68 km after Car A begins to travel.

Example 2: FASHION CYCLES

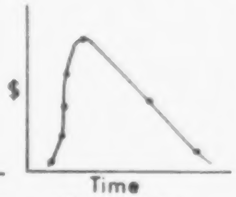


When a new product is created, it is shown to the manufacturers in the fashion houses of Paris/London/New York/ etc. Exclusive shops carry new styles for a period of time until the general public adopts the fashion. Mass production reduces the cost and the market becomes saturated, causing decline and eventual abandonment.

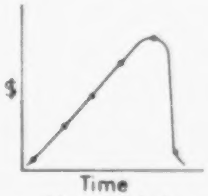
Comment on the following variations, explaining why the curves are shaped as they are:



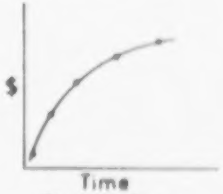
Disco Dance



Mini Skirts

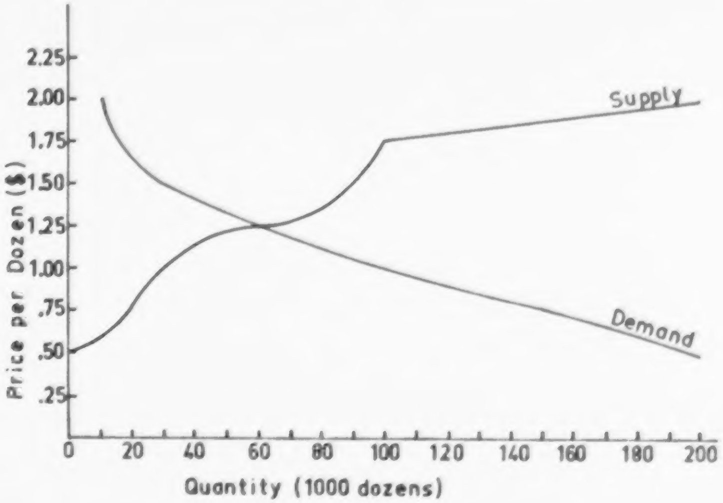


Maxi Skirts



Classic Cars

Example 3: MARKET (EQUILIBRIUM) PRICE



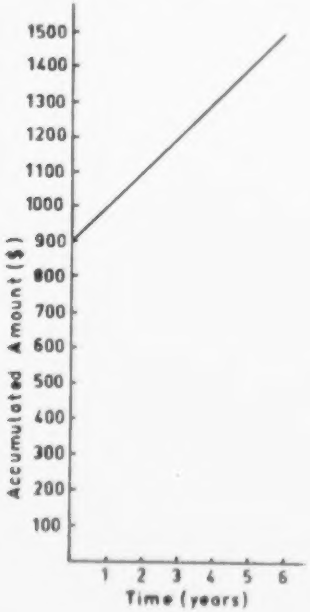
Schedule

Produce	Price Per Dozen	Quantity Demanded by Consumer (000)	Quantity Supplied by Farmer (000)
Eggs	$ 2.00	10	200
	1.75	15	100
	1.50	30	90
	1.25	60	60
	1.00	100	30
	.75	150	20
	.50	200	nil

In a free market economy, the price of a product is largely determined at the point where the quantity demanded equals the quantity supplied.

1. If the price were $2.00 per dozen, farmers would be willing to supply more (or more farmers would become suppliers), and yet the customers would not want many more eggs.
2. Why would farmers produce none at a price of 50¢ per dozen? (They would go broke.)

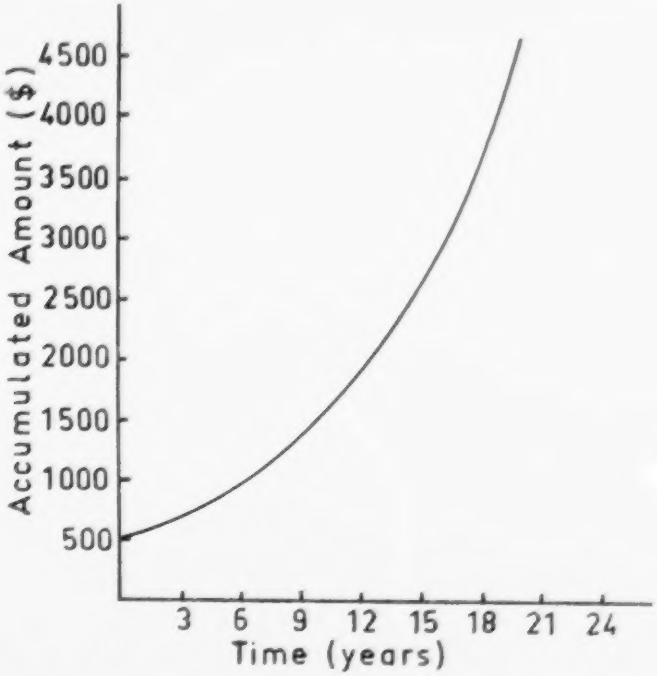
Example 4: GROWTH OF MONEY OWED



The Generous Finance Company lends Sheila Bright $900 for her tuition at 11% per annum (simple interest). What is the total amount that Miss Bright owes after x years?

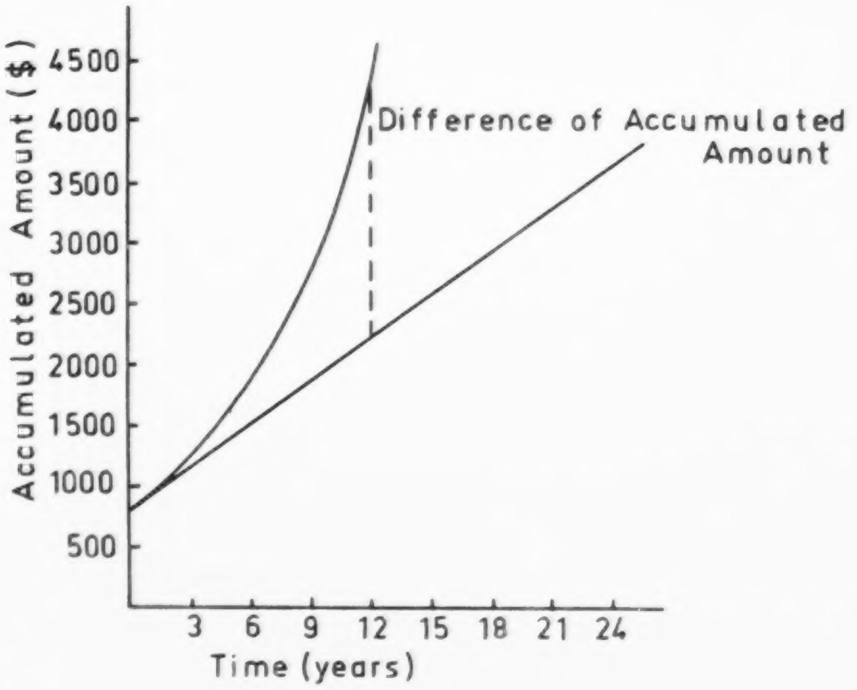
OBJECTIVES	NOTES TO TEACHERS

GROWTH OF MONEY SAVED



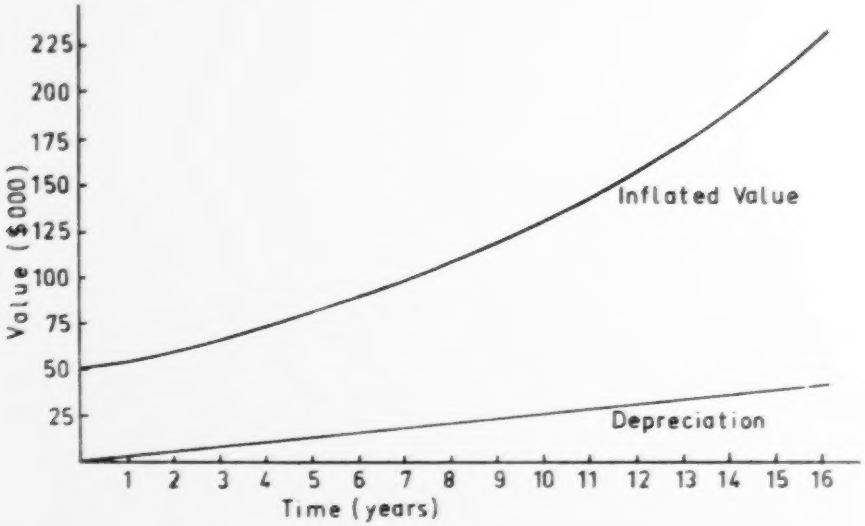
$500 is invested in a savings certificate on which interest at the rate of 12% per annum is compounded. What amount has accumulated after x years?

COMPARISON OF ACCUMULATED AMOUNTS BY SIMPLE INTEREST AND COMPOUND INTEREST



$800 is available to invest. How much more will accumulate in x years if it is invested at 15% compounded annually than if it is invested at 15% simple interest?

Example 5: APPRECIATION VALUE OF A HOUSE



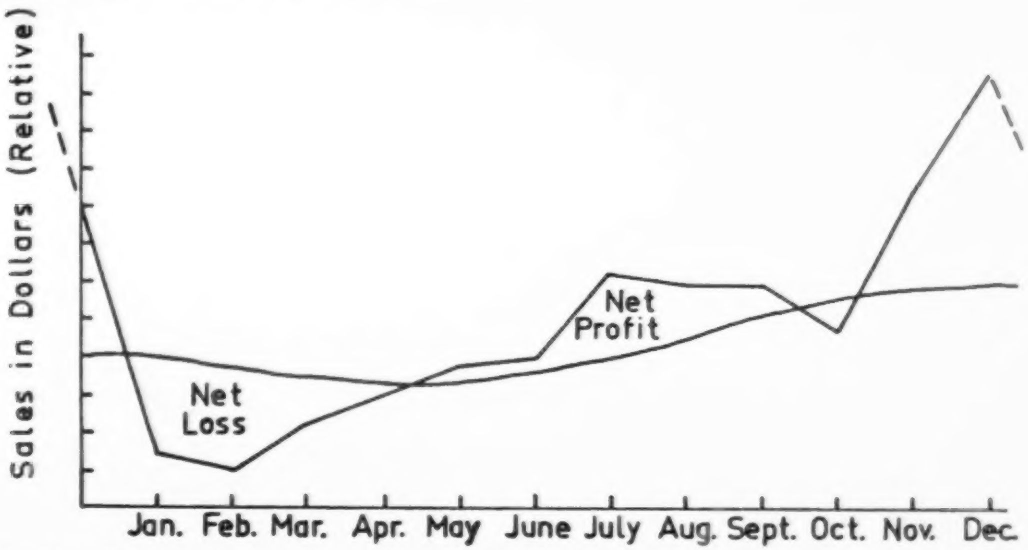
What will be the value of a $50 000 house in x years, given an inflation rate of 10% and a depreciation rate of 5%?

Usually, depreciation is expressed as a percent of the original value of the house, so is written as $50 000 x .05 x t.

Inflation, however, is expressed as a percent of the present value. Therefore, it has a compounding effect.

The difference between the two values should be the value of the house.

Example 6: YEARLY SALES CYCLE

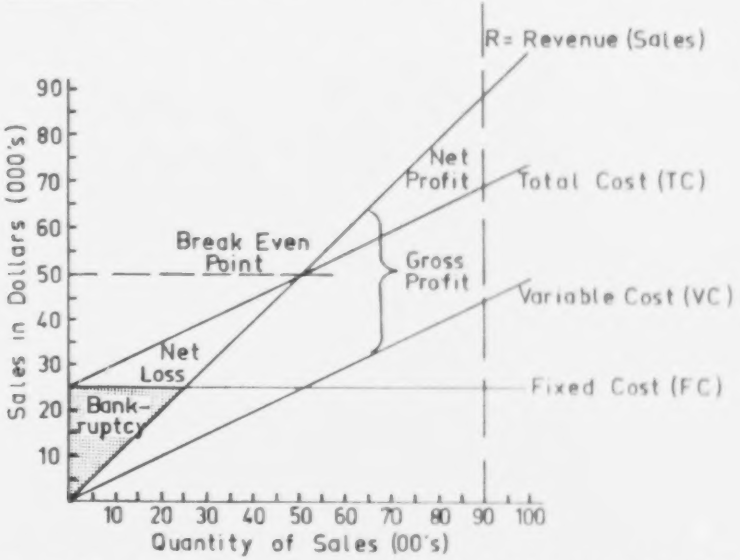


A department store may experience "peaks" and "troughs" in its revenue (sales). Christmas season and summer holidays are the two strongest periods. January to April can be the weakest period.

1. If net profits are greater than net losses over the year, the business stays in operation.
2. During periods of net loss, what does the business do for finances? (trade credit, line of credit, reduce staff, promote sales events, etc.)
3. Which of the two curves (Revenue or Sales, and Total Cost) does the business have the most control to regulate? (Total costs, especially the operating expenses or fixed costs.)

Enter numbers on the graph and have students answer specific questions about net profit in May, etc.

Example 7: BREAK-EVEN ANALYSIS



A small store in a shopping mall sells neckties for $10 each. The ties cost the merchant $5.00 each. Yearly operating expenses (wages, rent, utilities, insurance, etc.) are $25 000.

VC + FC = TC, R - TC = NP (or NL), R - VC = GP, GP - FC = NP

1. If the store sold 100 ties, the sales (R) would not pay for the expenses, therefore bankruptcy would result.
2. At $50 000 sales, the store's sales just cover all cost-of-goods sold (VC) and expenses (FC). Therefore, the store just breaks even.
3. What is the net profit if the store sells 9 000 ties in a year?
   What is the gross profit?
   What is the fixed cost?

   Answers: $20 000, $45 000, $25 000

Op.VI QUADRATIC EQUATIONS (½ MODULE)

OBJECTIVES	NOTES TO TEACHERS
The student should be able:	
1. to recognize quadratic equations;	Quadratic equations are second degree equations in one variable. The standard form of the quadratic equation is: $ax^2 + bx + c = 0$ where $a \neq 0$.
2. to solve a quadratic equation of the type $ax^2+c = 0$;	Examples:

1. $4x^2 - 64 = 0$

Solution:
$$\begin{aligned} 4x^2 &= 64 \\ x^2 &= 16 \\ x &= \pm 4 \end{aligned}$$

2. $25x^2 = 4$

Solution:
$$\begin{aligned} 25x^2 &= 4 \\ x^2 &= \frac{4}{25} \\ x &= \pm\frac{2}{5} \end{aligned}$$

3. $3x^2 - 15 = 0$

Solution:
$$\begin{aligned} 3x^2 &= 15 \\ x^2 &= 5 \\ x &= \pm\sqrt{5} \\ &\doteq \pm 2.24 \end{aligned}$$

OBJECTIVES	NOTES TO TEACHERS
3. to express quadratic equations in standard form: $ax^2+bx+c=0$;	Example: Express $-7 + 3x^2 = 5x$ in standard form. Solution: $3x^2 - 5x - 7 = 0$
4. to identify the values of a, b, c, in quadratic equations;	Examples: 1. $2x^2 = 4$ Solution: $2x^2 - 4 = 0$ $a = 2,\ b = 0,\ c = -4$ 2. $5x - 6x^2 = 7$ Solution: $-6x^2 + 5x - 7 = 0$ $a = -6,\ b = 5,\ c = -7$ OR $6x^2 - 5x + 7 = 0$ $a = 6,\ b = -5,\ c = 7$
5. to solve quadratic equations of the type $ax^2 + bx + c = 0$, using the quadratic formula; $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$;	Note: Students should not be expected to derive the quadratic formula. Examples: 1. $x^2 - 4x - 5 = 0$

Solution:
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-4) \pm \sqrt{(-4)^2 - (4)(1)(-5)}}{2(1)}$$
$$= \frac{4 \pm \sqrt{36}}{2}$$
$$= \frac{4 \pm 6}{2}$$
$$= 5, -1$$

2. $2x^2 + 4x - 5 = 0$

Solution:
$$x = \frac{-4 \pm \sqrt{(4)^2 - (4)(2)(-5)}}{2(2)}$$
$$= \frac{4 \pm \sqrt{56}}{4}$$
$$\doteq 0.87 \text{ or } -3.87$$

3. $x^2 - 10x = 0$

Solution:
$$x = \frac{10 \pm \sqrt{100 - (4)(1)(0)}}{2(1)}$$
$$= \frac{10 \pm \sqrt{100}}{2}$$
$$= \frac{10 \pm 10}{2}$$
$$= 0, 10$$

4. $x^2 - 8 = 0$

Solution:

$$x = \frac{-(0) \pm \sqrt{0-(4)(1)(-8)}}{2(1)}$$

$$= \frac{\pm\sqrt{32}}{2}$$

$$\doteq \pm 2.8$$

5. $x^2 - 4x + 4 = 0$

$$x = \frac{4 \pm \sqrt{0}}{2}$$

$$= 2$$

Note: Sometimes a quadratic equation has only one real number solution.

6. $x^2 + 2x + 5 = 0$

Solution:

$$x = \frac{-2 \pm \sqrt{(2)^2-(4)(1)(5)}}{2(1)}$$

$$= \frac{-2 \pm \sqrt{-16}}{2}$$

Since $\sqrt{-16}$ is not a real number, there is no real number solution.

Note: In some situations, there are no real number solutions.

6. to solve problems involving quadratic equations.

Examples:

1. The product of two consecutive numbers is 306. Find the numbers.

Solution:

Two consecutive numbers are x and $x + 1$.

$(x)(x+1) = 306$

$x^2+x-306 = 0$

$$x = \frac{-1\pm\sqrt{1^2-(4)(1)(-306)}}{2}$$

$x = -18$ or 17

If $x = -18$, then the consecutive numbers are -18 and -17.

If $x = 17$, then the numbers are 17 and 18.

2. The length of a rectangle is 4 m more than its width. If the area of the rectangle is 77 m^2, determine the length and width.

width = x
length = $x+4$

Area = length x width

$= (x+4)(x) = x^2+4x$

$A = 77 = x^2 + 4x$

$x^2 + 4x - 77 = 0$

$a = 1, b = 4, c = -77$

$$x = \frac{-4 \pm \sqrt{16 - (4)(1)(-77)}}{2(1)}$$

$$= \frac{-4 \pm \sqrt{324}}{2}$$

$$= \frac{-4 \pm 18}{2}$$

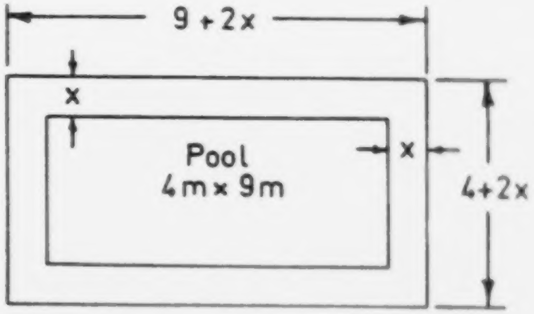
$= -11$ or 7 Disregard -11 because lengths are not negative.

∴ Width is 7 m.

Length is $7+4 = 11$ m.

3. A swimming pool 4m x 9m is to have a concrete walk of uniform width constructed around it. If you could afford to cover an area of 30m^2, how wide is the walk around the pool?

Solution: Let x = width of the walk.



OBJECTIVES NOTES TO TEACHERS

$L \times W = A$

$(9+2x)(4+2x) = 36+30$

$36+26x+4x^2 = 66$

$2x^2+13x-15 = 0$

$$x = \frac{-13 \pm \sqrt{(13)^2-(4)(2)(-15)}}{2(2)}$$

$$= \frac{-13 \pm \sqrt{289}}{4}$$

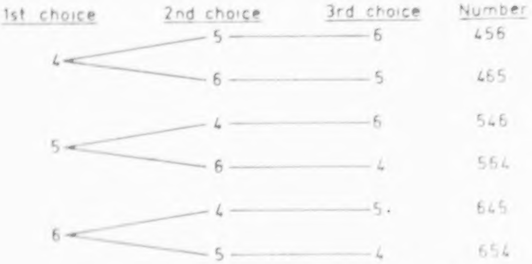
$$= \frac{-13 \pm 17}{4}$$

$= 1$ or -7.5 (Disregard -7.5)

∴ Width of walk is 1 m.

Op.VII. PERMUTATIONS AND COMBINATIONS (½ MODULE)

How many different passenger vehicle license plates can the province manufacture using three letters and three numerals? How many different radio stations in Canada could be licensed if the call letters of the stations must consist of four letters and the first letter must be a C? In how many ways can a person get a bridge hand of thirteen face cards from a deck of fifty-two cards? These and similar questions can be answered through a study of permutations and combinations, a sophisticated way of counting.

OBJECTIVES	NOTES TO TEACHERS
A. PERMUTATIONS	
The student should be able:	
1. to define a permutation;	A "permutation" is an arrangement of "r" objects from a group of n objects in which order counts and where $r \leq n$. Example: TAR is a different permutation than RAT of the three letters. The letters used are the same but their order is different. Therefore, the arrangements constitute two different permutations.
2. to construct a tree diagram to illustrate possible permutations;	Example: Construct a tree diagram to show the permutations of the numerals 4, 5 and 6. Solution: 

OBJECTIVES	NOTES TO TEACHERS
3. to define the Fundamental Principle;	"The Fundamental Principle": If one task can be done in just m different ways, and if, after it has been done, a second task can be done in just n different ways, the the two tasks can be done in just m•n different ways.
4. to apply the Fundamental Principle;	Examples: 1. How many different three-digit numbers can be formed from the digits 4, 5, 6, 7 and 8 if no digit can be repeated in any number? Solution: $\underline{5} \times \underline{4} \times \underline{3} = 60$ possible numbers. 2. How many even three-digit numbers can be formed from the digits 4, 5, 6, 7 and 8 if repetitions are not allowed? Note the restriction in the units place. Solution: Since the number must be an even number, there are only three possibilities for the units digit, either 4, 6 or 8. Having chosen one of these, there are four possibilities for the hundreds digit. For the tens digit there are three remaining digits. Hence $\underline{4} \times \underline{3} \times \underline{3} = 36$ possible numbers. 3. How many three-digit numerals can be formed from the digits 4, 5, 6, 7 and 8 if repetitions are permitted? Solution: $\underline{5} \times \underline{5} \times \underline{5} = 125$

4. How many different licence plates can be manufactured if each licence plate must consist of three letters followed by three digits?

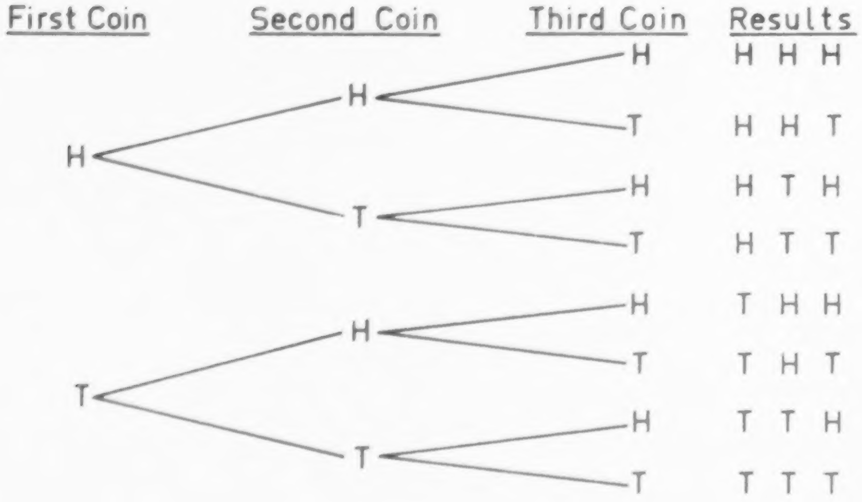
Solution: $\underline{26} \times \underline{26} \times \underline{26} \times \underline{10} \times \underline{10} \times \underline{10}$

5. In how many ways can three tossed coins land?

Since each coin can land heads up or tails up, there are only two possible ways each coin can be tossed.

Solution: $\underline{2} \times \underline{2} \times \underline{2} = 8$

or


First Coin
Second Coin
Third Coin
Results
H
T
H
T
H
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H H H
H H T
H T H
H T T
T H H
T H T
T T H
T T T

OBJECTIVES	NOTES TO TEACHERS
5. to simplify factorial notation;	The product of 4 x 3 x 2 x 1 can be expressed as 4! (four factorial). 0! by definition is 1. __Example:__ Evaluate 6! __Solution:__ 6 x 5 x 4 x 3 x 2 x 1 = 720. __Example:__ Simplify $\frac{6!}{(6-3)!} = 6 \times 5 \times 4 = \underline{120}$
6. to solve permutations using the formula $_nP_r = \frac{n!}{(n-r)!}$;	$_nP_r = \frac{n!}{(n-r)!}$ where r represents the number of things to choose out of n objects. __Examples:__ 1. How many different three-digit numbers can be formed from the numerals 1, 2, 3, 4, 5 and 6 if repetitions are not permitted? __Solution:__ $_6P_3 = \frac{6!}{(6-3)!} = 6 \times 5 \times 4 = \underline{120}$ 2. In how many ways can a class elect a president and secretary from a class of 50 students? __Solution:__ $_{50}P_2 = \frac{50}{(50-2)!} = 50 \times 49 = \underline{2450}.$

7. to solve permutations of "n" objects, some of which are the same."

Since some of the objects are the same, the total number of permutations must be divided by the number of permutations of the objects which are the same.

Given the letters, S,T,A,R,E,T,S, if all letters were different, then the possible permutations of the seven letters would be 7!.

However, in every one of the 7! permutations, the two S's could be interchanged. Therefore, the 7! permutations must be divided by 2!. Similarly, the two T's could be interchanged so the 7! permutations must be divided by another 2!.

The number of permutations of the letters STARETS is $\frac{7!}{2!2!} = 1260$.

In general, the number of permutations of n objects of which p things are alike, and q things are alike is $\frac{n!}{p!q!}$.

Examples:

1. In how many ways can the letters of the word TRESSES be arranged?

   Solution: $\frac{7!}{2!3!} = \underline{420}$ possible arrangements.

2. In how many ways can three red balls, four green balls and two white balls be arranged?

   Solution: $\frac{9!}{3!4!2!} = \underline{1260}$

This course does not consider circular permutations or key chain permutations or permutations in which two people insist on sitting together.

B. COMBINATIONS

The student should be able:

1. to define the term "combination";

A "combination" is a selection of r objects from a group of n objects in which order does not count; e.g., xy is the same combination of letters as yx although they are two different permutations.

Example:

In selecting class representatives order does not count. Electing Tom and then Sue is the same combination as electing Sue and then Tom. However, in elections for class president and class secretary, the order does count and would, therefore, be a permutation.

2. to solve combinations using the formula:

$C(n,r)$ or ${}_nC_r = \frac{n!}{(n-r)!r!}$.

Examples:

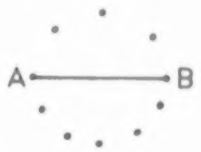
1. From a group of twelve people, in how many ways can a committee of 6 people be formed?

Solution: ${}_{12}C_6 = \frac{12!}{(12-6)!6!} = \frac{12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6!}{6 \times 5 \times 4 \times 3 \times 2 \times 1 \times 6!} = \underline{924}$

2. How many segments can be drawn joining any two of ten points, no three of which are in the same straight line?

Solution:

Since segment AB is the same as segment BA, the problem is a combination:



$$\begin{aligned} C &= \frac{10!}{(10-2)!2!} \\ &= \frac{10 \times 9 \times 8!}{8!2!} \\ &= \frac{10 \times 9}{2} \\ &= 45 \end{aligned}$$

Students should not be expected to take up the study of "at least" or "majority" problems in combinations.

Op.VIII. STATISTICS (STANDARD DEVIATION AND THE NORMAL CURVE) (½ MODULE)

Teachers who select this option topic should be aware that the C.IV Statistics must have been taught previously.

If students are required to collect data for analyses, the information should be available when this option is begun.

OBJECTIVES

The student should be able:

1. to calculate for a set of ungrouped data:

   a) the variance,
   b) the standard deviation;

NOTES TO TEACHERS

In statistics, the most common measures of dispersion or variability are range, mean deviation, variance and standard deviation. (The first two measures were discussed in the C.IV Statistics.)

The "variance" of a set of measures is the average of the squares of the differences of these measures from the mean.

(Variance = $\frac{\sum(x - \overline{x})^2}{N}$, where

x = a measure
$\overline{x}$ = the mean for the set
N = the number of measures

Example:

The number of hours per day that Jane practiced playing the piano last week was 3, 2, 5, 4, 6, 6, 2. Find the variance.

Number of Hours (x)	Difference from Mean $(x - \overline{x})$	Square of the Difference $(x - \overline{x})^2$
3	-1	1
2	-2	4
5	1	1
4	0	0
6	2	4
6	2	4
2	-2	4
		18

$$\text{Mean} = \overline{x} = \frac{3 + 2 + 5 + 4 + 6 + 6 + 2}{7} = \frac{28}{7} = 4$$

$$\text{Variance} = \frac{18}{7} = 2.57$$

The "standard deviation" is the square root of the variance.

Example:

An appliance store received a shipment of four refrigerators. The mass of each (in km) is: 230, 190, 200, 220. Determine the standard deviation for these measures.

OBJECTIVES | NOTES TO TEACHERS

Mass x	Deviation $(x - \bar{x})$	Square of Deviation $(x - \bar{x})^2$
230	+20	400
190	-20	400
200	-10	100
220	+10	100
		1000

$$\text{Variance} = \frac{1000}{4} = 250$$

$$\text{Standard deviation} = \sqrt{250} = 15.81$$

2. to calculate for a set of grouped data:

   a) the variance,
   b) the standard deviation;

Example:

The heights of 100 children have been tallied with the resulting frequency distribution table shown below. Calculate the variance and standard deviation from this table.

Class (cm)	Class Mark (x)	Frequency (f)	Deviation $x - \bar{x}$	$(x-\bar{x})^2 f$
90-92	91	5	-6.45	208.012 5
93-95	94	18	-3.45	214.245 0
96-98	97	42	-0.45	8.505 0
99-101	100	27	2.55	175.567 5
102-104	103	8	5.55	246.420 0
		100		852.750 0

Note: $\bar{x}$ = mean = 97.45

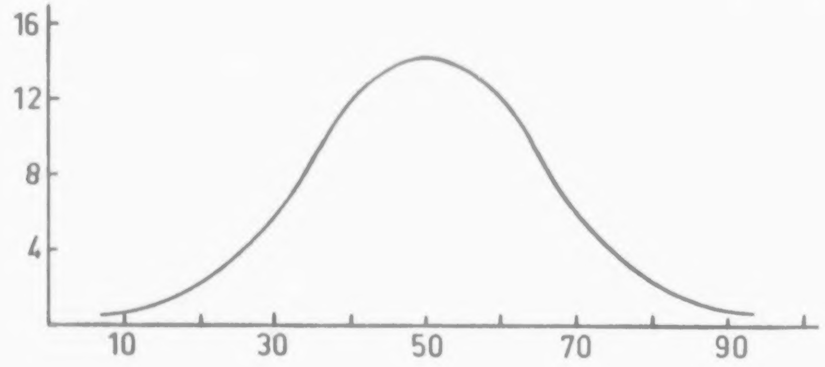
Variance = $\frac{852.750}{100}$ = 8.527 5

Standard deviation = $\sqrt{\text{variance}}$ = $\sqrt{8.5275}$ = 2.920

Standard deviation is usually denoted by σ when dealing with a population and 'S' when dealing with a sample.

3. to examine some characteristics of the curve of normal distribution.

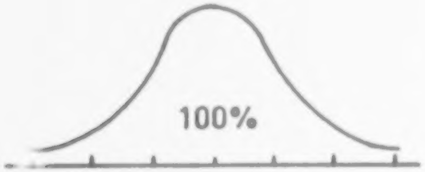
To introduce the normal distribution curve, ask students to sketch the graph of a frequency distribution with class boundaries denoted on the horizontal axis and frequency on the vertical axis. If the graph resembles a bell-shaped curve, the distribution is said to be "normal".



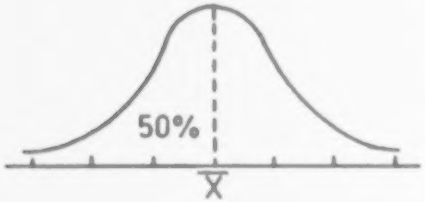
Note: The frequency in any class interval can be represented by the area under the curve between the upper and lower limits of that interval.

The following diagrams are useful in showing the relationship between a normal distribution and standard deviation (σ).

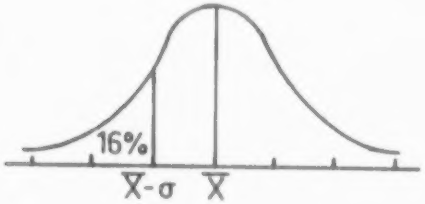
1. The total area under the normal curve represents the total number of items to be considered.



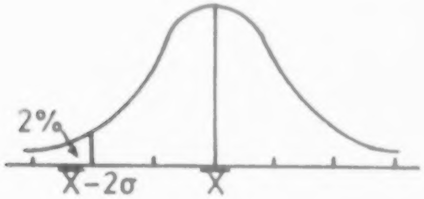
2. The mean ($\overline{x}$) corresponds to the 50th percentile; i.e., 50% of the items tested have test values less than the mean x.



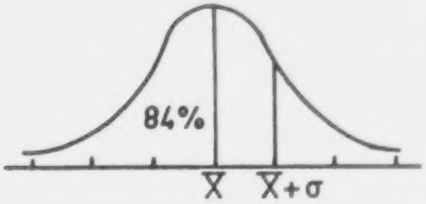
3. $\bar{x} - \sigma$ corresponds to the 16th percentile, 16% of the items tested have test values less than $\bar{x} - \sigma$. Conversely, 84% of the tested items have test values greater than $\bar{x} - \sigma$.



4. $\bar{x} - 2\sigma$ corresponds to the 2nd percentile, where 2% of the items tested have test values less than $\bar{x} - 2\sigma$. Conversely, 98% of the tested items have test values greater than $\bar{x} - 2\sigma$.

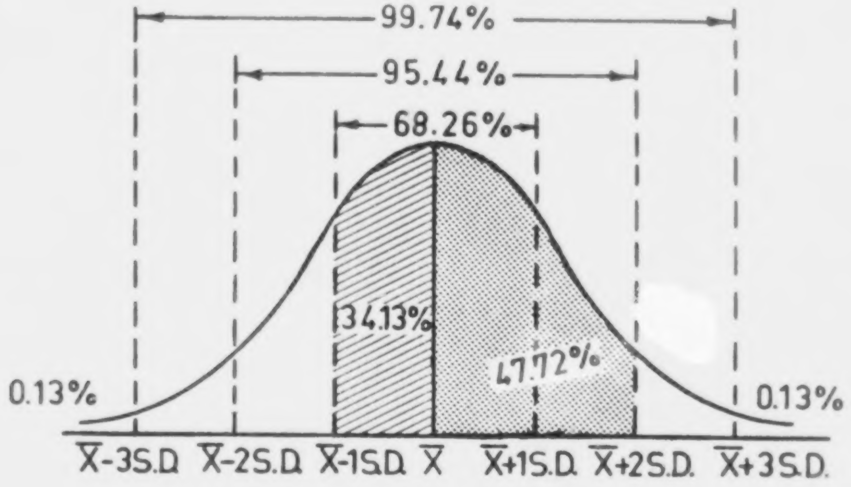


5. $\overline{x} + \sigma$ corresponds to the 84th percentile, where 84% of the items tested have test values less than $\overline{x} + \sigma$. Conversely, 16% of the tested items have test values greater than $\overline{x} + \sigma$.



The following table summarizes the relationship for various test values in terms of $\overline{x}$ and σ:

Test Value	Percentage of Items with Test Values less than $\overline{x} - k\sigma$
$\overline{x} - 3\sigma$	.13%
$\overline{x} - 2\sigma$	2
$\overline{x} - 1.5\sigma$	7
$\overline{x} - 1\sigma$	16
$\overline{x} - 0.5\sigma$	30
$\overline{x}$	50
$\overline{x} + 0.5\sigma$	70
$\overline{x} + 1\sigma$	84
$\overline{x} + 1.5\sigma$	93
$\overline{x} + 2\sigma$	98
$\overline{x} + 3\sigma$	99.87%


99.74%
95.44%
68.26%
34.13%
47.72%
0.13%
0.13%
$\overline{X}$-3S.D.
$\overline{X}$-2S.D.
$\overline{X}$-1S.D.
$\overline{X}$
$\overline{X}$+1S.D.
$\overline{X}$+2S.D.
$\overline{X}$+3S.D.

$\bar{x}$ = mean; S.D. = Standard Deviation

In the above sketch, 68.26% of the measures lie within one standard deviation of the mean. This implies that if a set of marks has a mean of 80 and a standard deviation of 9, then 68.26% of the marks are between 71 and 89.

Examples:

1. The Sparky Electric Company advertises bulbs to burn at least 1000 hours. A sample of bulbs has a mean life of 1050 hours and a standard deviation (σ) of 25 hours.

   a) What percent of bulbs will last at least 1000 hours?
   b) What percent of bulbs will last more than 1075 hours?

   Solution:

   a) $\overline{x} = 1050 \qquad \sigma = 25$
   1000 hours is $1050 - 50 = x - 2\sigma$
   but $\overline{x} - 2\sigma$ (see table) represents the 2nd percentile
   ∴ 2% of the bulbs will last less than 1000 hours
   ∴ 98% of the bulbs will last at least 1000 hours

   b) 1075 hours is $1050 + 25 = \overline{x} + 1\sigma$, which represents the 84th percentile
   ∴ 84% will last less than 1075 hours
   ∴ 16% will last more than 1075 hours

2. A student on an aptitude test scored 690.
   The mean of the test is 600 with a standard deviation of 60.
   What percent of the students scored lower than 690 on the test?

   Solution:

   $\overline{x} = 600, \qquad \sigma = 60$
   690 is $600 + 90 = \overline{x} + 1.5\sigma$
   $\overline{x} + 1.5\sigma$ corresponds to the 93rd percentile.
   93% of the students scored lower than this student.
   7% of the students scored higher than this student.

mathematics 300

THE 00 MATHEMATICS PROGRAM

Mathematics 100, 200, 300 are designed for students wishing to pursue further studies in mathematics or science-related programs at post-secondary institutions, which makes these the most "vocational" of all mathematics courses in Manitoba. So that the 00 programs will be more meaningful, greater emphasis should be placed on problem solving and practical applications in order to enhance the thinking skills of students.

Mathematics 100, 200 each contain 50% algebra, 40% geometry and 10% consumer mathematics. Mathematics 300 contains 80% core and 20% option topics selected from algebra, analytic geometry and trigonometry.

GENERAL GOALS OF THE 00 PROGRAM

In identifying the goals of the 00 mathematics curriculum, upon which this guide is based, the present needs of individuals and society have been considered. The student goals that follow are designed to develop:

1. a proficiency in problem solving;

2. an understanding and a mastery of basic algebra skills;

3. the ability to observe and to identify geometric relationships involving two-dimensional figures;

4. the practice of deductive and inductive reasoning;

5. the ability to interpret graphs and to display data graphically;

6. the ability to read mathematics with understanding;

7. the realization that mathematics is important for all consumers;

8. a positive attitude towards, and a curiosity about, mathematics;

9. the capability for post-secondary training in mathematics.

STRUCTURE OF THE 100 COURSE

Each module accounts for 14 periods (40 minutes each) of new work in a non-semestered system. This should allow approximately four periods per module for review, testing, diagnosis or enrichment. Ten modules comprise a full credit. Core modules are compulsory, and Mathematics 100 has the equivalent of eight and one-half core modules. The remaining equivalent of one and one-half option modules are to be selected from geometry topics (one-half module) and consumer mathematics (one module).

Note: Core topics are to be completed to the level specified, bearing in mind the suggested time allotments. Time must be scheduled in order that concepts from each of the option topic areas receive adequate coverage.

The following chart lists the topics identified for the Mathematics 100 course and indicates the allotted time in terms of modules.

CORE TOPICS

Algebra (5 modules)

C.I.	Equations	(1 module)
C.II.	Polynomial Operations	(1 module)
C.III.	Factorization and Rational Expressions	(1½ module)
C.IV.	Radicals	(1 module)
C.V.	Simultaneous Equations	(½ module)

Geometry (3½ modules)

C.VI.	Coordinate Geometry	(½ module)
C.VII.	Synthetic Geometry	(3 modules)
	Descriptive, Inductive, and Measurement Geometry	(minimum ½ module)
	Deductive Geometry	(minimum 1½ modules)

OPTION TOPICS

Geometry (Chooose one of the following)

Op. I.	Vectors	(½ module)
Op. II.	Orienteering	(½ module)

Consumer Mathematics (Choose one of the following)

Op. III.	Money Management	(1 module)
Op. IV.	Consumer Statistics	(1 module)

MATHEMATICS 200 OUTLINE

CORE TOPICS

Algebra (5 modules)

C.I.	Coordinate Geometry and Systems of Equations	(1 module)
C.II.	Linear Inequalities	(½ module)
C.III.	Graphs of Equations and Functions	(1 module)
C.IV.	Quadratic Functions	(1 module)
C.V.	Quadratics	(1½ modules)

Geometry (3½ modules)

C.VI.	Parallels	(½ module)
C.VII.	Quadrilaterals	(1½ modules)
C.VIII.	Circles	(½ module)
C.IX.	Similarity	(½ module)
C.X.	Trigonometry	(½ module)

OPTION TOPICS

Geometry Option (½ module)

Do one of the following:

Op.I.	Indirect Proof	(½ module)
Op.II.	Mathematical Reasoning	(½ module)
Op.III.	Vectors	(½ module)

Consumer Mathematics (1 module)

Do one of the following:

Op.IV.	Consumer Geometry	(1 module)
Op.V.	Investments and Annuities	(1 module)

MATHEMATICS 300 PROGRAM

CORE TOPICS (80%)

Do each of the following:

			Page References
C.I.	Polynomial and Rational Functions	(2 modules)	409
C.II.	Circular Functions and Trigonometry	(3 modules)	433
C.III.	Analytic Geometry	(1 module)	451
C.IV.	Exponents and Logarithms	(1 module)	459
C.V.	Sequences and Series	(1 module)	475

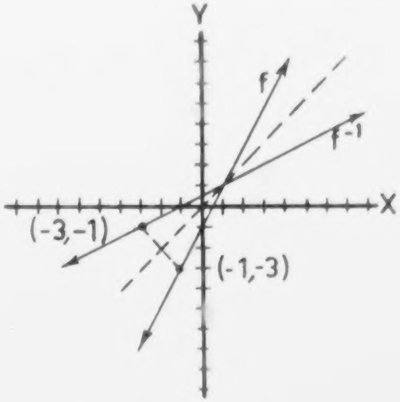
OPTION TOPICS (20%)

Select the equivalent of 2 modules from the following:

Op.I.	Binomial Theorem	(1 module)	481
Op.II.	Compound Interest, Annuities, Amortization and Mortgages	(½ module)	485
Op.III.	Graphs of Sinusoidal Functions	(½ module)	493
Op.IV.	Circular Functions of Two Angles	(½ module)	499
Op.V.	Permutations and Combinations	(1 module)	505
Op.VI.	Conic Sections in Standard Position	(1 module)	509
Op.VII.	Transformations of Conics from Standard Position	(½ module)	517

C.I. POLYNOMIAL AND RATIONAL FUNCTIONS AND THEIR GRAPHS (2 MODULES)

Note: Objectives 1, 2, and 6 are review of work from Math 200.

OBJECTIVES	NOTES TO TEACHERS
The student should be able:	
1. to solve problems using function terminology learned in Math 200;	Terms/symbols include: relation, function, domain, range, $f(x)$, $f:x \rightarrow f(x)$, 1-to-1. Students should be able to determine whether or not a relation is a function.
2. to define the expression "graph of a relation";	The "graph of a relation" is the set of points plotted from the ordered pairs of the relation. By convention, the letter "x" is used for the first member of an ordered pair belonging to a given relation and "y" is used for the second member.
3. to define the term "inverse" for a relation;	If we interchange the first and second members of each ordered pair in a relation, we obtain a relation called the inverse of the original relation. If the original relation is f, its inverse is denoted by f^{-1}. If we interchange x and y in the equation of a relation, we get an equation of the inverse relation.
4. to find the inverse of a given relation;	Example: If the relation, f, is given by the equation $y = 2x - 1$, find an equation which gives the inverse relation, f^{-1}. We interchange the variables in $y = 2x - 1$ to obtain $x = 2y - 1$. The relation f^{-1} is thus given by the equation $x = 2y - 1$, or (as we see by solving for y) the relation f^{-1} is given by the equation $y = \frac{x+1}{2}$. In this case, the relations f and f^{-1} are both functions, so, using the usual functional notation, we may write $f(x) = 2x - 1$ and $f^{-1}(x) = \frac{x+1}{2}$. 

OBJECTIVES

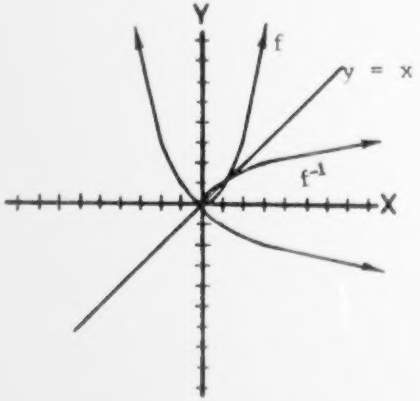
5. to graph a relation and its inverse using one set of axes;

NOTES TO TEACHERS

Example:

Suppose the relation, f, is given by the equation $y = x^2$.

a) Find an equation which gives the inverse relation f^{-1}.
b) Graph the relations f and f^{-1}.
c) Is the relation, f, a function?
d) Is the relation, f^{-1}, a function?



Solutions:

a) The function, f^{-1}, is given by the equation $x = y^2$.
b) Draw the graph shown above.
c) Yes.
d) No.

Students should observe that the graphs of f and f^{-1} are reflections of each other in the line $y = x$. They should also be able to tell by looking at the graphs whether or not f and f^{-1} are functions. (Recall vertical line list from Mathematics 200.)

Note: Since f^{-1} is NOT a function, we do NOT use the notation $f^{-1}(x)$ in this case.

6. to describe how various transformations affect the graph of a function;

Note: This objective is included here for the purpose of reviewing material covered in Math 200.

Given function "f" and its graph:

1) The graph of function g where $g(x) = af(x)$, with "a" a constant, is found by stretching (if $|a| > 1$) or shrinking (if $|a| < 1$) the graph of f vertically. Every point (x,y) on the graph of f yields a point (x,ay) on the graph of g.

   If $a < 0$, the graph of f must also be flipped upside down about the x-axis.

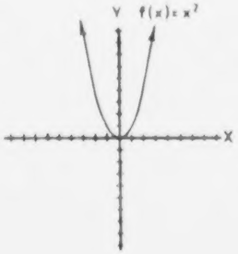
2) The graph of function p where $p(x) = f(x-h)$, with "h" a constant, is found by moving the graph of f right (if $h > 0$) or left (if $h < 0$) h units.

3) The graph of function a where $a(x) = f(x) + k$, with "k" a constant, is found by moving the graph of f up (if $k > 0$) or down (if $k < 0$) k units.

Example 1:

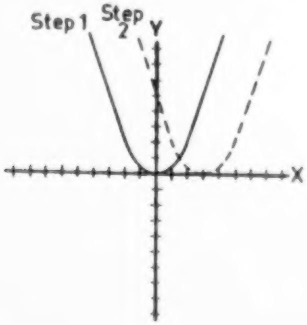
Given the graph determined by the equation $f(x) = x^2$, graph $g(x) = \frac{1}{2}(x-3)^2 + 1$.

Note: Several simpler examples can be done first as required.

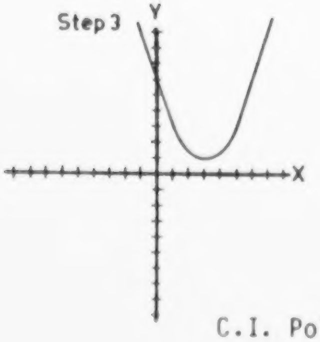


Step 1. When $y = \frac{1}{2}x^2$, $a = \frac{1}{2}$, so the curve will rise at half the first rate.

Step 2. When $y = \frac{1}{2}x^2-3$, $b = 3$, so the new curve will be shifted 3 units to the right.



Step 3. When $c = 1$, the new curve will be shifted 1 unit up.

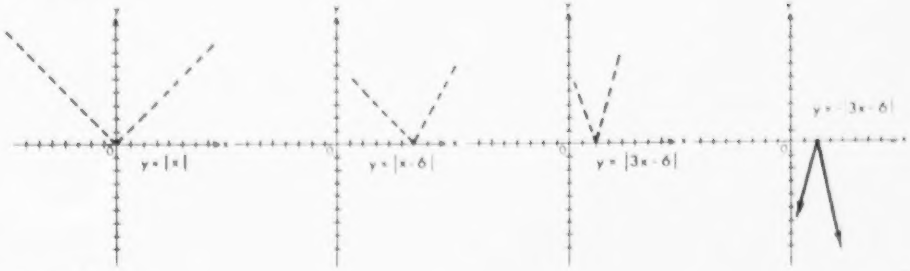


$g(x) = \frac{1}{2}(x-3)^2 + 1$

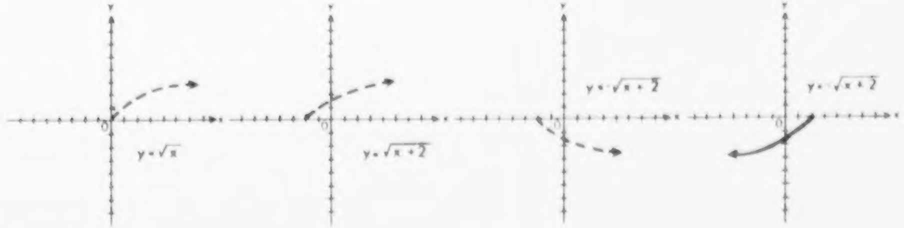
Example 2:

Graph the functions:

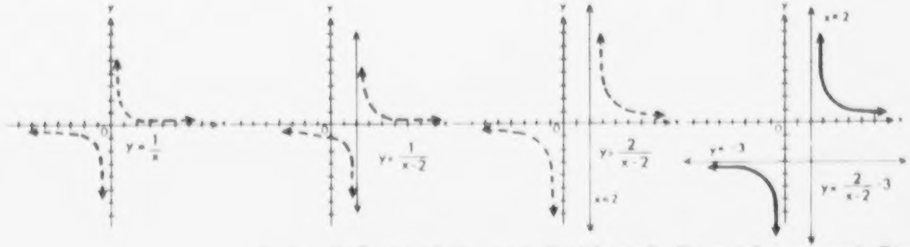
a) $y = -|3x - 6|$



b) $g(x) = -\sqrt{-x+2}$



c) $h(x) = \frac{2}{x-2} - 3$

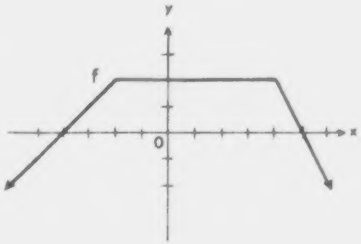


OBJECTIVES | NOTES TO TEACHERS

Example 3:

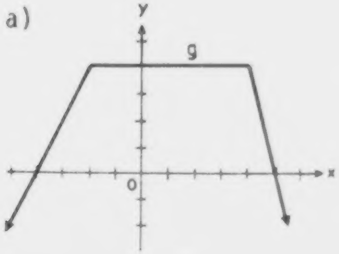
Given function, f, draw the graph of:

a) g where $g(x) = 2f(x)$
b) $h: x \to f(x)+3$
c) $n: x \to f(-x)+3$
d) p where $p(x) = f(2x)$

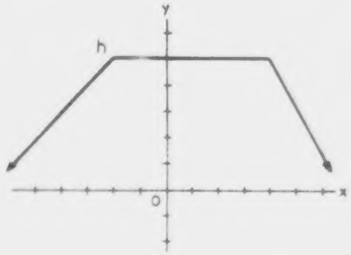


Solution:

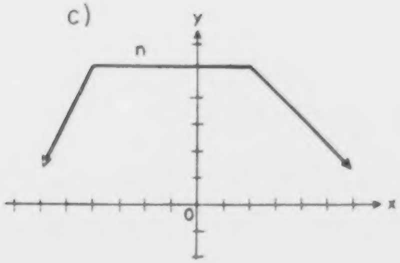
a)



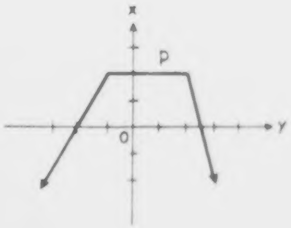
b)



c)



d)



OBJECTIVES	NOTES TO TEACHERS
7. to identify linear and quadratic functions and to sketch their graphs;	The equations for linear functions, as well as the features of their graphs (slope, intercepts), should be well known. Graphs of quadratic functions and the formulae and methods for finding zeroes and vertices were studied in Math 200. Example: Three equations are given below: 1. $2x - 3y = 5$ 2. $x^2 + 2x + 1 = y$ 3. $3x^2 + 2xy + 1 = y$ a) Identify the equation representing y as a linear function of x and sketch its graph. b) Identify the equation representing y as a quadratic function of x and sketch its graph. Note: The third equation represents a skewed hyperbola and is given as a distractor.
8. to determine algebraically the range of a quadratic function;	Example: Function f is represented by the equation $f(x) = x^2 + 3x + 2$. Find the range of f. Solution: Complete the square so that the function will be in the form studied in Objective 6.

$$
\begin{aligned}
f(x) &= x^2 + 3x + 2 \\
&= x^2 + 3x \qquad\quad + 2 \\
&= x^2 + 3x + \frac{9}{4} - \frac{9}{4} + 2 \\
&= (x^2 + 3x + \frac{9}{4}) - \frac{9}{4} + 2 \\
&= (x + \frac{3}{2})^2 - \frac{1}{4}
\end{aligned}
$$

From the work done on transformation, students will know that the graph of the function is a parabola opening upward from vertex $V(\frac{-3}{2}, \frac{-1}{4})$.

Thus, the range of f is $\{y | y \geq \frac{-1}{4}\}$.

ENRICHMENT:

The general quadratic function $f(x) = ax^2 + bx + c$ can be solved by completing the square so that students can use the formula for the vertex.

$$V\left(\frac{-b}{2a}, \; f(\frac{-b}{2a})\right).$$

The range can then be found as above.

9. to find a polynomial function f which has a given set of numbers as its zeroes;

Example:

Find a polynomial function of degree 3 whose zeroes are 1, 2 and -3.

Solution:

Let $f(x) = (x-1)(x-2)(x+3) = x^3 - 7x + 6$.
Then, f is a polynomial function of degree 3 whose zeroes are 1, 2 and -3.

Notes:

1. If $g(x) = a(x-1)(x-2)(x+3)$ where a is any non-zero number, then g is also a function whose zeroes are 1, 2 and -3. (If $a = 1$ then $g = f$.)
2. Students should grasp intuitively that if "f" has degree "n," then there are at most "n" distinct zeroes.
3. Do not spend time working examples where the degree of "f" is greater than 4. Problems with excessive calculation will tend to obscure the purpose of the exercise.

10. to find f(a) for a polynomial function f by substitution;

Review as necessary.

Example:

If $f(x) = x^3 - 7x + 6$, find f(2).

$$f(2) = 2^3 - 7(2) + 6$$
$$= 0$$

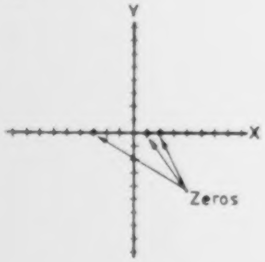
11. to sketch the graph of a polynomial function given in factored form;

Examples should be restricted to polynomials of degree 3.

Examples:

1. Sketch the graph $f(x) = (x-1)(x-2)(x+3)$

Step 1: Find the zeroes.
In this example they are 1, 2, and -3.



Step 2: Substitute to find values for intermediate choices of x.

$$f(0) = (0-1)(0-2)(0+3)$$
$$= 6.$$

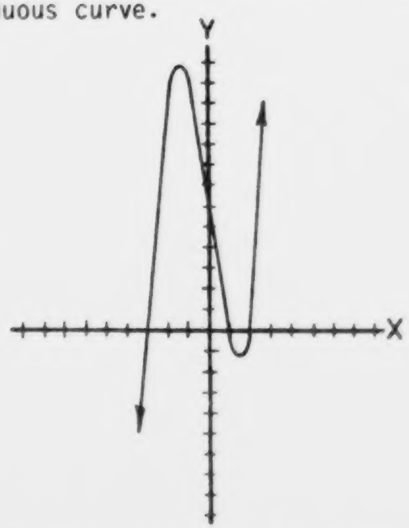
Similarly, $f(1.5) = -1.125$, $f(3) = 12$, $f(-4) = -30$.

Note: These intermediate values may be found more easily on a calculator by using the simplified form of the function: $f(x) = x^3-7x+6$.

Then, $f(0) = 0^3-7(0)+6$

$= 6$, and so on.

Step 3: Plot the points determined (in step 2) on a graph and connect them with a continuous curve.



Note: Undue time should not be spent on detailed graphing. The intention is to have the students sketch some curves using the zeroes and sufficient other points to illustrate the general shape. The sketch should show where the function is positive and where it is negative. (Exact maximum/minimum values are not required.)

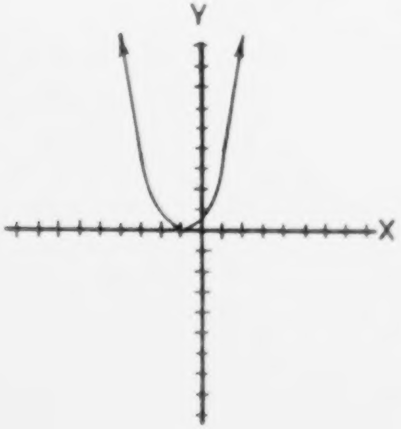
2) Graph $f(x) = (2x-3)(x-1)(3x+2)$.

Note that the non-factored form of this function, $6x^3-11x-x+6$, would not easily yield the zeroes by substitution. It is for this reason that functions are to be given in factored form only. The sketch would be done as in Example 1.

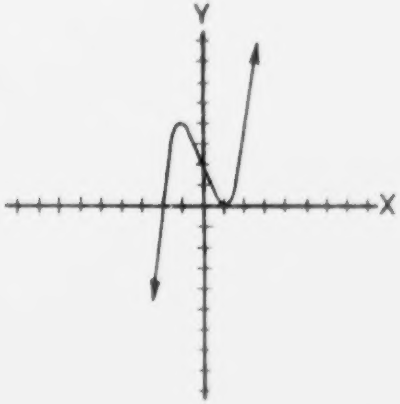
It may be of interest to observe the effect of multiple zeroes on a graph. In both examples given above, the three distinct zeroes imply three separate points at which the curve crossed the x-axis. Students should be able to see that curves having double zeroes do not cross the x-axis at those values of x, but touch it and bend back.

3) $f(x) = x^2+2x+1$. Graph f.

Note: Observe that the function $f(x)$ if factored becomes $f(x) = (x+1)^2$, with a double zero, $x = -1$. The curve is the one drawn below.



4) f(x) = (x-1)(x-1)(x+2). Graph f. This function has double zero x = 1. The shape of the curve can be verified by substitution to find several points.



Note: Teachers with access to microcomputers could use prepared software to verify the shapes of the sketches.

12. to review operations on polynomials and rational expressions;

Students should:

a) review the four algebraic operations on polynomials
b) review the types of factoring studied in Math 100
c) practise simplification of rational expressions

OBJECTIVES	NOTES TO TEACHERS

13. to factor sums and differences of cubes:
$a^3+b^3 = (a+b)(a^2-ab+b^2)$
$a^3-b^3 = (a-b)(a^2+ab+b^2)$

Examples:

1. $(x^3+1) = (x+1)(x^2-x+1)$
2. $(x^3-1) = (x-1)(x^2+x+1)$

Note: These two objectives (#12 and #13) could be incorporated into the study of trigonometric identities. The types of factoring listed in Objective #13 and Enrichment are new and will have to be taught.

ENRICHMENT:

Have students factor polynomials which can be grouped to yield a difference of squares.

Example:

$$x^2+2x+1-4z^2 = (x+1)^2 - (2z)^2$$
$$= (x+1-2z)(x+1+2z)$$

14. to understand the statement of the Factor Theorem;

The Factor Theorem:

Let f(x) be a polynomial. If f(a) = 0 then x-a is a factor of f(x).

Example:

If $f(x) = x^3-7x^2+9x+2$

$f(2) = 8-28+18+2$

$= 0$

$\therefore$ $x-2$ is a factor

Dividing $x-2$ into x^3-7x^2+9x+2 gives x^2-5x+1

Thus, $f(x) = (x - 2)(x^2 - 5x + 1)$

15. to use the Factor Theorem to factor certain polynomials;

Note: At this stage it should be sufficient to consider the application of the Factor Theorem only to the factoring of polynomials of degree 3 with integral coefficients and having at least one zero which is an integer. Students should realize that the Factor Theorem is not useful in factoring a polynomial such as x^3-x-2, which does not happen to have a rational zero.

Examples:

1. Factor $x^3-6x^2+11x-6$

By inspection or by trial we find that when $x=1$ is substituted into the above expression, we obtain zero, thus: $(1)^3-6(1)^2+11(1)-6 = 0$. The Factor Theorem indicates that $x-1$ is a factor of the above expression. By long division, $(x^3-6x^2+11x-6) \div (x-1) = x^2-5x+6$.

Therefore $x^3-6x^2+11x-6 = (x-1)(x^2-5x+6)$

$= (x-1)(x-2)(x-3)$

2) Factor x^3+2x^2+x+2.

Answer: $(x+2)(x^2+1)$ (this polynomial can also be factored by grouping). See Math 100 Enrichment.

ENRICHMENT:

The Factor Theorem may be stated without proof. As enrichment, a proof for the case in which $f(x)$ is a cubic could be given.

Factor Theorem for a cubic: Let $f(x) = px^3+qx^2+rx+s$. If $f(a) = 0$, then $x-a$ is a factor of $f(x)$.

Proof:
$$\begin{aligned} f(x) - f(a) &= (px^3+qx^2+rx+s) - (pa^3+qa^2+ra+s) \\ &= p(x^3-a^3)+q(x^2-a^2)+r(x-a) \end{aligned}$$

However, $x-a$ is a factor of x^3-a^3 and of x^2-a^2.

Thus $x-a$ is a factor of $f(x)-f(a)$ and therefore, if $f(a) = 0$, $x-a$ is a factor of $f(x)$. (This type of proof holds for a polynomial of any degree.)

OBJECTIVES

16. to sketch the graphs of simple polynomial functions which are readily factorable;

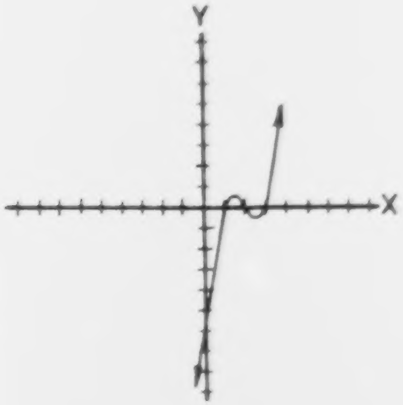
NOTES TO TEACHERS

Note: Limit the problems to those with integral zeroes only.

Examples: $f(x) = x^3-6x^2+11x-6$

1. Factor to get $f(x) = (x-1)(x-2)(x-3)$

2. Sketch by finding zeroes and determining height of curve at intermediate points. It is not necessary to find the exact value of the maxima and minima.



ENRICHMENT:

Graphs of polynomial functions with non-integral zeroes could be sketched.

OBJECTIVES

17. to sketch the graphs of functions involving quotients of polynomials;

NOTES TO TEACHERS

Functions which are quotients of polynomials are called rational functions.

Notes:

1. While this is a core topic, teachers should not spend too much time on this objective. The graphing of such functions is dealt with extensively in post-secondary mathematics courses.
2. Restrict numerators and denominators to linear and quadratic polynomials.

Examples:

1) Given $f(x) = \frac{2x}{x+1}$. Sketch the graph of f.

Step 1. Several points on the curve can be found by substitution. For this function,

$$f(0) = \frac{2(0)}{0+1} = 0$$

Similarly, we find that

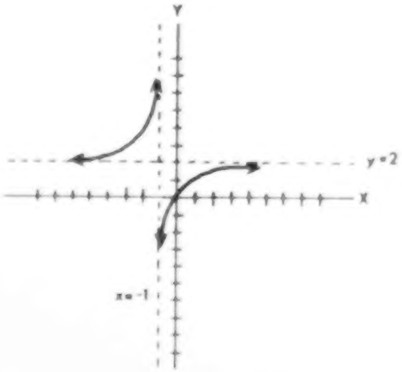
$$f(1) = 1$$
$$f(-2) = 4$$
$$f(-3) = 3$$

This gives us the points (0,0), (1,1), (-2,4), (-3,3).

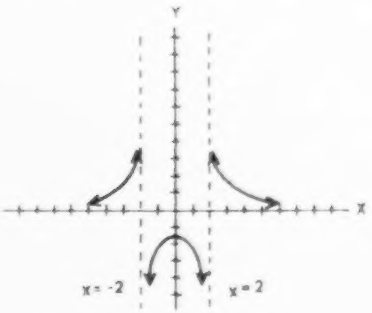
Warning: Students may try to make the graph continuous for all real values of x. When they have discovered that this is impossible, they should be encouraged to use values of x close to -1 in their substitution. Values larger and smaller than -1 should be used. Similarly, they should use large positive and negative values for x in their substitution.

Step 2. The curve can be sketched. Since it cannot touch the lines $x = -1$ and $y = 2$, these lines can be dotted in and labelled. Refer to text for discussion of horizontal and vertical asymptotes.

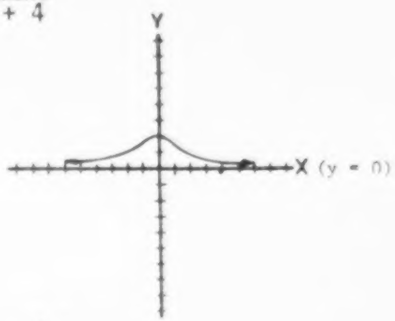
If greater accuracy is needed, more points can be found.



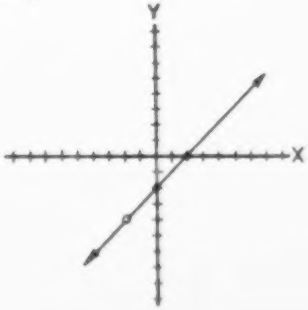
2) Graph: a) $f(x) = \frac{8}{x^2 - 4}$



b) $g(x) = \frac{8}{x^2 + 4}$



c) $h(x) = \frac{x^2 - 4}{x + 2}$



It is interesting that the second function has a graph with no vertical asymptotes while the first function has a graph with two vertical asymptotes. In the third graph a point will be missing at $x = -2$.

18. to determine algebraically the equations of asymptotes for functions which are quotients of polynomials.

Teachers should not spend excessive time determining horizontal asymptotes algebraically. The determination of asymptotes of graphs is dealt with extensively in post-secondary mathematics courses.

Let $p(x)$ and $q(x)$ be polynomials.

If $f(x) = \frac{p(x)}{q(x)}$, and if $q(x)$ is factorable into binomials, students should be able to state values of x for which $f(x)$ has no value. For any factor of $q(x)$, for example, $(x-a)$, the equation $x = a$ represents a vertical asymptote for the graph of $f(x)$ provided $p(a) \neq 0$. (If $p(a) = 0$ and $q(a) = 0$, there is a hole in the graph at $x = a$.)

Example:

$f(x) = \frac{2x}{x+1}$. Find the equations of the vertical and horizontal asymptotes for the graph of this function.

Step 1: Vertical asymptote:

If $f(x) = \frac{2x}{x+1}$, then $x = -1$ is a vertical asymptote for the graph of the function, since the numerator is not zero and the denominator is zero when $x = -1$.

Step 2: Horizontal asymptote:

If $f(x) = \frac{2x}{x+1}$, then on the graph of f, $y = \frac{2x}{x+1}$.

This implies $y(x+1) = 2x$

$$xy+y = 2x$$

$$xy-2x = -y$$

$$2x-xy = y$$

$$x(2-y) = y$$

$$x = \frac{y}{2-y}$$

When $y = 2$ the numerator of $\frac{y}{2-y}$ is not zero and the denominator is zero. Thus, the equation of the horizontal asymptote for the graph of "f" is $y = 2$.

Warning: Teachers should choose function f such that $y = f(x)$ can be solved for x giving a quotient of two polynomials in y.

Note: Future graphing of functions of this type may be simplified by finding and plotting asymptotes first.

Note: Alternative method.

Once the asymptote equations have been found and the asymptotes plotted, one could determine the location of the graph of the function as follows:

Step 1. Perform the division $x + 1\overline{)2x}$

Thus, $y = 2 + \frac{-2}{x + 1}$

i.e., $y = \frac{-2}{x + 1} + 2$

Step 2. We will recognize this to be of the function type $y = \frac{1}{x}$ which was graphed earlier under Objective 6. For $\frac{-2}{x + 1} + 2$, the graph of $y = \frac{1}{x}$ has been:

a) translated horizontally one unit to left;
b) reflected over the x-axis;
c) stretched by a factor of 2;
d) translated vertically two units up.

ENRICHMENT:

Have students find algebraically the domain and range for functions which are quotients of polynomials.

Examples:

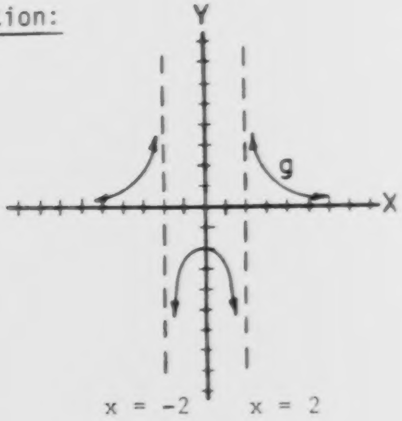
1) If $f(x) = \frac{2x}{x+1}$ find the domain and range for the function.

Solution:

From the solution of the example in Objective 18, we can say that the domain is $\{x | x \neq -1\}$ and the range is $\{y | y \neq 2\}$.

2) $g(x) = \frac{8}{x^2-4}$. Find the domain and range for this function. (Same as Example 2(a), Objective 17.)

Graphical Solution:



Careful plotting enables one to find the range graphically. It is:

$R = \{y | y < -2 \text{ or } y > 0\}$

OBJECTIVES NOTES TO TEACHERS

The equation $y = \frac{8}{x^2 - 4}$ is equivivlent to $x^2 = \frac{4y + 8}{y}$.

$\frac{4y + 8}{y} \geq 0$ if an only if $y > 0$ or $y \leq -2$

Thus, there exists a real number x such that $x^2 = \frac{4y + 8}{y}$ if and only if $y > 0$ or $y \leq -2$.

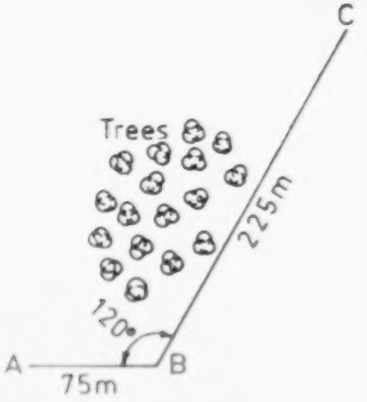
It follows that the range of the function, g, is the set

$\{y \mid y \leq -2 \text{ or } y > 0\}$

C.II. CIRCULAR FUNCTIONS AND TRIGONOMETRY (3 MODULES)

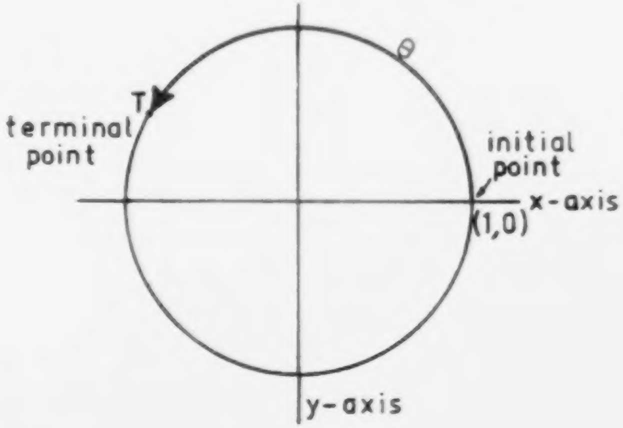
Introductory Example:

A section of a golf course is shaped as shown in the diagram to the right. If AB = 75 m, ∠ABC = 120°, and BC = 225 m, how far is it from the tee-off point A to the hole at C? Answer: 270 m.



OBJECTIVES	NOTES TO TEACHERS
The student should be able:	
1. to write the equation and sketch the graph of the unit circle;	Review C.X, Math 200.
2. to calculate the circumference of the unit circle;	$C = 2\pi r = 2\pi$ for a unit circle. Review as necessary.
3. to change an arc length into the equivalent number of revolutions for unit circles;	2π units = 1 revolution OR 6.2832 units $\doteq$ 1 revolution Students should be able to handle arc lengths given as multiples of π or other forms of a real number. For example: $\frac{\pi}{4}$ units = $\frac{1}{8}$ revolution. 1.5708 units $\doteq$ $\frac{1}{4}$ revolution.

4. to sketch an arc on the unit circle where length is not necessarily less than one revolution and to show the terminal point T;



The standard position is shown above. At this point, introduce the symbol θ (theta) for an arc length.

Discuss the concept of positive arcs (anti-clockwise) and negative arcs (clockwise).

Some attempt should be made to estimate the coordinates of T for various quadrants.

$P(\theta)$ is used to designate the terminal point of arc length θ, placed in standard position on the unit circle. The function $P:\theta \longrightarrow P(\theta)$ is sometimes called the wrapping function.

OBJECTIVES

NOTES TO TEACHERS

5. to find the coordinates of $P(\theta)$ for:
$\theta = 0, \pm\frac{\pi}{2}, \pm\pi, \pm\frac{3\pi}{2}, \pm 2\pi,$
and for larger multiples of $\frac{\pi}{2}$

Note: Other exact values are dealt with in Objectives 15 and 16.

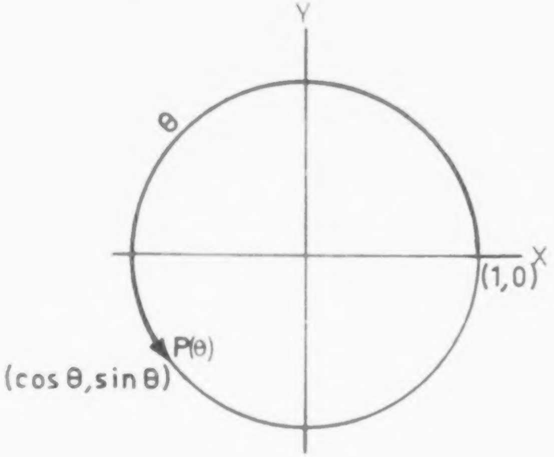
Example: $P(-\pi) = (-1,0)$

$P(\frac{3\pi}{2}) = (0,-1)$

$P(8\pi) = (1,0)$

6. to define:
   - a) the function cosine - $\cos: \theta \to \cos\theta$ where $\cos\theta$ is the x-coordinate of the point $P(\theta)$,
   - b) the function sine - $\sin: \theta \to \sin\theta$ where $\sin\theta$ is the y-coordinate of the $P(\theta)$;

Note: "sin" is the abbreviation for sine and "cos" is the abbreviation for cosine.



Since $\cos: \theta \to x$ is equivalent to $\cos\theta = x$, the point $P(\theta)$, therefore, has coordinates $(\cos\theta, \sin\theta)$.

OBJECTIVES	NOTES TO TEACHERS

7. to find sin θ, cos θ when $\theta = 0, \pm\frac{\pi}{2}, \pm\pi, \pm\frac{3\pi}{2}, \pm 2\pi$;

These should be set up into a table:

θ	Sin θ	Cos θ
0	0	1
$\frac{\pi}{2}$	1	0
π	0	-1
$\frac{3\pi}{2}$	-1	0
2π	0	1
. .		

Encourage students to look at the range of each function.

Show the periodicity of each function by examining values of θ such that $\theta > 2\pi$.

8. to show that $\cos^2\theta + \sin^2\theta = 1$;

Since the equation of the unit circle is $x^2+y^2 = 1$ and (x,y) is $(\cos\theta, \sin\theta)$, then $\cos^2\theta + \sin^2\theta = 1$. This is frequently called a Pythagorean Relation. It can also be illustrated by substituting the values from the table in (7).

9. to find sin θ or cos θ, given the value of one of them;

These problems can be made more challenging by the use of restricted sets.

10. to define:

a) $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$,

b) $\csc\theta = \dfrac{1}{\sin\theta}$,

c) $\sec\theta = \dfrac{1}{\cos\theta}$,

d) $\cot\theta = \dfrac{1}{\tan\theta}$,

e) $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

Example: Given that $\cos\theta = \frac{4}{5}$, find $\sin\theta$ if $\frac{3\pi}{2} < \theta < 2\pi$.

Solution:

$$\cos^2\theta + \sin^2\theta = 1$$

$$\left(\frac{4}{5}\right)^2 + \sin^2\theta = 1$$

$$\sin^2\theta = \frac{9}{25}$$

$$\sin\theta = \pm\frac{3}{5}$$

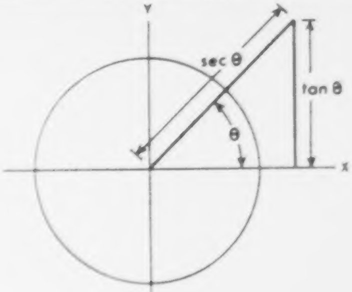
Since the sine function is negative in the 4th quadrant, $\sin\theta = \frac{-3}{5}$

$\text{Tan}\ \theta = \frac{\sin\theta}{\cos\theta}$ and $\cot\theta = \frac{\cos\theta}{\sin\theta}$ may be called the "quotient properties."

The abbreviation for tangent is "tan."
The abbreviation for cosecant is "csc."
The abbreviation for secant is "sec."
The abbreviation for cotangent is "cot."

Students frequently make errors in these definitions, so emphasize the definitions carefully.

OBJECTIVES	NOTES TO TEACHERS



For interest, point out that tan gets its name because it is the length of the tangent while sec is the length of the secant.

11. to find tan θ, csc θ, sec θ, and cot θ when $\theta = 0$, $\pm\frac{\pi}{2}$, $\pm\pi$, $\pm\frac{3\pi}{2}$, $\pm 2\pi$;

A table of values would be helpful here. Additionally, some general comments about the range of each function would be useful.

12. to identify the quadrants in which sin θ, cos θ, and tan θ are positive or negative;

sin = + cos = - tan = -	sin = + cos = + tan = +
sin = - cos = - tan = +	sin = - cos = + tan = -

13. to find the values of the other five circular functions if given one value;

Example:

If tan $\theta = \frac{4}{3}$, find the values of the five other circular functions.

Note: Teachers might prefer to cover objective 21 for use in solutions of this question.

Solution:

Since $\tan\theta = \frac{4}{3}$, $\frac{\sin\theta}{\cos\theta} = \frac{4}{3}$ and $\sin\theta = \frac{4\cos\theta}{3}$.

Substituting this value for $\sin\theta$ in the relation $\cos^2\theta + \sin^2\theta = 1$, gives

$\cos^2\theta + \frac{16\cos^2\theta}{9} = 1$ and $\frac{25\cos^2\theta}{9} = 1$

$\cos^2\theta = \frac{9}{25}$ so that $\cos\theta = \pm\frac{3}{5}$.

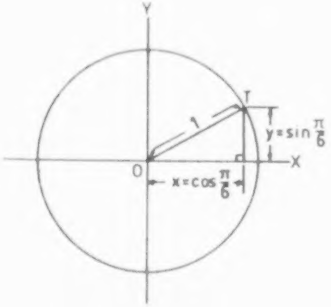
Since $\sin\theta = \frac{4\cos\theta}{3}$, $\sin\theta = \frac{4}{3} \cdot \pm\frac{3}{5} = \pm\frac{4}{5}$.

Consequently, $\csc\theta = \pm\frac{5}{4}$, $\sec\theta = \pm\frac{5}{3}$ and $\cot\theta = \frac{3}{4}$.

14. to determine the exact values of $\sin\theta$, $\cos\theta$, and $\tan\theta$ when $\theta = \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}$;

At this point, review that in a 30-60-90 triangle, the shortest side is one half the hypotenuse. Also, in a 45-45-90 triangle, the shorter sides are each $\frac{\sqrt{2}}{2}$ hypotenuse. (See C.X, Math 200.)

Example:

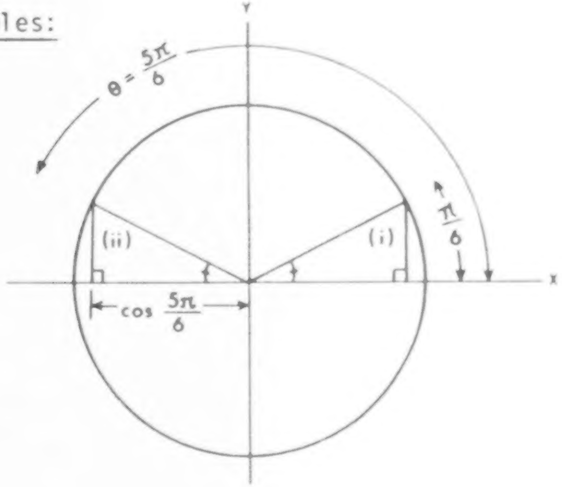


Since $\frac{\pi}{6}$ is $\frac{1}{12}$ of a revolution and one revolution is 360°, the cental angle XOT is 30°.

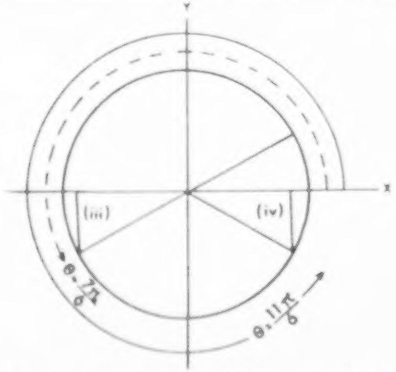
From the triangle, $\sin\frac{\pi}{6} = \frac{1}{2} \cdot 1$ or, $\sin\frac{\pi}{6} = \frac{1}{2}$, and substituting into the Pythagorean Relation, $\frac{1}{4} + \cos^2(\frac{\pi}{6}) = 1$ and $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$ (positive value only).

15. to determine the exact values of $\sin\theta$, $\cos\theta$, and $\tan\theta$ where θ = multiples of $\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}$ and $-2\pi \leq \theta \leq 2\pi$;

Examples:



The triangle used in Objective 14 is flipped from position (i) to (ii).

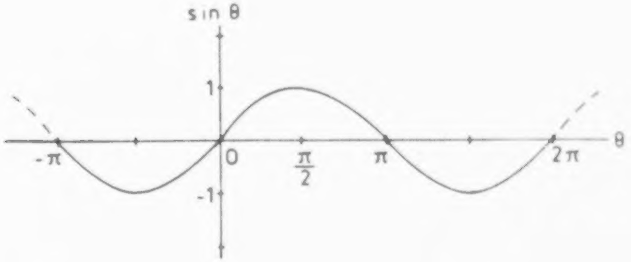


The triangle in position (ii) is flipped again twice, to produce triangles in positions (iii) and (iv). From these triangles, sin $\frac{7\pi}{6}$, cos $\frac{7\pi}{6}$ and sin $\frac{11\pi}{6}$, or cos $\frac{11\pi}{6}$ can be found directly. Set up a table of the various values.

16. to draw the graphs of sin Θ, cos Θ, tan Θ vs Θ in the interval $-2\pi \leq \Theta \leq 4\pi$;

Customarily, Θ values are placed on the horizontal axis.

Use circular functions values from tables.



Similarly, draw graphs of cos Θ and tan Θ.

OBJECTIVES | NOTES TO TEACHERS

17. to draw graphs in the internal $-2\pi \leq \theta \leq 4\pi$, of functions similar to those shown in the accompanying examples;

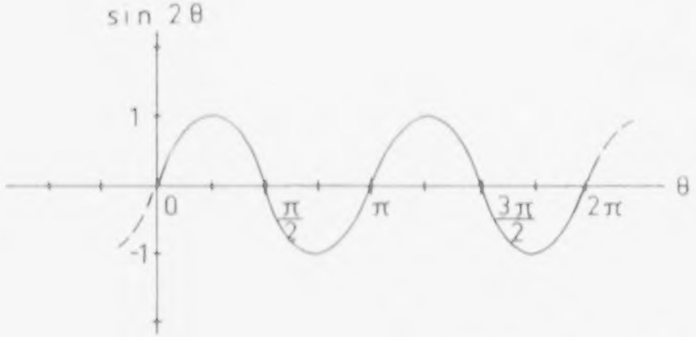
Function values for varous θ can be obtained from Math Tables or by using a calculator and/or computer.

Prepare a table of values and use it to draw the graph.

Examples:

1. Draw the graph of $y = \sin 2\theta$.

x	θ	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
y	$\sin 2\theta$	0	1	0	-1	0	1	0	-1	0



OBJECTIVES

NOTES TO TEACHERS

Continue with some of the following:

2. $2 \sin \theta$

3. $\sin \theta + 1$

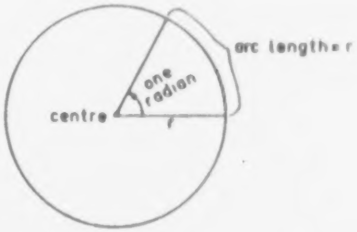
4. $\sin (\theta - \pi)$

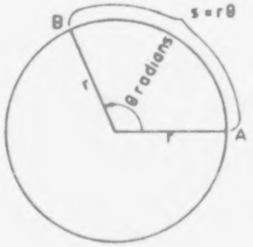
5. $\sin (\theta + \pi)$

18. to define radian measure and to obtain the approximate values for the six circular functions in the interval $0 \leq \theta \leq 2\pi$;

One radian is the measure of a central angle that intercepts an arc equal in length to the radius of the circle. Explain the specific relationship in a unit circle (i.e., arc length = radian value).

Obtain approximate values for the circular functions using a calculator.





S = arc length

θ = central angle (radians)

$s = r\theta$

$\theta = \frac{s}{r}$

OBJECTIVES	NOTES TO TEACHERS
	Example: If radius is r = 3 metres and central angle $\theta = 4.75$ radians, solve for s. $s = r\theta = 3 \times 4.75 = 14.25$ m. Example: If the arc length s = 81 cm and the radius r = 27 cm, find the measure of the central angle θ in radians. $\theta = \frac{s}{r} = \frac{81}{27} = 3$ radians.
19. to define inverse sin w, inverse cos w and inverse tan w;	The inverse sine or arcsine function, denoted by $\sin^{-1}$ or arcsin, is defined by $\sin^{-1} w = \theta$ if and only if $w = \sin\theta$ and $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$.
20. to find all values of θ in the interval $0<\theta<2\pi$ when given $\sin\theta = w$, $\cos\theta = w$ or $\tan\theta = w$ where w is a real number;	Note: It will be necessary to introduce the concept of "related arc." Any arc with $P(Q)$ in quadrants II, III, or IV, except for sign, has the same x, y coordinates for $P(\theta)$ as a first quadrant arc for which $0 \le \theta \le \frac{\pi}{2}$. For positive ratios, calculators always yield $0 \le \theta \le \frac{\pi}{2}$ for inverse functions. For some questions the calculator value will be one of the solutions. To find the desired arc lengths in quadrants II, III, or IV one could use:

quad. II: π - rel. arc

quad. III: π + rel. arc

quad IV: π - rel. arc

π - rel. arc	rel. arc
π + rel. arc	2π - rel. arc.

Example 1: Find all values of θ given $\sin\theta = 0.3246$ and $0 \le \theta \le 2\pi$.

Solution: Our two solutions are in quads I and II where $\sin\theta > 0$.

On calculator: 0.3246 $\boxed{\text{INV}}$ $\boxed{\text{SIN}}$ $\theta = 0.3306$ which is our quad I answer.
For quad II: $\theta = \pi - 0.3306 = 2.8110$.

Example 2: Given $\cos\theta = -0.7885$, find all values of θ if $0 \le \theta \le 2\pi$.

Solution: Our two solutions are in quads II and III where $\cos\theta < 0$.

Cos (rel arc) = 0.7885
rel arc = 0.6624

quad II: $\theta = \pi - 0.6624 = 2.4792$
quad III: $\theta = \pi + 0.6624 = 3.8040$

21. to derive the three Pythagorean Relations:

$\sin^2\theta + \cos^2\theta = 1$,

$1 + \tan^2\theta = \sec^2\theta$,

$1 + \cot^2\theta = \csc^2\theta$;

From the unit circle, $x^2+y^2 = 1$ so that $\cos^2\theta + \sin^2\theta = 1$ (review). By dividing this equation by $\cos^2\theta$, we get $1 + \tan^2\theta = \sec^2\theta$. On dividing the original equation by $\sin^2\theta$, we get $\cot^2\theta+1 = \csc^2\theta$.

OBJECTIVES

NOTES TO TEACHERS

22. to use the reciprocal properties, the quotient properties and the Pythagorean Relations to prove circular function identities;

At this point it would be wise to spend a little time reviewing factoring. (See C.I., Objective 13.) Many questions are available from any text. The time available will determine the number and difficulty of the questions attempted. Refer to C.I., Objectives 12 and 13.

Examples:

1. $\frac{\sin x}{1+\cos x} = \frac{1-\cos x}{\sin x}$

2. $\frac{\sec^2 x - 6\tan x + 7}{\sec^2 x - 5} = \frac{\tan x - 4}{\tan x + 2}$

23. to solve "linear" circular function equations;

Examples:

Solve for θ:

(1) $5 + 3\sin\theta = 6$

(2) $5 + 3\sin 2\theta = 6$ if $0 \leq \theta \leq 2\pi$

Solution for (2):

$5 + 3\sin 2\theta = 6$

$3\sin 2\theta = 1$

$\sin 2\theta = \frac{1}{3}$

By calculator:

$2\theta = \text{Arc}\sin(\frac{1}{3})$

$2\theta = 0.3398$ (quad I)

and $2\theta = 2.8018$ (quad II)

Since $0 \leq \theta \leq 2\pi$ then $0 \leq 2\theta \leq 4\pi$ and
$2\theta = 2\pi + 0.3398$
$= 6.6230$

$2\theta = 2\pi + 2.801818$
$= 9.0850$

The values for 2θ are 0.3398, 2.8018, 6.6230, 9.0850 and the values for θ are $\theta = 0.1699, 1.4009, 3.3115, 4.5425$.

24. to solve "quadratic" equations involving circular functions;

Example:

Solve for θ:

$4 \cos^2\theta + \cos \theta = 0$ if $0 \leq \theta \leq 2\pi$.

ENRICHMENT:

Double angle quadratics can be attempted with a capable class:

Solve for θ:

$\sec^2 2\theta + \tan 2\theta - 13 = 0$ if $0 \leq \theta \leq 2\pi$.

Solution:

Substitute $\sec^2 2\theta = 1 + \tan^2 2\theta$, so that $1 + \tan^2 2\theta + \tan 2\theta - 13 = 0$.

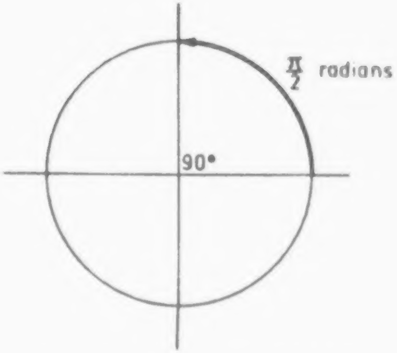
$$\tan^2 2\theta + \tan 2\theta - 12 = 0$$

$$(\tan 2\theta + 4)(\tan 2\theta - 3) = 0$$

This gives $\tan 2\theta = -4$ and $\tan 2\theta = 3$, each of which can be solved by the methods illustrated in Objective 23. Eight values of θ will result.

OBJECTIVES	NOTES TO TEACHERS

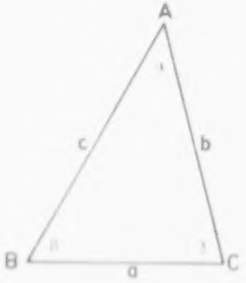
25. to define degree measure;



Illustrate the relationship between degrees and radians by showing that 90° is the measure of the central angle which is subtended by an arc of $\frac{\pi}{2}$ radians. Similarly, point out the relation between 180° and π radians, etc.

26. to solve oblique triangles;

Define:



The Sine Rule

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

and The Cosine Rule

$$a^2 = b^2 + c^2 - 2\,bc\cos\alpha$$

Students should see derivations of the sine and cosine laws.

Also, explain that the relations in a right triangle are a special case of these. With these, they can solve triangles given SAS, ASA and SSS.

The SSA triangle presents an ambiguous or inconsistent situation and must be explained carefully to students.

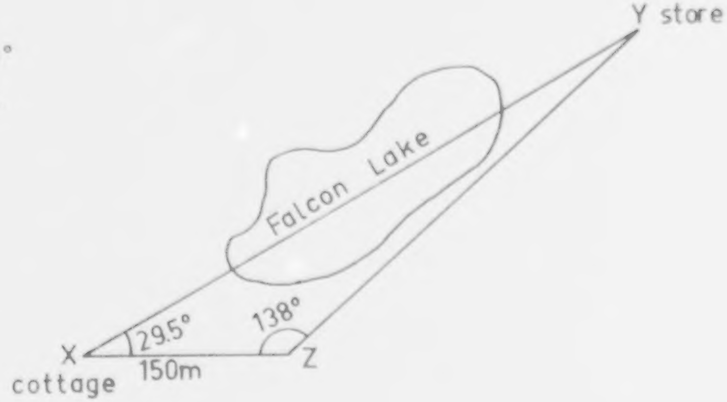
OBJECTIVES	NOTES TO TEACHERS
26. to solve problems involving solution of triangles.	Examples:

1. A cottage owner wishes to determine the distance from his cottage located at point X in the adjacent diagram and a grocery store at point Y on the opposite side of the lake. He locates a third point Z which is 150 m from X and determines ∠ZXY to be 29.5° and ∠XZY to be 138°.

$\angle X + \angle Y + \angle Z = 180^\circ$

$29.5^\circ + \angle Y + 138^\circ = 180^\circ$

$\angle Y = 12.5^\circ$



Law of Sines

$$\frac{XZ}{\sin Y} = \frac{XY}{\sin Z}$$

$$\frac{XY}{\sin 138^\circ} = \frac{150}{\sin 12.5^\circ}$$

$$XY = \frac{150 \sin 138^\circ}{\sin 12.5^\circ}$$

Second Quad: $\sin 138^\circ = \sin(180^\circ - 138^\circ)$

$= \sin 42^\circ$

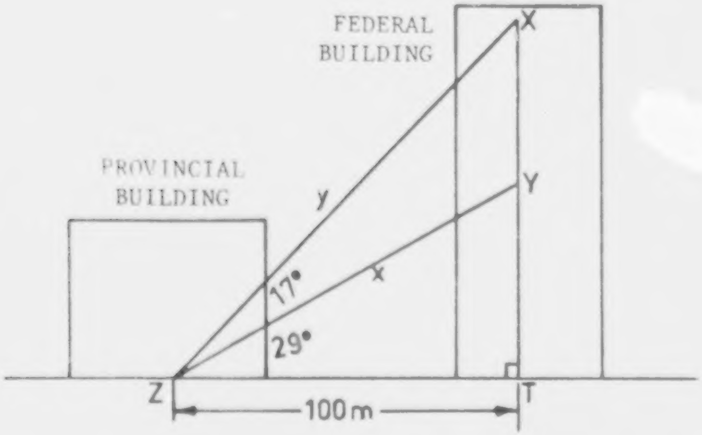
$\doteq 0.66913$

$$XY \doteq \frac{150(0.66913)}{0.21644}$$

$\doteq 463.72898$

XY = 464 m (to the nearest metre)

OBJECTIVES	NOTES TO TEACHERS
	2. Direct phone lines connect offices X and Y in the Federal Building to office Z in the Provincial Building as shown in the accompanying diagram. If an extra metre of wire is required at each end of each line for going through the wall, how much line is required to reach from office X to office Z? The horizontal distance from office Z to office X is given as 100 m and the size of ∠XZY is 17° and ∠YZT is 29°.


FEDERAL BUILDING
X
PROVINCIAL BUILDING
y
Y
17°
x
29°
Z
100 m
T

C.III. ANALYTIC GEOMETRY (1 MODULE)

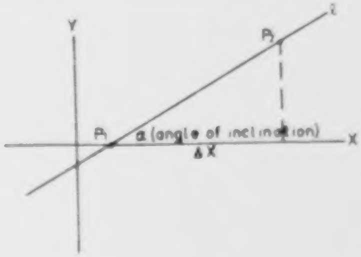
OBJECTIVES

The student should be able:

1. to define the angle of inclination of a line and to determine the relationship between the angle of inclination and the slope of a line;

2. to determine the angle of inclination of a line, given the equation of the line;

NOTES TO TEACHERS



For any two points, P_1 and P_2 on line ℓ it can be shown that:

$$\tan \alpha = \frac{\Delta y}{\Delta x}$$

where α is the angle of inclination of the line. Therefore, the slope of line ℓ is equal to $\tan \alpha$; $m = \tan \alpha$.

Note: Angle of inclination α satisfies $0 \leqslant \alpha \leqslant 180°$.

Example:

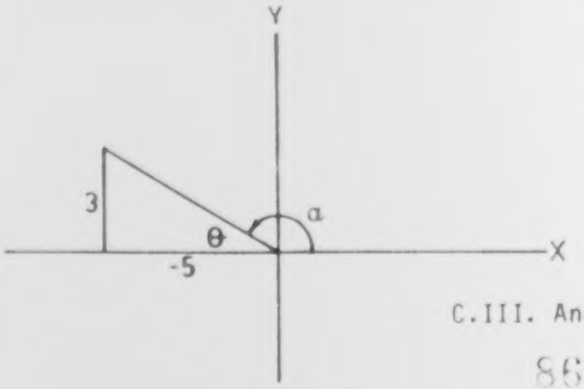
Determine the angle of inclination for the line whose equation is $3x+5y-7=0$.

Solution:

$m = \frac{-3}{5}$

$\tan \alpha = -\frac{3}{5} = -0.6$

$\alpha = 149°$

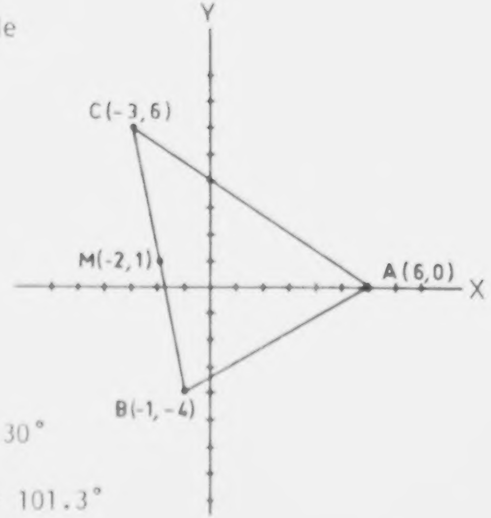


3. to solve problems involving midpoint, distance and slope;

Examples:

In $\triangle ABC$ where A (6, 0), B(-1, -4) and C(-3, 6), determine:

a) the length of the median from A
b) the length of each side
c) the angle of inclination of each side



Solution:

a) mid-point of BC is M(-2,1)

$AM = \sqrt{65}$

b) $AB = \sqrt{65}$

$BC = \sqrt{104}$

$CA = \sqrt{117}$

c) For AB: $m = \frac{4}{7}$, $\therefore \tan \alpha = \frac{4}{7}$, $\alpha \doteq 30°$

For BC: $m = -5$, $\therefore \tan \alpha = -5$, $\alpha \doteq 101.3°$

For AC: $m = \frac{-6}{9}$, $\therefore \tan \alpha = \frac{-6}{9}$, $\alpha \doteq 146.3°$

2. Determine whether the following points are collinear: D(-2, 5), E(3, -4) and F(0, 1).

DF = 4.5, FE = 5.8, DE = 10.3. Since DF + FE = DE the three points are collinear.

OBJECTIVES	NOTES TO TEACHERS
4. to apply the formula for the distance from a point to a line;	Show students the derivation of the formula $d = \frac{\lvert Ax_1+By_1+C\rvert}{\sqrt{A^2+B^2}}$ for the distance from a point $P(x_1, y_1)$ to a line $Ax+By+C = 0$.
5. to determine the equation of a set of points, given the conditions which govern their locus;	The following examples include a line, a circle and each of the other conic sections in standard position. Examples: 1. Find an equation describing the locus of all points equidistant from (3, 7) and (-2, 4). Solution: $\sqrt{(x-3)^2 + (y-7)^2} = \sqrt{(x+2)^2 + (y-4)^2}$ $x^2 -6x + 9 + y^2 - 14y + 49 = x^2 + 4x +4 + y^2 - 8y + 16$ $10x + 6y = 38$ $5x + 3y = 19$ 2. Find an equation describing the locus of all points 4 units from (2, -3). Solution: $\sqrt{(x-2)^2 + (y+3)^2} = 4$ $(x-2)^2 + (y+3)^2 = 16$

3. Find an equation of the locus of all points such that for each point the sum of its distances from (-4,0) and (4,0) is 10.

Solution:

$$\sqrt{(x+4)^2 + y^2} + \sqrt{(x-4)^2 + y^2} = 10$$

$$\sqrt{(x-4)^2 + y^2} = 10 - \sqrt{(x+4)^2 + y^2}$$

$$4x + 25 = 5\sqrt{(x+4)^2+y^2}$$

$$9x^2 + 25y^2 = 225$$

$$\frac{x^2}{25} + \frac{y^2}{9} = 1.$$

Note: We have now shown that the co-ordinates of any point on the locus satisfy the equation $\frac{x^2}{25} + \frac{y^2}{9} = 1$. Conversely (by reversing the steps), it can be shown that each point whose co-ordinates satisfy this equation must lie on the locus. (Notice that if $\frac{x^2}{25} + \frac{y^2}{9} = 1$, then $\frac{x^2}{25} \leq 1$ so $-5 \leq x \leq 5$ and $5 \leq 4x + 25 \leq 45$, so that $4x + 25$ is positive. Also, from $4x + 25 = 5\sqrt{(x+y)^2 + y^2}$ and $4x + 25 \leq 45$, it follows that $\sqrt{(x+y)^2 + y^2} \leq 9$ and $10-\sqrt{(x+y)^2 + y^2} \geq 1$ so that $10-\sqrt{(x+y)^2 + y^2}$ is positive.)

4. Find an equation of the locus of all points such that for each point the difference of its distances from (-5,0) and (5,0) is 8.

Solution:

Using reasoning similar to that used in the preceding example, it can be shown that each of the following equations are equivalent:

$$\sqrt{(x+5)^2 + y^2} - \sqrt{(x-5)^2 + y^2} = 8$$

$$\sqrt{(x+5)^2 + y^2} = 8 + \sqrt{(x-5)^2+y^2}$$

$$20x - 64 = 16\sqrt{(x-5)^2 + y^2}$$

$$25x^2 - 160x + 256 = 16x^2 - 160x + 400 + 16y^2$$

$$9x^2 - 16y^2 = 144$$

$$\frac{x^2}{16} - \frac{y^2}{9} = 1$$

5. Find an equation describing the locus of all points equidistant from (0, 2) and the line whose equation is y + 2 = 0.

Solution:

$$\sqrt{x^2 + (y-2)^2} = \frac{|y + 2|}{\sqrt{1}}$$

$$x^2 + y^2 - 4y + 4 = y^2 + 4y + 4$$

$$x^2 = 8y$$

OBJECTIVES	NOTES TO TEACHERS
6. to sketch the graph of the equation of a given locus;	Examples 1 and 2 in Objective 5 are easily sketched. Sketches for examples 3-5 will require substitution of values to determine the shape of the locus. The definitions of the conic sections should be given after the appropriate curves in Objective 5 have been sketched. Circle: locus of all points equidistant from a fixed point. Parabola: locus of all points equidistant from a fixed line and a fixed point not on the line. Hyperbola: locus of all points such that for each point the difference of its distances from two fixed points is constant. Ellipse: locus of all points such that for each point the sum of its distances from two fixed points is constant. The use of microcomputers for illustrating the shapes of the conic sections will be useful at this stage.
7. to recognize the conic section, given its equation in standard form;	Standard forms: Circle: $x^2 + y^2 = r^2$ Ellipse: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ or $\frac{y^2}{a^2} + \frac{x^2}{b^2} = 1$ Example: $9x^2 + 25y^2 = 225$ becomes $\frac{x^2}{25} + \frac{y^2}{9} = 1$ Hyperbola: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ or $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$ Parabola: $x^2 = 4py$ or $y^2 = 4px$

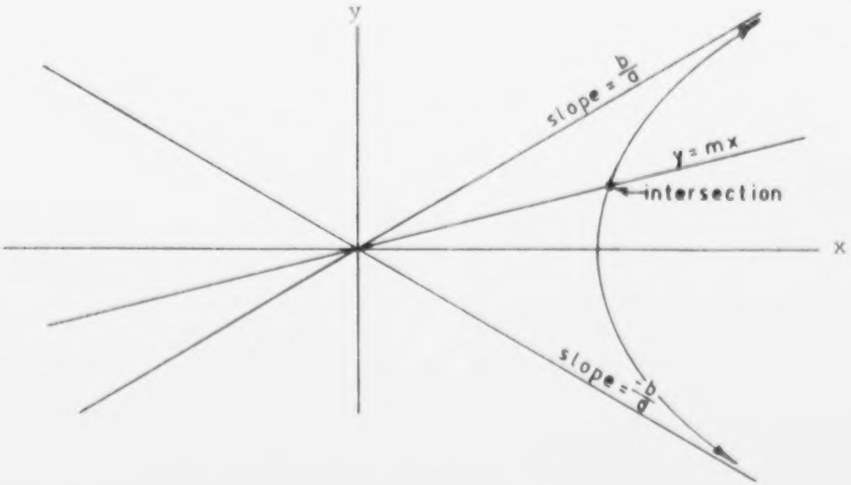
Note: If we solve the equations $y = mx$ and $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, simultaneously, we obtain as a solution for "x",

$$x = \pm \frac{ab}{\sqrt{b^2-a^2m^2}}$$

The solution is real if and only if $b^2 > a^2m^2$.

That is, if and only if $|m| < \frac{b}{a}$ (where a and b are positive real numbers).

Thus, the line intersects the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ if and only if $|m| < \frac{b}{a}$.



OBJECTIVES

8. to sketch the conic section, given its equation in standard form;

9. to determine the radius and centre of a circle, given its equation.

NOTES TO TEACHERS

Make students aware of the effect on the graph of changing the constants in the equations for conics in standard form.

Note: It may be necessary to consider the concept of asymptotes which are neither vertical nor horizontal.

Examples:

1. Find the radius and centre of the circle whose equation is $x^2 + y^2 = 25$.

   Solution:

   radius = 5; centre (0,0)

2. Find the radius and centre of the circle whose equation is

   $(x-3)^2 + (y+2)^2 = 36$.

   Solution:

   radius = 6; centre (3,-2)

3. Find the radius and centre of the circle whose equation is $x^2 + y^2 + 8x - 12y + 2 = 0$.

   Solution:

   $x^2 + 8x + 16 + y^2 - 12y + 36 = -2 + 16 + 36$

   $(x+4)^2 + (y-6)^2 = 50$

   radius = $\sqrt{50}$; centre (-4,6)

C.IV. EXPONENTS AND LOGARITHMS (1 MODULE)

Introductory Problems:

1. In 1980 the total population of the world was approximately 5 billion. If this population doubles every 35 years, what should the world population be in the year 2120? (Answer: about 80 billion).

2. The Japanese earthquake of May 25, 1982 measured 4 on the Richter scale while a Pacific Island earthquake of June 22, 1934 measured 6.75 on the Richter scale. By definition, the Richter energy number (n) of an earthquake is $\eta = \log_{10}(x)$ where x = severity of tremor.

   How many times greater was the severity of the Pacific Island earthquake than the severity of the Japanese earthquake? (Answer: 562)

In this unit, students should use scientific calculators rather than tables to aid in computations involving exponential and logarithmic functions. Students still need to understand and to use logarithms in the solution of exponential equations and appropriate problem applications. However, little or no time should be spent using logarithms to perform arithmetic calculations as calculators and computers are much more efficient for this purpose.

OBJECTIVES	NOTES TO TEACHERS
A. EXPONENTS/EXPONENTIAL FUNCTIONS	
The student should be able:	
1. to understand and to use the following terminology: a) base, b) exponent, c) power, d) coefficient e) term/monomial, f) laws of exponents;	Review as necessary.

OBJECTIVES

NOTES TO TEACHERS

2. to express a term containing negative exponents in its equivalent positive form and vice-versa.

Define $a^{-n} = \frac{1}{a^n}$ where "n" is a positive integer so that $\frac{1}{a^{-n}} = a^n$.

Examples:

1) Simplify the following by removing negative exponents: $\frac{27x^{-3}}{-14yz^{-1}}$.

Solution: $\frac{27z}{-14x^3y}$

2) Express x^5y so that each power has a negative exponent.

Solution: $\frac{1}{x^{-5}y^{-1}}$

3. to express powers containing fractional exponents as radicals and vice-versa;

New course outlines for Mathematics 100, 200 have omitted fractional exponents in favour of their introduction at this point as new work whose application is found in logarithms.

Define $x^{\frac{m}{n}} = \sqrt[n]{x^m}$ or $(\sqrt[n]{x})^m$ where "m" and "n" are integers with $m>0$ and $n>0$. (We exclude the case where n is even and x is negative.) Show students that this is a reasonable definition and is compatible with the Laws of Exponents for integers.

For example, $(x^{\frac{1}{2}})(x^{\frac{1}{2}}) = x^1$ or x and $\sqrt{x}\sqrt{x} = x$ so that $x^{\frac{1}{2}} = \sqrt{x}$.

Examples:

1) Express $ab^{\frac{2}{3}}$ without using fractional exponents.

Solution: $a\sqrt[3]{b^2}$ or $a(\sqrt[3]{b})^2$

2) Express $\sqrt{8\sqrt{t}}$ using fractional exponents.

Solution: $(8t^{\frac{1}{2}})^{\frac{1}{2}}$ or $8^{\frac{1}{2}}t^{\frac{1}{4}}$

4. to evaluate and to simplify expressions involving zero exponents, negative exponents and fractional exponents;

Review basic algebraic operations involving powers. Use both numerical and algebraic examples.

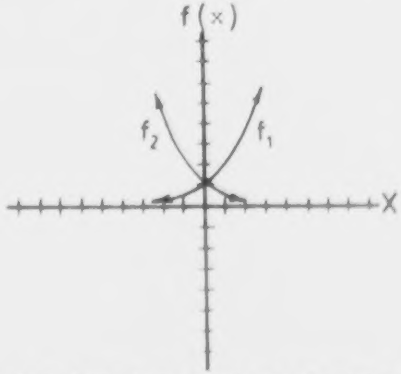
Example:

1) Find the numerical value of $27^{\frac{-2}{3}} + x^0$.

Solution: $\frac{1}{9} + 1 = 1\frac{1}{9}$

2) Without using calculators, evaluate $9^{-2.5}$.

Solution: $9^{-2.5} = 9^{\frac{-5}{2}} = \frac{1}{9^{\frac{5}{2}}} = \frac{1}{(\sqrt{9})^5} = \frac{1}{3^5} = \frac{1}{243}$

OBJECTIVES	NOTES TO TEACHERS
5. to locate points on the graph of a specific exponential function such as $y = 2^x$ where x is real;	2^x can be defined for real x in such a way that the graph of $y = 2^x$ is continuous. Any function of the form, $f:x \to a^x$ or $y = a^x$ where $a > 0$ and $a \neq 1$, is an exponential function.
6. to graph given exponential functions and to determine domain, range, zeroes and when the function increases/decreases;	Examples should include graphing two exponential functions such as $f_1:x \to 2^x$ and $f_2:x \to 2^{-x}$ with constant bases. Note that f_2 can be expressed as $f_2:x \to (\frac{1}{2})^x$.  Properties for the exponential function $f:x \to a^x$ where $a > 0$ include: domain: $\{x \mid x \in R\}$ or real numbers range: $\{y \mid y > 0\}$ or positive reals zeroes: none increasing function when $a > 1$ decreasing function when $a < 1$ (i.e., $0 < a < 1$)

ENRICHMENT:

Graph the following functions using transformation principles developed in C.I.

1. $f(x) = 2^x + 5$
2. $f(x) = 3(2^x) - 1$
3. $f(x) = 2^{x-1}$
4. $f(x) = 2(3^{x-1}) + 4$

7. to solve simple exponential equations without using calculators.

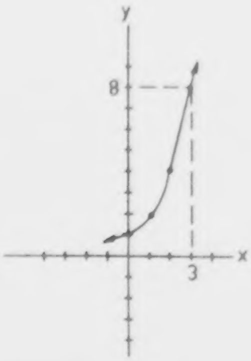
Consider exponential equations of the form $y = a^x$ where $a>0$.

Include examples such as:

1) exponent unknown

$2^x = 8$

Refer to the graph of $y = 2^x$ to show that when y is 8, x is 3.



Solution: $x = 3$

2) base unknown

$$a^{\frac{2}{3}} = 16$$

Solution:

$$\left(a^{\frac{2}{3}}\right)^{\frac{3}{2}} = 16^{\frac{3}{2}}$$

$$a = 16^{\frac{3}{2}}$$

$$a = \sqrt{16}^{3}$$

$$a = 64$$

Alternate Solution

$$a^{\frac{2}{3}} = 16$$

$$(\sqrt[3]{a})^{2} = 16$$

$$\sqrt[3]{a} = 4$$

$$a = 64$$

3) numerical value of power unknown

$$4^{\frac{-3}{2}} = y$$

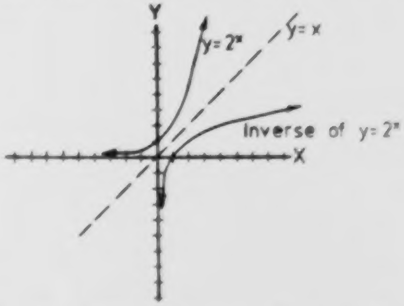
Solution:

$$y = 4^{\frac{-3}{2}}$$

$$y = \frac{1}{4^{\frac{3}{2}}}$$

$$y = \frac{1}{\sqrt{4}^{3}}$$

$$y = \frac{1}{8}$$

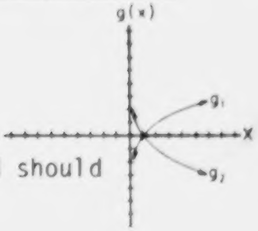
OBJECTIVES	NOTES TO TEACHERS
B. LOGARITHMS/LOGARITHMIC FUNCTIONS	Teachers should indicate that logarithms are used to measure real-world quantities such as Richter scale (earthquakes), decibels (sound), pH (Chemistry), slide-rule tuning scale (radio), and the absorption of x-rays. Application examples can be found in: Foerster, Paul A., Algebra and Trigonometry: Functions and Applications (Menlo Park, CA: Addison-Wesley Publishing Company, 1980).
The student should be able:	
1. to graph the inverse of a specific exponential function such as $y = 2^x$;	Two functions are said to be inverses if, for each point (a,b) on the graph of either function, the point (b,a) is on the graph of the other function.  The graphs of two inverses are symmetric with respect to the line whose equation is $y = x$.
2. to define a logarithm as an exponent;	$y = \log_b x$ means $b^y = x$ where $b > 0$ and $b \neq 1$ (i.e., a logarithm is simply an exponent). Note: if x is negative or zero the logarithm of x is undefined. Terminology associated with $y = \log_b x$ includes: y is the logarithm (abbreviated log) b is the base x is the argument

OBJECTIVES | NOTES TO TEACHERS

3. to graph given logarithmic functions and to determine domain, range, zeroes and when the function increases/decreases;

Example:

Graph the two logarithmic functions,
$g_1 : x \rightarrow \log_2 x$ and $g_2 : \rightarrow \log_{\frac{1}{2}} x$, on the same axis.
Students should graph $f\colon x \rightarrow 2^x$ and $g\colon x \rightarrow \log_2 x$ on the same graph and should realize that they are inverses.



ENRICHMENT:

Graph the following functions using transformation principles developed in C.I.

1. $f(x) = \log_2 (x-5)$
2. $f(x) = 3 \log_2 x$
3. $f(x) = \log_2 x + 5$
4. $f(x) = 4 \log_2 (x+1)-3$

4. to convert equations from logarithmic to exponential form and vice-versa;

$\log_a t = p \leftrightarrow a^p = t \qquad a > 0$

5. to identify special types of logarithms;

Include common logarithms with a base of 10 and natural logarithms with a base of e. Note that e is a special constant which is irrational and e ≈ 2.718 281 8.

Type	Abbreviation	Base
common natural	log ln	10 e

6. to use the logarithmic theorems to express products, quotients or powers in the form $\log x = m \log a \pm n \log b \pm p \log c$ where $a, b, c > 0$;

Guide students in the derivation of the logarithmic theorems.

1) Logarithm of a Product

$$\log_a(MN) = \log_a M + \log_a N$$

2) Logarithm of a Quotient

$$\log_a\left(\frac{M}{N}\right) = \log_a M - \log_a N$$

3) Logarithm of a Power

$$\log_a M^n = n \log_a M.$$

Examples:

Express each of the following equations in the form $\log x = n \log a \pm m \log b \pm p \log c$ where $a, b, c, > 0$. Do not evaluate.

1. $x = \dfrac{(2.4)(6.49)}{1.8}$

   Solution: $\log x = \log 2.4 + \log 6.49 - \log 1.8$

2. $x = \dfrac{\sqrt{58.2}}{(14.91)^3}$

   Solution: $\log x = \frac{1}{2} \log 58.2 - 3 \log 14.91$

OBJECTIVES

NOTES TO TEACHERS

3) $x = \frac{m^3}{t\sqrt{p}}$

Solution: $\log x = 3 \log m - (\log t + \frac{1}{2} \log p)$

or $\log x = 3 \log m - \log t - \frac{1}{2} \log p$.

In the preceding examples, there is no intention of having students perform arithmetic computations using logarithms. The purpose of this exercise is to familiarize the students with logarithmic theorems. Students taking post-secondary mathematics courses must have knowledge of these theorems.

7. to write an expression of the form $\log p = n \log a \pm m \log b$ as $p = a^n b^{\pm m}$ and solve for p;

In each of the following examples solve for x:

1) $\log x = \log 4 - \log 2$

Solution: $\log x = \log (\frac{4}{2})$; $x = 2$

2) $\log x = \frac{\log 81}{\log 9}$

Solution: $\log x = \frac{\log 9^2}{\log 9}$ or $\frac{2 \log 9}{\log 9}$

$\log x = 2$

$\therefore x = 10^2$ or 100

Note: Teachers may find it advantageous to introduce "inverse logarithm," abbreviated "Inv. log," at this time.

Examples:

1. Solve $\log_4(x+2) - \log_4(x-3) = \log_4 4$ for x.

Solution: Take the inverse logarithm (base 4) of both sides of the equation to get:

$$\log_4\left(\frac{x+2}{x-3}\right) = \log_4 4$$

$$\frac{x+2}{x-3} = 4$$

$$x + 2 = 4x - 12$$

$$-3x = -14$$

$$\therefore x = \frac{14}{3}.$$

2. $\log(x + 3) - \log(x - 4) = \log(x + 3)$

Solution: $\frac{x+3}{x-4} = x + 3$

$$x + 3 = x^2 - x - 12$$

$$x^2 - 2x - 15 = 0$$

$$x = 5 \text{ or } x = -3$$

Check: -3 is an extraneous root since $\log(x-3)$ is undefined when $x = -3$.

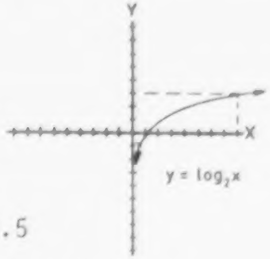
Also, $\log(x+3)$ is undefined when $x = -3$ since $\log 0$ is undefined.

OBJECTIVES | NOTES TO TEACHERS

8. to solve simple logarithmic equations without using calculators;

Examples could include solving for x in the following:

1) $\log_2 x = 3$. Refer to graph of $y = \log_2 x$ to show that if $y = 3$, then $x = 8$.



2) $\log_x 27 = 1.5$

Solution: $x = 9$

3) $\log_5 125 = x$

Solution: $x = 3$

4) $\log_2 75 = x$ given $\log_2 3 = 1.585$ and $\log_2 5 = 2.322$

Solution:

$$\begin{aligned} x = \log_2 75 &= \log_2(3 \times 5^2) \\ &= \log_2 3 + \log_2 5^2 \\ &= \log_2 3 + 2\log_2 5 \\ &= 1.585 + 2(2.322) \\ &= 1.585 + 4.6444 \\ &= 6.229 \end{aligned}$$

OBJECTIVES

NOTES TO TEACHERS

9. to evaluate logarithms and inverse logarithms using calculators;

Students should have a scientific calculator for use in this section. All time considerations in this module are based on the students using calculators.

Examples could include:

1) Evaluate log 284
2) Evaluate ln 284
3) Evaluate Inv. log 1.75

10. to solve exponential equations using logarithms;

Examples could include:

1) Solve for x in $\log_e x = 0.591$

2) Solve for x in $\log_4 28 = x$

3) Solve for x in $19^{x-5} = 3^{x+2}$

Solution:

$$(x-5) \log 19 = (x+2) \log 3$$

$$x \log 19 - 5 \log 19 = x \log 3 + 2 \log 3$$

$$x \log 19 - x \log 3 = 2 \log 3 + 5 \log 19$$

$$x (\log 19 - \log 3) = 2 \log 3 + 5 \log 19$$

$$x = \frac{2 \log 3 + 5 \log 19}{\log 19 - \log 3}$$

$$\therefore x = 9.166$$

Logarithms in any base would give the same solution.

Note: In the previous example, the algebraic manipulation is much more important than the answer. To emphasize the process of solving rather than the final answer, ask multiple-choice questions.

Example: The value of x in the equation $3^x = 5^{x-4}$ is:

a) $\dfrac{4 \log 5}{\log 3 - \log 5}$

b) $\dfrac{4 \log 5}{\log 5 - \log 3}$

c) $\dfrac{\log 5 - \log 3}{\log 5}$

d) $\dfrac{\log 6 - \log 3}{4 \log 5}$

e) $\dfrac{\log 5}{\log 3} - 4$

Solution: b

11. to solve applications involving exponential or logarithmic equations.

Resource:

Foerster, Paul A., Algebra and Trigonometry: Functions and Applications (Menlo Park, CA: Addison-Wesley Publishing Co., 1980).

Examples could include compound interest, the law of natural growth, radioactive decay, carbon dating, cell reproduction, Richter scale, decibels, pH, slide-rule tuning on radio dial, and absorption of x-rays.

Examples:

1) In chemistry, pH is defined as the negative of the common logarithm of the hydrogen ion concentration in moles per litre. Find the pH for a solution if the hydrogen ion concentration is about 1.6×10^{-8} moles per litre.

Solution:

$\text{pH} = -\log (H^+)$ where (H^+) represents the hydrogen ion concentration.

$$\text{pH} = -\log (1.6 \times 10^{-8})$$

$$= -(\log 1.6 + \log 10^{-8})$$

$$= -(\log 1.6 + (-8))$$

$$= -\log 1.6 + 8$$

$$= -0.204 + 8 \text{ or } 7.8$$

∴ the pH for this solution is 7.8 (i.e., slightly basic)

2) The loudness of sound (L) in decibels is given by the formula $L = 10 \log I$ where I is the intensity of the sound waves. A sound which is just audible is given an intensity of 1 and, therefore, has a decibel reading of 0.

a) Calculate to the nearest decibel the loudness of the sound of a truck for which the intensity is 10^9.

Solution:

$L = 10 \log I$

$L = 10 \log (10^9)$

$= 10 \times 9$

$= 90$

Loudness is 90 decibels.

b) Calculate the intensity of a background noise if its loudness is measured as 35 decibels.

Solution:

$L = 10 \log I$

$35 = 10 \log I$

$3.5 = \log I$

$I = \text{Inv. } \log 3.5$

$I = 3162$

The intensity of the background noise is 3162 times the strength of a sound which is just audible.

C.V. SEQUENCES AND SERIES (1 MODULE)

This unit is intended to develop the ideas found in sequences and series and as an application to introduce amortization and mortgage calculations.

OBJECTIVES | NOTES TO TEACHERS

The student should be able:

1. to write a given sequence of numbers as a function rule;

Sequences may be finite or infinite. Conventionally, the term "sequence" without a modifier means an infinite sequence.

For 200, 220, 240,...,380 any term, $t_k = 200 + (k-1)(20) = 180+20k$, where

k is the position of the term in the sequence.

Examples:

2. to represent a finite sequence in three alternate forms: t_k or $f(k)$ or $f{:}k\rightarrow$;

1) for 1, 4, 7, ..., 28

$$t_k = 3k-2$$

2) for 3, 9, 19, ..., 201

$$t_k = 2k^2+1$$

3) for 8, 11, 14, ..., 35

$$f(k) = 5+3k$$

4) for 2, 5, 8, ..., 29

$$f{:}k\rightarrow 3k-1$$

OBJECTIVES	NOTES TO TEACHERS
3. to represent a finite sequence via a recursion formula;	Example: for 2, 5, 8, ..., 29 $t_1 = 2$ and $t_{k+1} = t_k + 3$
4. to use a recursion formula and to generate the first "m" terms;	Examples: 1) Find the first 4 terms if the sequence is generated by: $t_1 = 4; \; t_k = t_{k-1} + 2 \qquad (k>2)$ 2) Find the first 5 terms if the sequence is generated by: $t_1 = -3; \; t_{k+1} = t_k + (3k+2)$ 3) Find the first 7 terms in the Fibonacci sequence: $t_1 = t_2 = 1$ and $t_k = t_{k-1} + t_{k-2} \qquad (k>2)$
5. to identify arithmetic and geometric sequences (A.P.'s and G.P.'s) and understand terminology associated with them;	Examples: 1) Identify: 3, 8, 13, ..., 53 An A.P. with $t_1 = 3$ and common difference = 5 $t_k = 3 + (k-1)(5) = 5k-2$ 2) Identify: 3, 9, 27, ..., 729 A G.P. with $t_1 = 3$, common ratio = 3 and $t_k = 3(3)^{k-1} = 3^k$

OBJECTIVES	NOTES TO TEACHERS
6. to find the fourth unknown given the other three in an A.P. (i.e., t_1, t_n, n,d);	Examples: 1) How many terms are there in -19, -14, ..., 126? 2) In an A.P. $t_7 = 32$ $t_{10} = 22$, find t_1, d and t_k.
7. to find the fourth unknown given the other three in a G.P. (i.e., t_1, t_n, n,r);	Example: 1) How many terms are there in 3, 6, 12, ..., 768? 2) In a G.P. $t_5 = 12$ $t_9 = 108$, find t_1, r and t_k.
8. to insert "k" arithmetic means in any A.P.;	Example: Insert 5 A.M.'s in the sequence: 5,__,__,__,__,__,29 (i.e., between 5 and 29).
9. to insert "m" geometric means in any G.P.;	Example: Insert 3 G.M.'s between 5 and 80; i.e., for what values is 5, __, __, __, 80 a G.P.?
10. to use 'sigma notation' in representing a sum;	Examples: 1) $S_8 = \sum_{k=1}^{8} k = 1+2+\ldots+8 = 36$ 2) $S_4 = \sum_{k=1}^{4} (3k+2) = 5+8+11+14 = 38$ 3) $\sum_{k=1}^{6} kx^k = x+2x^2+3x^3+4x^4+5x^5+6x^6$

OBJECTIVES	NOTES TO TEACHERS
	Note: A finite arithmetic series may be defined as the sum of all the terms in a finite A.P.
11. to derive and to apply the formula used for finite arithmetic series: $S_n = \frac{n}{2}(t_1 + t_n)$ or $S_n = \frac{n}{2}[2t_1 + (n-1)d]$;	$t_n = t_1 + (n-1)d$ Example: Find the following sums: 1) S for 2, 7, 12, ... 2) S_{40}^{200} for 4, 7, 10, ... 3) S_{30} for 6, 4, 2, ...
12. to derive and to use the formula for finite geometric series ($r \neq 1$): $S_n = \frac{t_1(1-r^n)}{1-r}$ or multiplying top, bottom by -1: $S_n = \frac{t_1(r^n - 1)}{r-1}$,	Note: A finite geometric series may be defined as the sum of all the terms in a finite G.P. Examples: 1. Find S_8 for 1, 3, 9, ... 2. Find the sum of 1+3+9+...+2187.
13. to verify intuitively and to use the formula for the sum of an infinite geometric series: $S = \frac{t_1}{1-r}$ if $\lvert r \rvert < 1$;	Teachers may want to examine r^n by studying r values raised to successively larger exponents: $(\frac{2}{3})^2, (\frac{2}{3})^5, (\frac{2}{3})^8, \ldots$ demonstrating intuitively that $\lim_{n \to \infty} r^n = 0$ if $\lvert r \rvert < 1$.

Example:

Find the sum of the infinite geometric series $1 + \frac{1}{2} + \frac{1}{4} + \ldots$

14. to solve problems involving A.P.'s, G.P.'s and infinite G.P.'s.

Examples:

1) Doubling your money each day for a month:

1¢ on the first day
2¢ on the second day
4¢ on the third day

At the end of 30 days, how much money has accumulated?

2) Ancestors for n generations back:

first generation back - parents: 2
second generation back - grandparents: 4
third generation back - great-grandparents: 8

How many ancestors do you have 15 generations back and how many in total for the entire 15 generations?

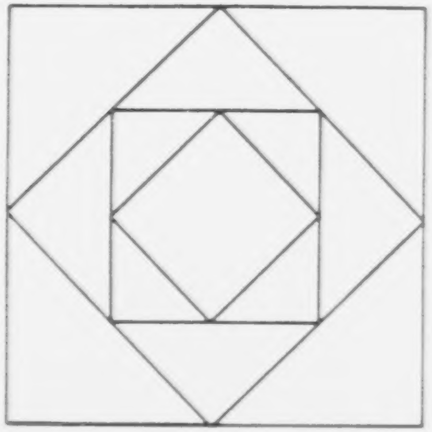
3) Sweepstakes problem:

Which is better: $40 000 now or $200 compounded monthly at 15% for the rest of your life? (Identify assumptions.)

4) Nesting squares at the midpoints of the former square:

Find the perimeter and area for the 8th nested square.

Find the sum of the perimeters and areas for the first 8 squares.



5) Solve problems involving an infinite G.P. such as:

A ball is dropped from 90 metres and rebounds 60 metres. It then drops 60 metres and rebounds 40 metres. It continues to rebound 2/3 of its former height. How many metres does it travel before coming to rest?

6) Find 3 numbers in an A.P. whose sum = 27 and whose product = 405.

7) A car costs $15 600. It depreciates 20% of its value per year. What is its value after 4 years?

Op. I. THE BINOMIAL THEOREM (½ MODULE)

OBJECTIVES	NOTES TO TEACHERS
The student should be able:	
1. to simplify factorial expressions;	$n! = n(n-1)(n-2)\ldots(3)(2)(1)$ $n! = n(n-1)!$ Define $0! = 1$ $\frac{9!}{5!} = (9)(8)(7)(6)$ $\frac{(n+2)!}{(n-1)!} = (n+2)(n+1)(n)$
2. to define $C(n,r) = \frac{n!}{r!(n-r)!}$;	The intent is to define $C(n,r)$ by means of a mathematical formula rather than a combinations approach. Other notations, such as ${}_nC_r$ or $\binom{n}{r}$, may be used. Compute $C(7,2)$ and $C(7,5)$. Both are equal to 21. What about $C(n,r)$ and $C(n,n-r)$?
3. to expand $(a+b)^n$ where n is a whole number, using Pascal's triangle to determine the coefficients;	Review as necessary. Pascal's Triangle $\begin{array}{ccccccccc l} &&&&1&&&& & \text{....coefficients for } (a+b)^0 \\ &&&1&&1&&& & \text{....coefficients for } (a+b)^1 \\ &&1&&2&&1&& & \text{....coefficients for } (a+b)^2 \\ &1&&3&&3&&1& & \text{....coefficients for } (a+b)^3 \\ 1&&4&&6&&4&&1 & \text{....coefficients for } (a+b)^4 \end{array}$

Examples:

1) $(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

2) $(x^2-3y^3)^3 = x^6 - 9x^4y^3 + 27x^2y^6 - 27y^9$

Students should realize that in the binomial expansion, $(a+b)^n$:
1) there are $(n+1)$ terms and for each term the sum of the exponents of a and b is n;
2) powers of "a" start at a^n and the exponent decreases by 1 for each term while powers of "b" start at b^0 and increase by 1 each term;
3) the numerical coefficients form a palindrome.

4. to determine the relationship between the C(n,r) values and the numbers in Pascal's triangle;

Students should conclude that the numbers in Pascal's triangle are equivalent to the following:

$$\begin{array}{ccccccc} & & & C(0,0) & & & \\ & & C(1,0) & & C(1,1) & & \\ & C(2,0) & & C(2,1) & & C(2,2) & \\ C(3,0) & & C(3,1) & & C(3,2) & & C(3,3) \\ & & & \cdots\cdots & & & \end{array}$$

5. to find a given term in any $(a+b)^n$ expansion;

Example:

Find the 9th term in the expansion of $(x^2-y)^{12}$.

Solution:

Answer is $C(12,8)\ (x^2)^4(-y)^8 = 495x^8y^8$. It is important for students to realize that the 9th term has a coefficient of C(12,8) and that the r'th term of $(a+b)^n$ has a coefficient of C(n, r-1).

6. to find the term containing a^k or b^k for any $(a+b)^n$ expansion.

Example:

Find the term containing x^6 in $(x - \frac{a}{x^2})^{15}$.

The solution could involve establishing a pattern of exponents in the simplified expansion or formally as follows:

Answer is $C(15,r)x^{15-r}(- \frac{a}{x^2})^r$

Combining powers of x: $x^{15-3r} = x^6$

So, $15-3r = 6$ or $r = 3$.

This is the 4th term or $C(15,3)x^{12}(- \frac{a}{x})^3$

$= -455x^6a^3$.

ENRICHMENT:

Teachers may wish to introduce the binomial expansion $(1+x)^n$ where $-1<x<1$ and n is a rational number.

Example:

Write the first four terms in the expansion of $(1+x)^{-\frac{1}{2}}$.

Op. II. COMPOUND INTEREST, ANNUITIES, AMORTIZATION AND MORTGAGES (1 MODULE)

Refer for the material found under "Consumer Mathematics" in the Math 100 and Math 200 outlines. The main objective here is to have the students apply the formulae intelligently and with precision. They should not be made to memorize these formulae. Care should be taken in developing the formulae to show the logical result from sequences and series. Teachers may wish to implement the use of computers in this topic.

Resources: Dottori, D. et al., FMT Senior (Toronto, ON: McGraw-Hill, Ryerson, 1979).
Carli, E.G., et al., Mathematics For A Modern World, Book 4 (Toronto, ON: 1975).

OBJECTIVES	NOTES TO TEACHERS
The student should be able:	
1. to use and to apply the compound interest formula;	Teachers are encouraged to use current interest rates. Review from Math 100, 200 outlines. The amount of money an investment P grows to after n periods of interest accumulation is given by: $$A = P(1+i)^n$$ This formula can be solved for each letter: 1) To solve for P: $P = A(1+i)^{-n}$ This is called the 'present value' of an investment. 2) To solve for i: $\frac{A}{P} = (1+i)^n$ $\sqrt[n]{\frac{A}{P}} = 1+i$ and so: $\sqrt[n]{\frac{A}{P}} - 1 = i$ 3) To solve for n, use 'logs': $\log A = \log P + n \log(1+i)$ $\frac{\log A - \log P}{\log(1+i)} = n$

OBJECTIVES NOTES TO TEACHERS

2. to use and to apply the annuity formulae;

Review from Math 100, 200 outlines.

The annuity formulae are:

1) $S = R\left[\frac{(1+i)^n - 1}{i}\right]$ where R is the periodic payment (rent) and S the accumulated amount of those payments.

2) $R = S\left[\frac{i}{(1+i)^n - 1}\right]$ when the periodic payment (rent) is to be found given the amount that must be accumulated after n payments.

3. to apply the concept of "present value" to problem situations;

This section will introduce the methods by which the payment for a mortgage or bank loan is determined.

Present Value of an Annuity

We have n payments of \$R each, made annually. The first payment is due in one year and will pay off a given amount of borrowed money, such as a bank loan or house mortgage.

To find the total present value of all the payments:

Present value of:

1st payment $\frac{R}{1+i}$

2nd payment $\frac{R}{(1+i)^2}$

3rd payment $\frac{R}{(1+i)^3}$

. . .

nth payment $\frac{R}{(1+i)^n}$

We need the sum: $\frac{R}{(1+i)} + \frac{R}{(1+i)^2} + \cdot \cdot \cdot + \frac{R}{(1+i)^n}$

This is a geometric series with the common ratio, $\frac{1}{(1+i)}$.

Since: $S_n = \frac{t_1(1-r^n)}{1-r}$

We have: Present Value of Annuity =

$$\frac{R}{(1+i)} \cdot \frac{1 - \frac{1}{(1+i)^n}}{1 - \frac{1}{1+i}}$$

$$= \frac{R}{(1+i)} \cdot \frac{1-(1+i)^{-n}}{\frac{1+i-1}{1+i}}$$

$$= R \cdot \frac{1-(1+i)^{-n}}{i}$$ after cancelling (1+i) in the denominator

Tables for $\frac{1-(1+i)^{-n}}{i}$ are available.

Examples:

1) An assembly line job at a company costs $8500 a year for labour. The same job can be performed by automated equipment for $44 625 payable now. In how many years will the automated equipment pay for itself if money can be invested at 16%, computed annually?

Solution:

In how many years will the $8 500 accrue to $44 625, using the rate of 0.16?

$$\text{Present Value of Annuity} = R \cdot \frac{1-(1+i)^{-n}}{i}$$

$$44\,625 = 8500 \cdot \frac{1-(1.16)^{-n}}{0.16}$$

$$.16\frac{(44\,625)}{8500} = 1-(1.16)^{-n}$$

$$(1.16)^{-n} = 1 - 0.16\frac{(44\,625)}{8500}$$

$$(1.16)^{-n} = 1 - 0.84$$

$$(1.16)^{-n} = 0.16$$

$$-n \log(1.16) = \log(0.16)$$

$$n = -\frac{\log(0.16)}{\log(1.16)} = -\frac{(-0.7959)}{0.0645}$$

$$= 12.33 \text{ years}$$

2) A condominium must replace its furnace element every 9 years. It costs $16 000. How much should be set aside each year, starting now, if funds can be invested at 18%, compounded annually?

Solution:

$$R = S\left[\frac{i}{(1+i)^n - 1}\right]$$

$$R = 16\,000\left[\frac{0.18}{(1.18)^9 - 1}\right]$$

$$= 16\,000\left[\frac{0.18}{3.435\,454}\right] = 16\,000\,(0.052\,394\,8) = \$838.32$$

3) A house is purchased with a $50 000 mortgage at 15%, compounded annually, to be paid off in annual payments in 12 years. What is the value of each payment?

Solution:

Here, 50 000 is the present value of the sequence of equal payments.

$$50\ 000 = R \cdot \frac{1 - (1+i)^{-n}}{i}$$

$$\text{So, } 50\ 000 = R \cdot \frac{1 - (1.15)^{-12}}{0.15}$$

$$50\ 000 = R \cdot \frac{1 - .186\ 907}{0.15}$$

$$50\ 000 = R \cdot \frac{.813\ 092\ 8}{0.15}$$

$$50\ 000 = R \cdot 5.420\ 619$$

$$\$9224.04 = R$$

4) A legacy of $150 000 is left to a widow. She wishes to receive equal payments of cash every 6 months for the next 8 years. What is the value of each payment if money can be invested at 14%, compounded annually?

Solution:

936

There are 16 payments, each at rate 0.07, for this annuity.

$$\text{So, } 150\ 000 = R \cdot \frac{1 - (1+i)^{-n}}{i}$$

$$150\ 000 = R \cdot \frac{1 - (1.07)^{-16}}{0.07}$$

$$150\ 000 = R \cdot 9.4466486$$

$$\$15\ 878.65 = R$$

4. to use the concept of "amortization" in problems such as loans and mortgages.

Example of Amortization:

Determine the amount of each payment to amortize a loan of $10 000 at 16½% per annum, compounded semi-annually, in monthly payments for 3 years.

Solution:

Find the effective monthly rate: $(1+i)^6 = 1.0825$

$$i = \sqrt[6]{1.0825} - 1 = 0.013\ 299\ 863$$

We will use this "i" value in our calculations. Each payment will accrue a different number of times and the present value of each payment is:

$$\frac{R}{(1.01)} + \frac{R}{(1.013..)^2} + \ldots + \frac{R}{(1.013299863)^{36}}$$

Writing this sum backwards and using $S_n = \dfrac{a(r^n-1)}{r-1}$, we have:

$$S_n = \frac{R}{(1+i)^n}\left[\frac{(1+i)^n - 1}{(1+i-1)}\right] = \frac{R\left[1 - \frac{1}{(1+i)^n}\right]}{i}$$

Since S_n = amount of the amortization we have:

$$A = R\,\frac{\left[1 - \frac{1}{(1+i)^n}\right]}{i} \quad \text{or} \quad R\left[\frac{1-(1+i)^{-n}}{i}\right]$$

Rearranging: $R = \dfrac{Ai}{1 - \dfrac{1}{(1+i)^n}}$ is the required monthly payment.

Substituting given values: $$R = \frac{10\ 000(0.013299863)}{1 - \dfrac{1}{(1.013299863)^{36}}} = \frac{132.99863}{1 - 0.621487758}$$

$$= \frac{132.99863}{0.378512242} = \$351.37$$

Example of a Mortgage

A home is purchased with a \$40 000 mortgage at 18%, amortized for 25 years in monthly payments. Find each payment if money is compounded semi-annually.

Solution: Find the effective rate of "i": $(1+i)^6 = 1.09$

$$R = \frac{40\ 000(0.014466592)}{\dfrac{1}{1 - (1.04466592)^{300}}}$$

$$i = \sqrt[6]{1.09} - 1 = .014466592$$

$$= \frac{0.57866368}{1 - 0.013448539} = \frac{578.66368}{0.98655146} = \$586.55 \text{ per month.}$$

Op.III. GRAPHS OF SINUSOIDAL FUNCTIONS (½ MODULE)

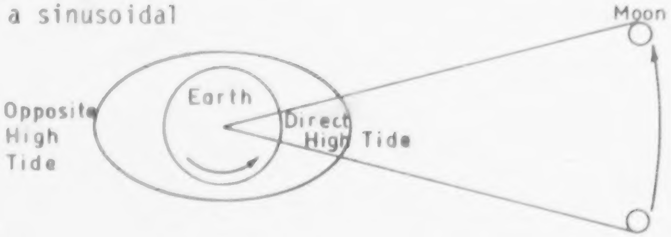
Students have graphed $f(x) = \sin x$ and $g(x) = \cos x$. Problems involving sinusoidal variations usually require some transformations of these functions.

Introductory Problem

Tides in a coastal area of Canada caused the water level to rise 1.5 m above mean sea level and to drop 1.5 m below mean sea level. The tide completes one cycle every 12 hours. If the height of the water with respect to mean sea level can be represented by a sinusoidal function:

a) Draw a graph for a 24-hour period.

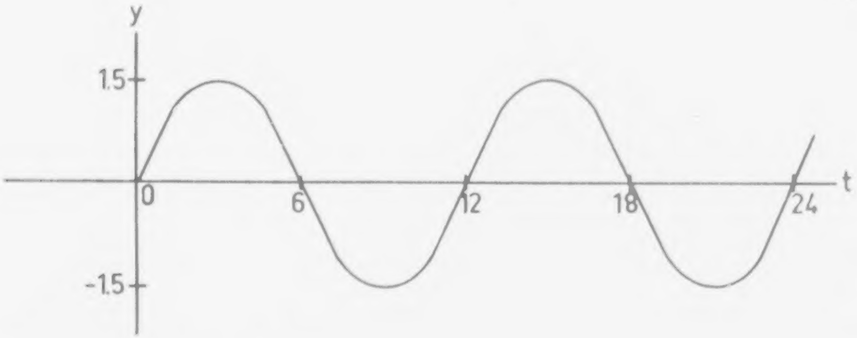
b) Find the equation of the graph.



Solution:

a) We will take mean sea level as our reference point and assume that the water is at sea level and rising at $t = 0$.

Note: The amount of rise or fall from mean sea level is called the amplitude.



b) Relating the graph to the general sine function $y = a \sin b(t + c) + d$,

we have amplitude: 1.5

period: $12 = \frac{2\pi}{b}$, $b = \frac{\pi}{6}$

phase shift: none

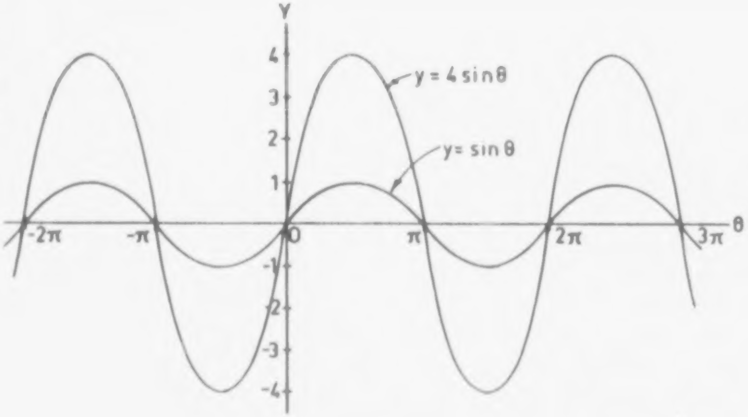
vertical translation: none

∴ the equation is $y = 1.5 \sin \frac{\pi}{6} t$

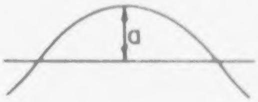
OBJECTIVES | NOTES TO TEACHERS

The student should be able:

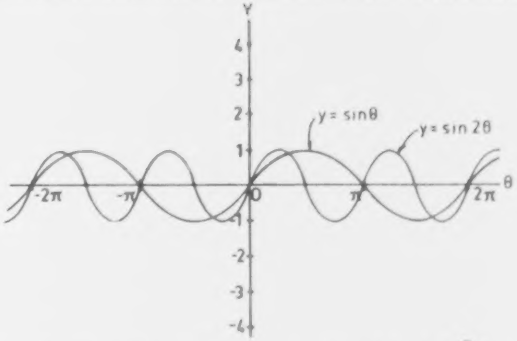
1. to sketch the graph of a sinusoidal function of the form $y = a \sin \Theta$;



The amplitude of the function $y = a \sin \Theta$ is $|a|$.

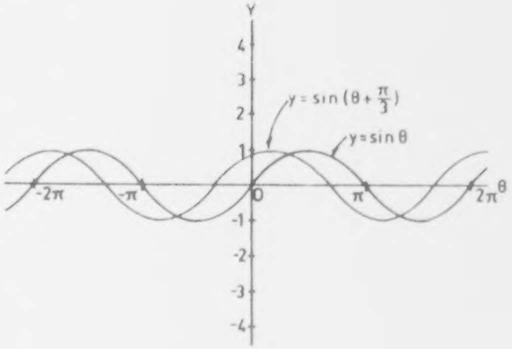


2. to sketch the graph of a sinusoidal function of the form $y = \sin b\theta$;



The period of the function $y = \sin b\theta$ is $\frac{2\pi}{b}$, $b > 0$

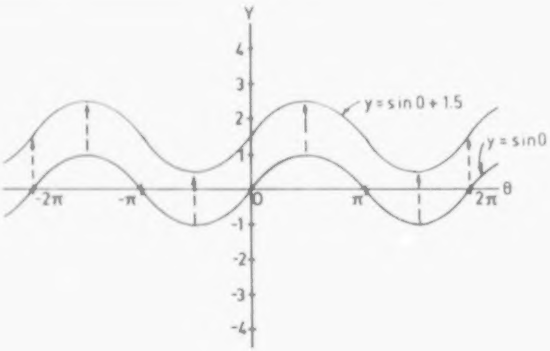
3. to sketch the graph of a sinusoidal function of the form $y = \sin(\theta + c)$;



The phase angle for $y = \sin(\theta + c)$ is the constant angle c. The phase shift is $|c|$ units to the right if c is negative and $|c|$ units to the left if c is positive.

OBJECTIVES	NOTES TO TEACHERS

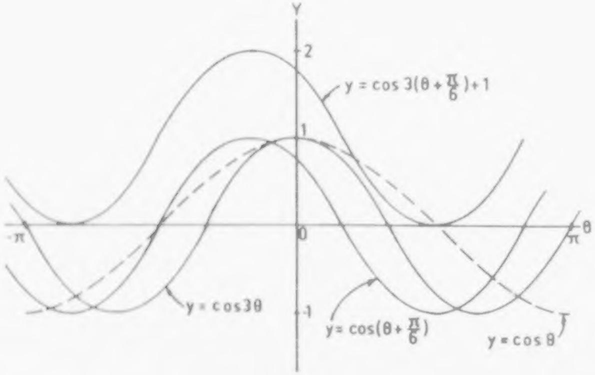
4. to sketch the graph of a sinusoidal function of the form $y = \sin\theta + d$;



The vertical translation for $y = \sin\theta + d$ is d units upwards if d is positive and d downwards if d is negative.

5. to sketch the graph of a sinusoidal function of the form $y = a \sin b(\theta + c) + d$;

Example: Sketch the graph of $y = \cos 3\left(\theta + \frac{\pi}{6}\right) + 1$ where $-\pi \leq \theta \leq \pi$

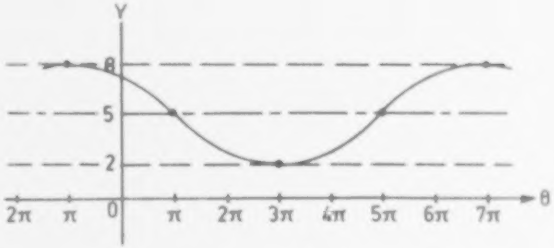


amplitude:	1
period:	$\frac{2\pi}{3}$
phase shift:	$\frac{\pi}{6}$ left
vertical translation:	1 up

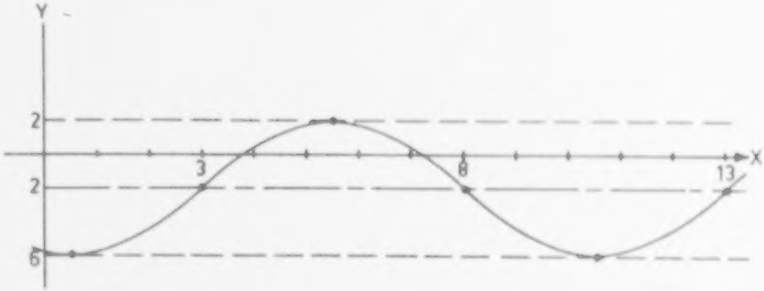
6. to determine the equation in the form $y = a \sin b(\theta+c) + d$ for the function represented by a sinusoidal curve;

Example:

Write the equation for each curve below given the equation in the form $y = a \sin b\,(\theta+c)$.



Solution: $y = 5 + 3 \cos \frac{1}{4}(\theta + \pi)$



Solution: $y = -2 + 4 \sin \frac{\pi}{5}\,(x-3)$

OBJECTIVES

NOTES TO TEACHERS

7. to write a function for a problem involving sinusoidal variation;

Example:

A ferris wheel with a radius of 5 m makes one complete revolution every 20 seconds. The bottom of the wheel is 1.5 m above the ground level.

Find an equation which shows how a person's height above ground varies with time as the wheel is turning. (Assume the timing begins when the person is at the bottom.

Solution:

Relate to the general sine function $y = a \sin b(t+c) + d$

where amplitude: $a = 5$

period: $20 = \frac{2\pi}{b}$ $\quad \therefore b = \frac{\pi}{10}$

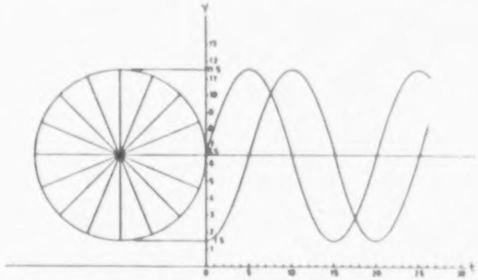
phase shift: 5 right, $\therefore c = -5$

vertical translation: $d = 1.5 + 5 = 6.5$

$\therefore$ an equation of the graph is $y = 5 \sin \frac{\pi}{10}(t - 5) + 6.5$

8. to sketch the graph of the function above in #7.

Draw the graph to show how a person's height above the ground varies with time. (Assume the timing begins when the person is at the bottom.)



OP.IV. CIRCULAR FUNCTIONS OF TWO ANGLES (½ MODULE)

OBJECTIVES	NOTES TO TEACHERS
The student should be able:	
1. to prove the formula $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$;	
2. to solve problems by means of the formula in Objective #1;	Examples: 1. Find cos 15° without the use of a calculator or tables. (Hint: $15° = 45° - 30°$) 2. Prove that $\cos(\frac{3\pi}{2} - \theta) = -\sin\theta$. 3. Simplify $\cos 6x \cos 5x + \sin 6x \sin 5x$. (Answer: $\cos x$)
3. to derive the following formulae from the formula for $\cos(\alpha - \beta)$: $\cos(\frac{\pi}{2} - \theta) = \sin\theta$, $\sin(\frac{\pi}{2} - \theta) = \cos\theta$, $\tan(\frac{\pi}{2} - \theta) = \cot\theta$, $\cot(\frac{\pi}{2} - \theta) = \tan\theta$,	
$\cos(-\theta) = \cos\theta$ $\sin(-\theta) = -\sin\theta$;	Substitute $\alpha = 0$ and $\beta = \theta$ into the formula in Objective #1 and simplify.

OBJECTIVES

4. to solve problems by means of the formulae in Objective #3;

5. to derive the following addition and subtraction formulae from the formula for cos $(\alpha - \beta)$:

$\cos(\alpha + \beta)$
$= \cos\alpha\cos\beta - \sin\alpha\sin\beta$,

$\sin(\alpha + \beta)$
$= \sin\alpha\cos\beta + \cos\alpha\sin\beta$,

$\sin(\alpha - \beta)$
$= \sin\alpha\cos\beta - \cos\alpha\sin\beta$,

NOTES TO TEACHERS

Examples:

1. What is the value of $\cos(-45°)$?

2. Prove that $\tan(-\theta) = -\tan\theta$.

Solution: $$\tan(-\theta) = \frac{\sin(-\theta)}{\cos(-\theta)} = \frac{-\sin\theta}{\cos\theta} = -\left(\frac{\sin\theta}{\cos\theta}\right) = -\tan\theta.$$

3. Given that $\sin 36.9° = 0.600$, find the values of each of the following without using tables or calculators: $\cos 53.1°$, $\sin 53.1°$, $\tan 53.1°$.

Solution: $$\cos 53.1° = \sin(90° - 53.1°) = \sin 36.9° = 0.600$$

$$\sin 53.1° = \sqrt{1 - \cos^2 53.1°} = \sqrt{1 - 0.360} = \sqrt{.640} = 0.800$$

$$\tan 53.1° = \frac{\sin 53.1°}{\cos 53.1°} = \frac{0.800}{0.600} = 1.33$$

$$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta};$$

$$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta};$$

6. to solve problems by means of formulae in Objective #5.

Examples:

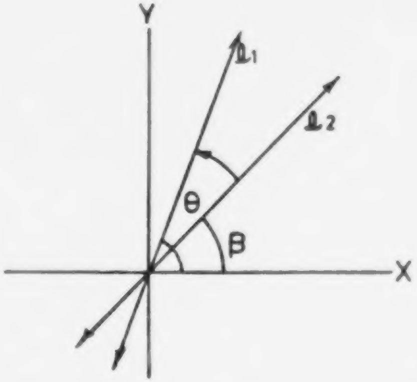
1. α and β both lie between 0° and 90°. If $\sin\alpha = \frac{4}{5}$ and $\sin\beta = \frac{5}{13}$, find the value of $\sin(\alpha + \beta)$ and $\tan(\alpha - \beta)$.
2. Without using tables or calculators, find the value of $\tan\frac{7\pi}{12}$.
3. Prove that $\tan(\pi - \theta) = -\tan\theta$.
4. Show that:

$$\frac{\sin(\theta + h) - \sin\theta}{h} = \sin\theta\left(\frac{\cos h - 1}{h}\right) + \cos\theta\left(\frac{\sin h}{h}\right).$$

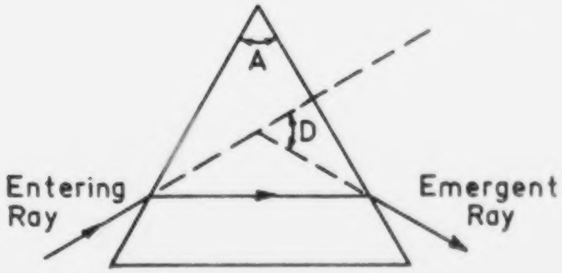
5. A line through the origin has slope 3. Another line through the origin has slope ½. Show that one of the angles between these two lines is 45°.

6. θ is the angle that the line ℓ_1 makes with the line ℓ_2 as shown in the diagram. If the slope of ℓ_1 is m_1 and the slope of ℓ_2 is m_2, prove that:

$$\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2}$$



7. The index of refraction n of the glass in a prism is given by the formula $n = \dfrac{\sin \frac{1}{2}(A + D)}{\sin \frac{1}{2} A}$ where A is the angle of the prism and D is the minimal angle of deviation. The index of refraction of the glass to be used in a prism is 1.500. What size should we make the angle of the prism in order to obtain a minimal angle of deviation of 30.00°?



Solution:

In the formula $n = \dfrac{\sin \frac{1}{2}(A + D)}{\sin \frac{1}{2} A}$, substitute the known values $n = 1.500$, and $D = 30.00°$, obtaining $1.500 = \dfrac{\sin (\frac{1}{2}A + 15°)}{\sin \frac{1}{2}A}$.

This can be written $1.500 = \dfrac{\sin \frac{1}{2}A \cos 15° + \cos \frac{1}{2}A \sin 15°}{\sin \frac{1}{2}A}$.

That is, $1.500 = \cos 15° + \cot \frac{1}{2}A \sin 15°$.

Thus, $\cos \frac{1}{2}A = \dfrac{1.500 - \cos 15°}{\sin 15°} = 2.0635$ (calculator).

$\therefore \frac{1}{2}A = 25.86°$ $\therefore A = 51.72°$

8. Prove that $\sin(\alpha + \beta) + \sin(\alpha - \beta) = 2 \sin \alpha \cos \beta$.

ENRICHMENT:

Have students derive the following double-angle formulae from the formulae in Objective #5.

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha$$

$$\begin{aligned}\cos 2\alpha &= \cos^2\alpha - \sin^2\alpha \\ &= 1 - 2 \sin^2\alpha \\ &= 2 \cos^2\alpha - 1\end{aligned}$$

$$\tan 2\alpha = \frac{2 \tan \alpha}{1 - \tan^2\alpha}$$

Examples:

1. α lies between 0° and 90° and $\sin \alpha = \frac{3}{5}$
   Find $\sin 2\alpha$, $\cos 2\alpha$, and $\tan 2\alpha$.

2. Prove that $\cos \frac{\theta}{2} = \sqrt{\frac{1 + \cos \theta}{2}}$

3. Prove that $\cos 165° = -\frac{1}{2}\sqrt{2 + \sqrt{3}}$

   One possible solution $165° = \frac{330°}{2}$

   Thus $\cos 165° = \cos \frac{330°}{2}$

   $$= \sqrt{\frac{1 + \cos 330°}{2}} = \frac{\sqrt{1 + \frac{\sqrt{3}}{2}}}{2}$$

   $$= \frac{1}{2}\sqrt{2 + \sqrt{3}}$$

   But 165° is in Quadrant II so $\cos 165° = -\frac{1}{2}\sqrt{2 + \sqrt{3}}$.

4. Prove that $\tan \theta = \frac{\sin 2\theta}{1 + \cos 2\theta}$

   Teachers may wish to use the double-angle formulae to solve trigonometric equations.

5. Solve the equation $\cos 2\theta = \cos \theta$ for all values of θ such that $0 \leq \theta < 2\pi$.

6. Solve the equation $4 \sin \theta \cos \theta = \sqrt{2}$ for all values of θ such that $0 \leq \theta < 2\pi$.

Op. V. PERMUTATIONS AND COMBINATIONS (1 MODULE)

OBJECTIVES	NOTES TO TEACHERS
The student should be able:	
1. to use the Fundamental Counting Principle to solve problems;	Examples: 1. How many ways can 4 books be arranged on a shelf? 2. In how many ways can m people be seated in a room with h chairs? a) when $m < h$ b) when $m = h$ c) when $m > h$ 3. In how many ways can 5 girls and 5 boys be arranged in a row if: a) a boy is at each end of the row? b) the boys and girls are arranged alternately?
2. to develop the formula: $P(n,r) = \frac{n!}{(n-r)!}$, $n > r$;	Although students should become familiar with this formula and be able to evaluate it for given values of n and r, the Fundamental Counting Principle is often more useful in the solution of permutation problems. 1. If $P(n,2) = 30$ find the value of n. Solution: $n = 6$ 2. How many different sets of mixed doubles for tennis can be formed from 10 ladies and 6 gentlemen? Solution: 127

OBJECTIVES	NOTES TO TEACHERS
3. to solve problems involving permutations of n things of which two or more are alike;	Example: How many different 5-digit numbers can be made by arranging the digits of 46 164?
4. to solve problems involving circular permutations;	Examples: 1. In how many ways can 8 people be seated at a round table? Solution: 7! or 5040 2) In how many ways can 10 persons be seated in a circle if: a) 2 given persons must be seated next to each other, b) these 2 persons must not be seated next to each other, c) there are 5 boys and 5 girls who must be seated alternately? Solution: a) 2 x 8! b) 7 x 8! c) 2 880
5. to develop the formula: $C(n,r) = \frac{n!}{(n-r)!r!}$;	Students should realize that $C(n,r) = C(n, n-r)$; i.e., $C(5,2) = C(5,3)$.
6. to solve combination problems using the formula in Objective 5.	Examples: 1. Luigi's Pizzeria offers green peppers, onions, mushrooms, pepperoni, bacon, shrimp and Italian sausage as toppings. How many different pizzas can be made? Answer: 128

2. If C(n,3) = 3, find n.

   Solution: n = 11

3. How many different sums of money can be made by taking 1 or more coins from a penny, a nickel, a dime, a quarter, a half dollar, a dollar and and a $5 gold piece?

   Solution: 127

4. How many different combinations and how many different permutations can be made with the letters of the word, "adaptable," taken 5 at a time?

   Solution: Combinations: 56 Permutations: 4 020.

5. How many arrangements can be formed from the letters of the word "formulate", taken: 1) 3 at a time 2) 5 at a time 3) all at a time?

   Solution: 1) 504 2) 15 120 3) 362 880

6. In how many ways can a baseball team of 9 players be chosen from 12 players of whom only 2 can catch and only 3 can pitch, if the pitchers and catchers cannot play any other position?

   Solution: 30 240.

Op. VI. CONIC SECTIONS IN STANDARD POSITION (1 MODULE)

OBJECTIVES

A. THE ELLIPSE

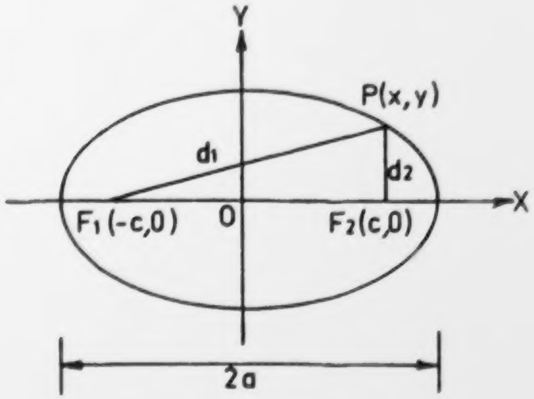
The student should be able:

1. to define an ellipse and associated terms such as: foci, principal axis, vertices, centre, major axis and minor axis;

NOTES TO TEACHERS

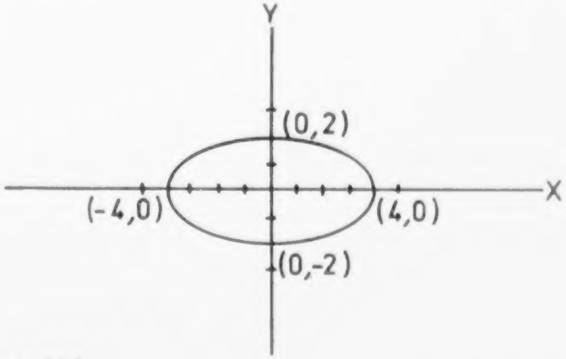
An "ellipse" is the locus of a point such that the sum of its distances to two fixed points (called the foci) remains constant.

Note: The distance from a point on the ellipse to a focus is called a "focal radius for the point."



$d_1 + d_2 = 2a$

Note: An ellipse is said to be in standard position if its principal axis is along the x-axis and its centre is at the origin.

OBJECTIVES	NOTES TO TEACHERS
2. to derive an equation of the ellipse with foci at (c, o) and (-c, o) and sum of focal radii 2a;	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is an equation of this ellipse, where $b^2 = a^2 - c^2$. Notice that when the ellipse is in standard position, the denominator of the x^2 term is greater than or equal to the denominator of y^2 term.
3. to sketch the graph of an ellipse having an equation of the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with $a>b$;	Example: Sketch the graph of $\frac{x^2}{16} + \frac{y^2}{4} = 1$ Solution: Compare with $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ x-intercept = ±4, y-intercept = ±2  Note: a and b are always taken as positive.

4. to solve problems involving ellipses having equations of the form

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

Examples:

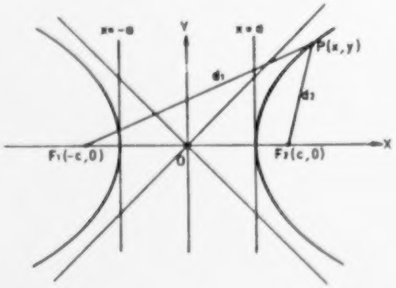
1. Find an equation for an ellipse with centre at the origin, foci at (3, 0) and (-3, 0), sum of focal radii = 10.
2. Find an equation of an ellipse with centre at the origin, principal axis along x-axis, major axis 4 and passing through the point(1, 2).
3. Find the foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{4} = 1$.
4. Find an equation of an ellipse with centre at the origin, one focus at (-5, 0) and one vertex at (13, 0).

B. THE HYPERBOLA

The student should be able:

1. to define an hyperbola and associated terms such as: foci, principal axis, vertices, centre, transverse axis, conjugate axis and asymptotes;

An "hyperbola" is the locus of a point such that the difference of its distances from two fixed points (called the foci) remains constant.



$d_1 - d_2 = 2a$

Note: An hyperbola is said to be in standard position if its principal axis is along the x-axis and its centre is at the origin.

OBJECTIVES	NOTES TO TEACHERS
2. to derive an equation of the hyperbola with foci at (c, 0) and (-c, 0) and difference of focal radii 2a;	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is an equation of this hyperbola, where $c^2 = a^2 + b^2$.
3. to sketch the graph of an hyperbola having an equation of the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$;	Examples: 1. Sketch the graph of $\frac{x^2}{16} - \frac{y^2}{9} = 1$. 2. Sketch the graph of $\frac{x^2}{4} - \frac{y^2}{9} = 1$.
4. to solve problems involving hyperbolas having equations of the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.	Examples: 1. Find an equation of an hyperbola with centre at the origin, foci at (5,0) and (-5, 0) and vertices at (4, 0) and (-4, 0). 2. Find the vertices, foci and asymptotes for the hyperbola $\frac{x^2}{4} - \frac{y^2}{9} = 1$. 3. Find an equation of an hyperbola which has $y = \frac{5}{4}x$ as an asymptote, centre at the origin and one focus at (10, 0). 4. Find an equation of an hyperbola with centre at the origin, one vertex at (3, 0) and passing through (4, 5).

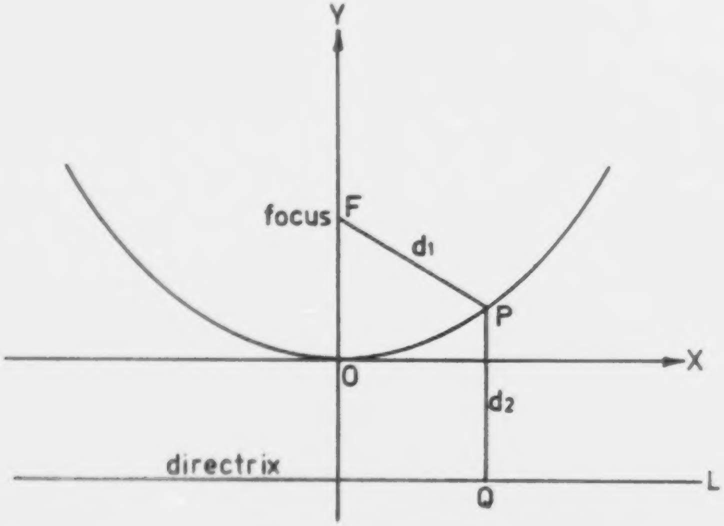
OBJECTIVES | NOTES TO TEACHERS

C. THE PARABOLA

The student should be able:

1. to define a parabola and associated terms such as: focus, directrix, principal axis and vertex;

A "parabola" is the locus of a point whose distance from a certain fixed point, called the focus, equals its distance from a certain fixed straight line called the directrix.



$d_1 = d_2$

Note: A parabola is said to be in standard position if its vertex is at the origin and principal axis along the x-axis.

OBJECTIVES	NOTES TO TEACHERS
2. to find an equation of a parabola given the focus and the directrix;	Examples: 1. Find an equation for the parabola whose directrix is the line $y = 2$ and whose focus is the point $(3, 5)$. (Note that this parabola is not in standard position.) 2. Find an equation for the parabola whose directrix is the line $3x + 4y - 5 = 0$ and whose focus is the point $(3, 5)$. (Note that this example is not in standard position.) 3. Find an equation for the parabola whose focus is at $(3, 0)$ and whose directrix is the line $x = -3$.
3. to derive an equation of the parabola whose vertex is at the origin and whose focus is at $(\frac{p}{2}, 0)$;	Use the definition of a parabola to show that $y^2 = 2px$ is an equation for this parabola.
4. to sketch the graph of a parabola having an equation of the form $y^2 = 2px$;	Examples: 1. Sketch the graph of $y^2 = 8x$, indicating the focus and directrix. 2. Sketch the graph of $y^2 = -6x$, indicating the focus and directrix.

OBJECTIVES	NOTES TO TEACHERS
5. to solve problems involving parabolas having equations of the form $y^2 = 2px$.	Examples: 1. Find the equation of the parabola with vertex at the origin whose directrix is $x = 3$. 2. Find the focus of the parabola having the equation $y^2 = -6x$. 3. Find the directrix of the parabola which passes through (2, 12), has the x-axis as its principal axis and its vertex at the origin.

Op. VII. TRANSFORMATIONS OF CONICS FROM STANDARD POSITION (½ MODULE)

OBJECTIVES	NOTES TO TEACHERS
The student should be able:	
1. to solve problems involving parabolas having equations of the form $x^2 = 2py$;	Examples: 1. Sketch the graph of the parabolas $x^2 = 6y$ and $x^2 = -8y$. 2. Find the focus and directrix of the parabola whose equation is $y = 4x^2$. 3. Find an equation for the parabola with vertex at the origin and focus at (0, -3).
2. to solve problems involving ellipses having equations of the form $\frac{y^2}{a^2} + \frac{x^2}{b^2} = 1$, where $a^2 > b^2$;	Examples: 1. Sketch the graph of the ellipse $\frac{y^2}{9} + \frac{x^2}{4} = 1$. 2. Find the foci of the ellipse $\frac{y^2}{4} + x^2 = 1$.

OBJECTIVES	NOTES TO TEACHERS
3. to solve problems involving hyperbolas having equations of the form: $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$;	Examples: 1. Sketch the graph of the hyperbola $\frac{y^2}{4} - \frac{x^2}{9} = 1$. 2. Find the vertices and foci of the hyperbola $\frac{y^2}{25} - \frac{x^2}{4} = 1$. 3. What is the equation of the hyperbola with centre at the origin, one vertex at (0, 3) and one focus at (0, 4)?
4. to solve problems involving parabolas having equations of the form $(y-k)^2 = 2p(x-h)$ or $(x-h)^2 = 2p(y-k)$;	Examples: 1. Sketch the graph of $(y-2)^2 = 6(x-3)$. 2. Sketch the graph of $y^2 - 6y - 2x + 5 = 0$. 3. Find an equation for the parabola whose vertex is at (2, 3) and whose focus is at (2, 7). 4. Find an equation for the parabola whose principal axis is parallel to the x-axis and which passes through (0, 0) and has its vertex at (1, 3).

OBJECTIVES	NOTES TO TEACHERS
5. to solve problems involving ellipses having equations of the form $\frac{(x-h)^2}{m^2} + \frac{(y-k)^2}{n^2} = 1$;	Examples: 1. Sketch the graph of $\frac{(x-2)^2}{9} + \frac{(y-3)^2}{16} = 1$. 2. Sketch the graph of the ellipse $4x^2+y^2-16x-2y+13 = 0$.
6. to solve problems involving hyperbolas having equations of the form $\frac{(x-h)^2}{m^2} - \frac{(y-k)^2}{n^2} = \pm 1$;	Examples: 1. Sketch the graph of $\frac{(x-2)^2}{4} - \frac{(y-3)^2}{9} = 1$. 2. Sketch the graph of the hyperbola $9x^2-4y^2+36x+8y+68 = 0$. Find the foci, vertices and asymptotes.
7. to classify an equation of the form $Ax^2+Cy^2+2Dx+2Ey+F = 0$ where A and C are not both zero.	Example: Determine whether $4x^2-9y^2-16x-18y-29 = 0$ represents an ellipse, a parabola or an hyperbola.

END

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